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Steady State Simulation of Ion Temperature Gradient Drift Instabilities*

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The effects of the velocity-space nonlinearity on the time evolution of the ion-temperature-gradeint (ITG) drift turbulence based on the adiabatic electron approximation have been studied using gyrokinetic particle simulation techniques. It is found that this often-neglected nonlinearity can provide another channel for the turbulence to reach its steady state at a faster rate. Moreover, in the steady state, a reduced level of ion thermal diffusivity and an enhanced level of zonal flow have been observed.

In this paper, the physics issues arising from the steady state simulations of ion temperature gradient (ITG) drift instability with adiabatic electrons are addressed. Specifically, we will report our recent investigations by including the important velocity-space nonlinearity in our global Gyrokinetic Toroidal Code (GTC) [1] based on the gyrokinetic particle simulation model [2]. So far, this term has largely been ignored in the turbulence simulation community, although it is responsible for energy conservation, and may also play a vital role for the entropy balance for the steady state transport in the form of collisionless dissipation [3]. Since GTC is a Particle-In-Cell code, the addition of this nonlinear term is rather trivial. Our results have shown that the velocity-space nonlinearity has negligible impact on the linear and early nonlinear stages of the simulation. However, it significantly enhances the level of zonal flow in the later stage of the nonlinear development and, in turn, greatly reduces the steady state thermal flux. The enhanced fluctuation of $\delta\phi(n=0, m=1)$ mode has also been also observed.

The equations of motion for pushing particles in terms of the gyrokinetic units of Ω_i, ρ_s for time and space, and $e\phi/T_e$ for the perturbed potential. [Note that the ion acoustic speed c_s is unity in these units]

$$\frac{d\mathbf{R}}{dt} = v_{\|}\hat{\mathbf{b}} + \mathbf{v}_d - \frac{\partial\bar{\phi}}{\partial\mathbf{R}} \times \hat{\mathbf{b}} \tag{1}$$

$$\frac{dv_{\|}}{dt} = -\hat{\mathbf{b}}^* \cdot \left(\frac{v_{\perp}^2}{2}\frac{\partial}{\partial\mathbf{R}} ln B + \frac{\partial\bar{\phi}}{\partial\mathbf{R}}\right), \tag{2}$$

$$\frac{dw}{dt} = -(1-w)\left(\kappa\frac{\partial\bar{\phi}}{\partial\mathbf{R}} \times \hat{\mathbf{b}} \cdot \hat{\mathbf{r}} + \frac{T_e}{T_i}(v_{\|}\hat{\mathbf{b}} + \mathbf{v}_d) \cdot \frac{\partial\bar{\phi}}{\partial\mathbf{R}}\right), \tag{3}$$

and

$$\mu_B \equiv \frac{v_\perp^2}{2B}(1 - v_\parallel \hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}} \times \hat{\mathbf{b}}) \approx const., \tag{4}$$

where

$$\hat{\mathbf{b}}^* = \hat{\mathbf{b}} + v_\parallel \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}},$$

$$\mathbf{v}_d = v_\parallel^2 \hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB,$$

where $\kappa = \kappa_n - (3/2 - v^2/2v_{ti}^2)\kappa_{Ti}$ is the background inhomogeneity with $\kappa_n \equiv 1/L_n$ and $\kappa_T \equiv 1/L_T$. The perturbed distribution is defined as

$$\delta f = \sum_{j=1}^{N} w_j \delta(\mathbf{R} - \mathbf{R}_{\alpha j})\delta(\mu - \mu_{\alpha j})\delta(v_\parallel - v_{\parallel \alpha j}),$$

where N is the total number of particle ions in the simulation, $F = F_0 + \delta f$, F_0 is the background Maxwellian with $\int F_0 d\mathbf{x} = 1$,

$$w \equiv \delta f / F \tag{5}$$

and $F \equiv \delta f(w_j = 1)$. The transformation between the gyrocenter coordinates $\mathbf{R}$ and the particle coordinates $\mathbf{x}$ are

$$\bar{\phi}(\mathbf{R}) = \langle \int \phi(\mathbf{x})\delta(\mathbf{x} - \mathbf{R} - \rho) d\mathbf{x} \rangle_\varphi,$$

where $\langle \cdots \rangle_\varphi$ is the average over the gyro-angle φ and ρ is the particle gyroradius. The gyrokinetic Poisson's equation can be written as

$$\tau[\phi(\mathbf{x}) - \tilde{\phi}(\mathbf{x})] + \phi(\mathbf{x}) = -\bar{n}_i(\mathbf{x}),$$

where

$$\tilde{\phi}(\mathbf{x}) \equiv < \int \bar{\phi}(\mathbf{R})\delta f_i(\mathbf{R}, \mu, v_\parallel)\delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} d\mu dv_\parallel >_\varphi,$$

$$\bar{n}_i(\mathbf{x}) = e\langle \int \delta f_i(\mathbf{R})\delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} dv_\parallel d\mu \rangle_\varphi,$$

and $\mu \equiv v_\perp^2/2$. Numerically, the transformation between $\mathbf{R}$ and $\mathbf{x}$ can be accomplished through a 4-point average process valid for $k_\perp \rho_i \leq 2$ [2].

We should remark here that the nonlinearity wich is the focus of the present paper is the last term on the right hand side of Eq. (2). This term has mostly been ignored in the microturbulence community. However, without this term, the energy conservation cannot be satisfied in the simulation [4]. It aslo plays a role of collisionless dissipation for the entropy balance as mentioned earlier [3]. From particle simulation point of view, the inclusion of this nonlinear

term is rather trivial. Since particle simulation turns a nonlinear partial differential equation, i.e., the Vlasov equation, in the Eulerian coordinates to a set of ordinary differential equations for the particles in the Lagrangian coordinates.

The simulation has been carried out using the GTC code [1]. This global toroidal code uses field-line-aligned magnetic coordinates for a plasma with circular cross section. Particle pushing and field solve are carried out in the configuration space. The relevant parameters are as follows: there are 64 toroidal grid with $a/\rho_i = 250$ on each poloidal plane. The code uses an unstructured grid of the size ρ_s, i.e., the ion thermal radius measured with the electron temperature. Thus, the shortest wavelength modes that can be resolved in the code is $k_\perp \rho_s \approx 1$. The other parameters are: $n_0 = 10$ (number of particles per cell) , $R/L_T = 6.9$, $R/a = 2.79$, $L_n/L_{Ti} = 3.13$, $\Omega_i \Delta t = 15$ and $T_e/T_i = 1$. The radial profile of the inhomogeneity is given by $(1/L)e^{-[(r-r_c)/r_w]^6}$, where L represents either the temperaure scale length L_{Ti} or the density scale length L_n with $r_c/a = 0.5$ and $r_w/a = 0.35$.

The simulation results for two runs, one without the parallel velocity space nonlinearity represented by red (or light) lines, and the other with the nonlinearity represented by blue (dark) lines are shown in Fig. 1. As we can see, there is no appreciable difference between the two cases in the linear stage of the development. However, the saturation amplitude of the spatially-averaged $e\phi/T_e$ at 1% for the case without this nonlinearity is about a factor of two higher than the case with the nonlinearity as indicated by Fig. 1(a). On the other hand, Fig. 1(b) shows that the zonal flow amplitude in term of v_{ExB}/c_s is about four times larger. Most strikingly is the evolution of the ion thermal diffusivity, given in Fig, 1(c), where the case with the velocity-space nonlinearity reaches its steady state value at much faster pace and at a level of 0.3 in GyroBohm units, which is a factor two smaller than the case without the term. Another interesting aspect the these simulation is the the time evolution of the particle weights defined in Eq. (5). As shown in Fig. 1(d), the weight gain between the two cases are quite different with $\sqrt{\langle w^2 \rangle} \approx 0.2$ at the end of the simulation for the case with the nonlinearity. The time derivatives of $\langle w^2 \rangle$ is related to the entropy production as first pointed out in Ref. [3]. With the presence of the velocity space nonlinearity, it can be written as

$$\frac{\partial}{\partial t} \sum_{j=1}^{N} (1 - \alpha/4) w_j^2 = \kappa_{Ti} \langle Q_{ir} \rangle,$$

where $\alpha \approx 1$ is related to the velocity space nonlinearity and κ_{Ti} denotes ion temperature inhomogeneity and

$$Q_{ir} \equiv \frac{1}{N} \sum_{j=1}^{N} w_j v_j^2 \mathbf{v}_{ExB} \cdot \hat{\mathbf{r}}$$

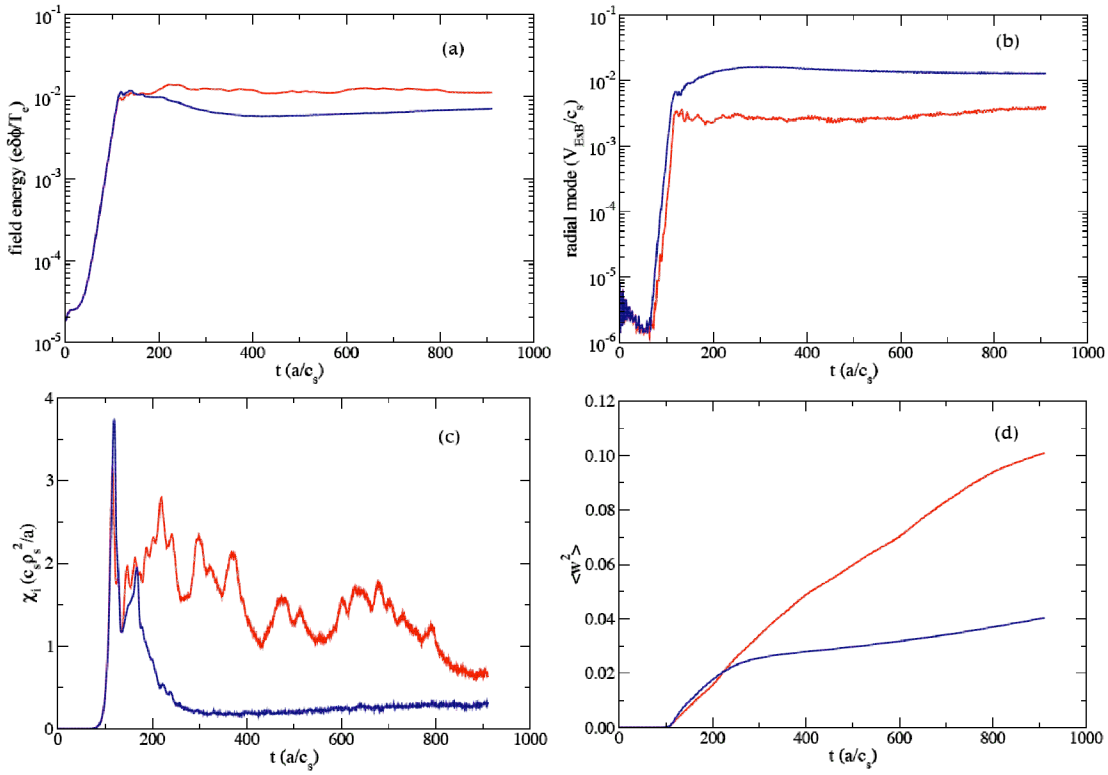

FIG. 1: *Time evolutions for the ITG turbulence for* $a/\rho_i = 250$ *with (blue or dark lines) and without (red or light lines) the velocity space nonlinearity for the (a) perturbed field energy, (d) radial mode amplitude, (c) ion thermal diffusivity, and (d) particle-weight-square.*

is the ion thermal flux. The noise level for the simulation plasma can be estimated by [5] $\phi \approx \sqrt{\langle w_j^2 \rangle / N}$, which is related to the high frequency wiggles depicted in Fig. 1(c). Similar trend relating to the velocity space nonlinearity has been observed in shaped plasma simulations as well as those with non-adiabatic electrons. Detailed will be reported elsewhere.

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