

# ON THE MOTION OF PLASMA PARTICLES IN THE FIELD OF A HIGH-POWER WAVE PROPAGATING NORMAL TO THE MAGNETIC FIELD

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## I. Introduction

In a previous work [1] we have proposed a mechanism for rotating a plasma, based on the normal injection of microwaves of very high power, in the electron cyclotron range. As the waves are resonantly absorbed the longitudinal gradient of the wave field gives rise to a ponderomotive (PM) force that may in turn produce a rotational instability when there is a poloidal asymmetry. Under certain circumstances, plasma rotation will be established, giving rise to possible improved confinement. The critical point in this model is that a radial PM force has to be able to produce a poloidal particle flow of a significant magnitude. In order to ascertain the appearance of a PM force under the conditions just mentioned, we have performed a study of the particle motion in the field of a resonant wave, propagating normally to a magnetic field, corresponding to a wave launched radially inwards in a tokamak.

With the latest developments of gyrotrons the powers are getting increasingly large, reaching a few megawatts, especially when more than one gyrotrons are combined. Actually, the power density delivered to the plasma must be quite large (of the order of  $5kW/cm^2$ ) in order to have a strong enough PM force to drive rotation. The high amplitude wave field accelerates electrons to very high speeds, nearing the speed of light. For this reason, we consider here the relativistic motion of particles. Previously, we have studied the average motion of the particles using a Lagrangian formalism [2] finding evidence for the action of the PM force, but the numerical study made there was non-relativistic. We will now present the relativistic simulations in three dimensions, showing interesting effects. Our treatment is most relevant for the case of intense lasers interacting with a plasma, and we discuss such a possibility as a means of rotating electrons. The effect of the plasma on single particles is also simulated using a simple model that includes the collective behavior through the plasma frequency [3].

## II. Particle orbital motion

The complete relativistic treatment of a particle trajectory can be obtained from the Hamilton-Jacobi equation for the action  $S(\mathbf{r}, t)$  as a function of the space-time coordinates. For a charged particle in the field of an electromagnetic wave with constant amplitude, the solution is well known [4]. It is also straightforward to find the solution when the particle moves in a constant, uniform magnetic field. However the problem gets complicated when the two fields ( $\mathbf{B}$  and wave) are taken together, and

even more if the wave amplitude is varying, as in our case of interest. We will consider a wave propagating along the  $x$ -axis and a background uniform magnetic field,  $B_0$ , pointing in the  $z$  direction, so that the fields are represented by the potential  $\mathbf{A}(\mathbf{r}, t) = a(x, t)\mathbf{A}_w(\eta) + B_0x\hat{y}$ , where the variable  $\eta = \omega t - \mathbf{k} \cdot \mathbf{r}$  describes a traveling wave with an envelope  $a(x, t)$ , that varies along the direction of propagation  $\mathbf{k} = k\hat{x} = (\omega/c)\hat{x}$  and vanishes before the wave is injected. This envelope contains the information about the absorption of the wave at the resonant surface. Taking the resonant surface at  $x = 0$  and the injection time at  $t = 0$ , the wave form will be  $a(x, t) = \Theta(t) \exp[-x/\Delta]$  if  $x < 0$ ;  $a(x) = \Theta(t)$ , if  $x > 0$ , with  $\Theta(t)$  being the Heaviside step function. The precise form of the wave potential is:

$$\mathbf{A}_w(\eta) = A_0[(1 - \delta^2)^{1/2} \sin \eta \hat{y} + \delta \cos \eta \hat{z}] \quad (1)$$

where  $\delta$  determines the wave polarization ( $\delta = 1$  for O-mode,  $\delta = 0$  for X-mode).

The relativistic equations of motion that we solve can be written as,

$$\frac{dv_x}{dt} = \frac{e}{m\gamma c} \left[ -(v_x E_y + v_z E_z) \frac{v_x}{c} + v_y B_z - v_z B_y \right] \quad (2)$$

$$\frac{dv_y}{dt} = \frac{e}{m\gamma} \left[ \left(1 - \frac{v_y^2}{c^2}\right) E_y - \frac{v_z v_y}{c^2} E_z + v_z B_x - v_x B_z \right] \quad (3)$$

$$\frac{dv_z}{dt} = \frac{e}{m\gamma} \left[ \left(1 - \frac{v_z^2}{c^2}\right) E_z - \frac{v_z v_y}{c^2} E_y + v_x B_y - v_y B_x \right] \quad (4)$$

$$\frac{d\gamma}{dt} = \frac{e}{mc^2} [v_y E_y + v_z E_z] \quad (5)$$

$$(6)$$

where, as usual,  $\gamma = (1 - (v/c)^2)^{-1/2}$  and the fields are obtained from  $\mathbf{A}$  as,  $\mathbf{E} = -\frac{1}{c}\partial\mathbf{A}/\partial t$  and  $\mathbf{B} = \nabla \times \mathbf{A}$ .

In the presentation of results all times are normalized to the inverse electron gyrofrequency in the field  $B_0$ , the lengths to the Larmor radius and velocities to the initial (thermal) speed. The kinetic energy ( $E_k = (\gamma - 1)mc^2$ ) is given in KeVs. The ratio of wave field to  $B_0$  is represented by the parameter  $E_w$ . We analyzed a wide range of parameters. In most cases a particle drift is observed along  $y$ , which can be attributed to the action of the PM force. This is verified because it disappears when  $\Delta \rightarrow \infty$ . However this effect is not noticeable for high wave frequencies and small amplitudes  $A_0$  (see Fig.(1)). In the figures we show 3D plots of the electron trajectory for  $\Delta = 2$ , showing the  $y$ -drift but also a field-aligned drift (along  $z$ ). This is attributable to the wave electric field which does not average to zero in the damping region (for an O-mode). When there is an initial velocity in  $z$ ,  $V_{z0}$ , the parallel motion is there from the onset and the direction is kept unless the wave intensity is quite high. But when  $V_{z0} = 0$  the particle acquires a parallel velocity (downwards in our case). An additional  $y$ -drift also appears as a result of the Lorentz force of the wave magnetic field, which is along  $x$  and is important for relativistic particles.

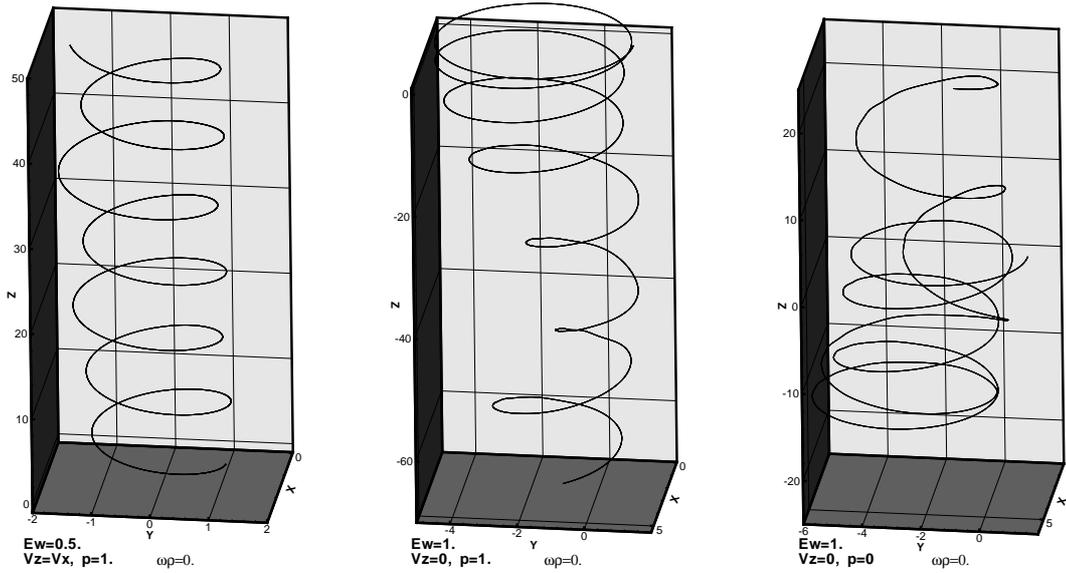


Figure 1: Electron orbit for Figure 2: Particle orbit for Figure 3: Particle orbit for  
 $V_{x0} = V_{z0}$ ,  $E_k = 300$ ,  $\omega = 5$ ,  $E_w = 1$ ,  $V_{z0} = 0$ ,  $\omega = 2$ ,  $\delta = 0$ ,  $V_{z0} = 0$ ,  $\omega = 2$ ,  $E_k =$   
 $E_w = .5$  and  $\Delta = 2$ .  $E_k = 1000$ ,  $\delta = 1$   $1000$ ,  $E_w = 1$

In Fig.(1) we show a case with low wave amplitude and intermediate electron energy, where a very mild  $y$ -drift is apparent, and there is an initial motion in  $z$ . In Figs.(2) and (3)  $V_{z0} = 0$ , but a finite  $V_z$  soon builds up, the first for the O-mode ( $\delta = 1$ ) and the second for the X-mode. These are high energy particles ( $E_k = 1000$ ). It is seen that the  $z$ -drift changes sign in the X-mode, since the E-field is not parallel to  $\mathbf{B}_0$ . On the other extreme of low energies the parallel motion shows an oscillatory behavior, as it is seen in Fig.(4), for an O-mode high wave amplitude ( $E_w = 2$ ), with  $E_k = 50$ .

### III. Plasma effects

The collective effect of the plasma on single particles is modeled by including a restoring force proportional to the plasma frequency,  $\omega_p$ , acting in the direction of the wave propagation:  $\mathbf{F} = -m\omega_p^2\mathbf{x}$ . This term is included on the right-hand-side of Eqs.(2-4) and in the energy equation (5) as the power term  $\mathbf{v} \cdot \mathbf{F} = v_x F$ . The presence of plasma can have a drastic effect on the particle motion, being able to change the direction of the drift along  $y$ . This by itself gives rise to a particle drift  $\mathbf{F} \times \mathbf{B}$  that competes with the one due to the PM force. The relative magnitude is proportional to  $w_p \equiv (\omega_p/kA_0)^2$ . We show in figs.(5) and (6) the resulting trajectories for the case of  $w_p = 1$ , when the wave is linear and circularly polarized, respectively. The electron kinetic energy is  $E_k = 500keV$ . It is clear that the drift direction is in the positive  $y$  direction, opposed to the case with  $w_p = 0$  with the same parameters.

It is to be noticed that for much higher electron energies the  $y$ -drift direction is again reversed, matching the case with no collective influence. We found that the transition total energy is about 1150 keV. Its presumably due to the dominance of relativistic effects at high energies. Another feature that remains is that even for initial null

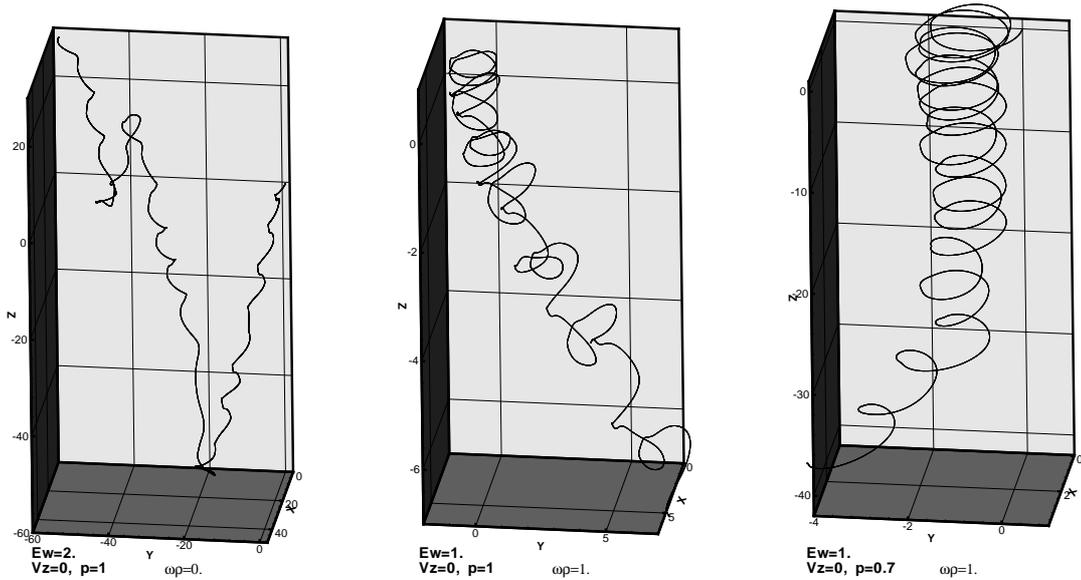


Figure 4: Particle orbit for Figure 5: Orbit with model Figure 6: Same case for  $\Delta = 2$ ,  $V_{z0} = 0$ ,  $\omega = 2$ , plasma for  $V_{x0} = 0$ ,  $E_k = E_w = 1$ ,  $V_{z0} = 0$ ,  $\omega = 2$ ,  $E_k = 50$ ,  $E_w = 2$   $500$ ,  $\omega = 2$ ,  $E_w = 1$ ,  $\delta = 1$ .  $E_k = 500$ ,  $\delta = 1/\sqrt{2}$

velocity along the magnetic field, it acquires a field-aligned speed that may be able to rotate the electrons in the toroidal direction mainly.

#### IV. Conclusions

We have shown that when a single particle in a magnetized plasma moves in a wave with variable amplitude, as in the case of resonant absorption, there are drift motions in both of the directions perpendicular to the wave propagation. They are due to the interplay of different effects that occur in this special situation. There is first the PM force due to the electric field gradient, which would produce a poloidal rotation in a toroidal plasma. There is also the contribution from the Lorentz force for the wave fields, which are also poloidal. The background field-aligned motion is also important and is also the result of the E-field gradient for the O-mode. These results suggest that the presence of both drifts may be used to drive current and/or plasma flows in the poloidal and toroidal directions in a tokamak. There is a requirement of high power in order for this effects to be relevant, but this could be addressed by the new generation of gyrotrons. It is possible to envision the use of high power lasers in order enhance the relativistic effects that are found here.

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#### References

- [1] J.J. Martinell and C. Gutierrez-Tapia, Phys. Plasmas **8** (2001) 2808.
- [2] J.J. Martinell, Rev. Mexicana Fis. **48** (2002) 239.
- [3] J.N. Bradsley, B.M. Penetrante and M.H. Mittleman, Phys. Rev. A **40** (1989) 3823.
- [4] E.S. Sarachik and G.T. Schappert, Phys. Rev. D **1** (1970) 2738.