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# Simulation and theory of the initial stage of floating-sheath formation

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**Abstract** The problem of sheath formation in front of a conductive planar absorbing plate collector is formulated. We consider a single-emitter plasma diode filled with an initially uniform plasma. Electron plasma (Langmuir) waves are excited [1] at the initial stage of the collector charging-up process. Assuming a simplified time dependence of the potential on the collector, the boundary value problem for this excitation of Langmuir waves is solved analytically.

## Introduction

Since the thermal velocity of the electrons is much higher then that of the ions, negative electric charge starts to accumulate on the collector, so that electrons are gradually repelled, ions are attracted, and a positive-space-charge sheath begins to form. Keeping the plates externally disconnected, the charging-up process ceases when the electron and ion currents onto the collector cancel each other, and the potential difference reaches its floating value. The establishment of the floating potential between the plates is a time-dependent process and its description represents a rather complex mathematical and physical problem.

## Analytic description and wave excitation

The collector and the emitter are placed at $x = 0$ and $x = L > 0$, respectively. The length $L$ is to be chosen large enough in order to fulfill the condition $L \gg \lambda_D$, where $\lambda_D$ is the Debye length. The emitter potential is chosen to be zero at all times. At the initial time, ($t = 0$), the electron and ion densities are equal and spatially uniform ($n_{e0} = n_{i0} = n_0$), the velocity distribution functions of both particle species are full Maxwellians, and the electric potential is zero everywhere in the system. Throughout the entire simulations the following boundary conditions are used: At the collector, all particles impinging are absorbed and no particles are emitted. At the emitter, all particles impinging are absorbed as well, but new particles with half Maxwellian distribution functions are injected.

The electric potential in the region considered is denoted by $V(x,t)$ or, in normalized form, $\Phi(x,t) = -eV(x,t)/T_e$. According to Fig. 1(a) the collector potential, $V_c(t) = V(0,t)$, decreases and simultaneously undergoes small oscillations, associated with plasma oscillations and a corresponding modulation of the electron current. Let us assume that the collector has the form of a thin disk with radius $a$ and thickness $h$, so that $a \gg h$. The one-dimensionality of the problem requires that $a \gg \lambda_D$. In CGS units the capacitance of the collector equals $C = 2a/\pi$ [2]. After integrating Ampère's law over the collector surface we find

$$\frac{\partial \Phi_c(t)}{\partial t} = \frac{\pi}{8} \frac{a}{\lambda_D^2 n_0} \{J_e(t) - J_i(t)\}, \tag{1}$$

where $\Phi_c(t) = \Phi_c(0,t) = -eV_c(t)/T_e$ is the normalized collector potential, $n_0$ is the particle number density in the unperturbed plasma ($x \to L$), $T_e$ is the electron temperature, $J_e(t)$ and $J_i(t)$ are the electron and ion particle fluxes onto the collector. During the initial stage of the process, characterized by the electron time scale, we can assume $J_i(t) \approx 0$ and use the approximate expression $J_e(t) \simeq n_0 V_{Te} \exp(-\Phi_c(t))$ for the electron flux ($V_{Te}$ is the electron thermal velocity). From Eq. (1) we find

$$\Phi_c(t) = \ln\left\{\frac{\pi}{8}\frac{a}{\lambda_D}\omega_{pe} t + 1\right\}, \tag{2}$$

where $\omega_{pe}$ is the electron plasma frequency. Obviously, the approximation used above for the electron flux leads to a monotonic decrease in the collector potential and implies neglect of the potential oscillations on the surface, presented in Fig. 1(a). At the initial stage of the collector charging-up process, the plasma response can be described in the linear approximation. From the hydrodynamic equations for the electrons (the ions are assumed to be immobile) we obtain

$$\frac{\partial^2 \Phi(x,t)}{\partial t^2} - V_{Te}^2 \frac{\partial^2 \Phi(x,t)}{\partial x^2} + \omega_{pe}^2 \Phi(x,t) = 0. \tag{3}$$

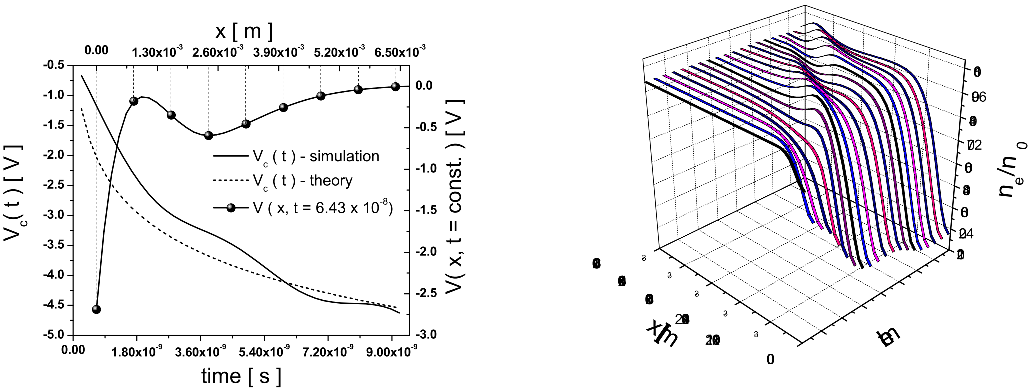
This equation we have to solve with the boundary condition (2), assuming the homogeneous initial conditions $\Phi(x,t=0) = 0$ and $(\partial \Phi(x,t)/\partial t)|_{t=0} = 0$. Equation (3) is of the Klein-Gordon type. Using the corresponding Green function [3] for the solution of Eq. (3) with $(a/\lambda_D) \gg 1$ we find

$$\Phi(x,t) = \Phi_c(t - \frac{x}{V_{Te}}) + V_{Te}\frac{\partial}{\partial x} J_0\left\{\omega_{pe}\sqrt{t^2 - (x/V_{Te})^2}\right\} \int_0^{t-(x/V_{Te})} dt' \Phi_c(t'), \tag{4}$$

where $J_0\{x\}$ is the zero-order Bessel function. Hence the front of the electron Langmuir oscillations moves with the electron thermal velocity in the positive $x$ direction ($t - x/V_{Te} \geq 0$) and the amplitude of oscillations decreases with the distance from the collector.

**Simulation results and conclusions**

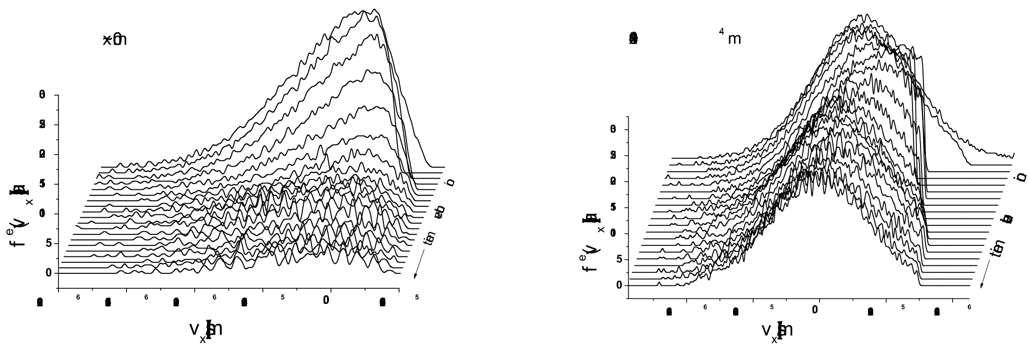
The simulations have been performed by means of the BIT1 code [4], developed on the basis of the XPDP1 code from U. C. Berkeley [5]. At this stage of investigation, the theory gives us only qualitative agreement for the potential evolution on the collector (Fig. 1(a)). In the same figure we present the potential in space at a given time $t = 6.43 \times 10^{-8}\,s$, the dots along the potential curve denoting the positions where we perform Electron Velocity Distribution Function (EVDF) diagnostics. Analyzing the distributions of the electron density in time and space (Fig. 1(b)) we clearly demonstrate the formation of electron Langmuir waves. In the unperturbed plasma region (i.e., $x \rightarrow L$) the electron velocity distribution function will not change appreciably, whereas close to the collector it will acquire a cut-off form (Figs. 2,3). With increasing




(a) (b)

Figure 1: *(a) Potential evolution vs. time on the collector $V_c(t)$, and in space at a given time $V(t = 6.43 \times 10^{-8}\,s, x)$; (b) Electron density evolution in time and space;*

negative collector surface charge, the potential drop in the region becomes higher (Fig. 1(a)) and more and more electrons are reflected by the potential barrier back towards the plasma.



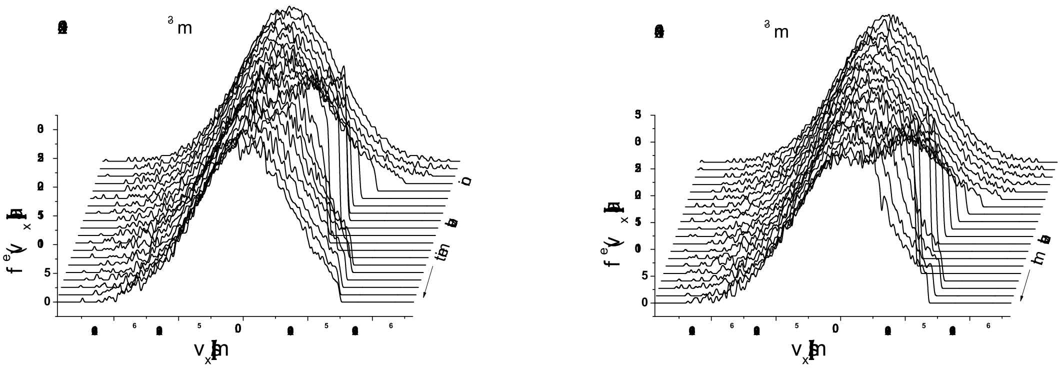
(a) (b)

Figure 2: *EVDF evolution in time at positions (a) $x \simeq 0.0\,m$ and (b) $x \simeq 8.4 \times 10^{-5}\,m$ from the conductive collector plate.*

The electrons density increases locally, forming a potential well in which, the slow electrons are trapped and transported with the wave velocity towards the bulk plasma.

A clear indication of the slow-electron trapping and transport is the shifting of the EVDF maximum (Figs. 2,3). Keeping a fixed position in space and analyzing the EVDF evolution in time, one can easily observe the moment when the wave perturbs the local electron distribution.

At the same time, the EVDF becomes a cut-off Maxwellian due to absorbtion of fast particles at the collector. Since the shape of the distribution function essentially depends on the potential between the plates, the cut-off of the EVDF changes in time. These perturbations in the region



(a) (b)

Figure 3: *EVDF evolution in time at positions (a)* $x \simeq 2.44 \times 10^{-3}\,m$ *and (b)* $x \simeq 4.84 \times 10^{-3}\,m$ *from the conductive plate.*

close to the collector, whose characteristic time scale is of the order of the electron plasma period, generate the electron Langmuir waves propagating into the plasma.

In conclusion we may state that the processes taking place in the plasma during the sheath formation reflect the time evolution of the collector potential and vice versa.

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**References**

[1] C. Lupu, D.D. Tskhakaya sr., S. Kuhn, M. Cercek, R. Schrittwieser and G. Popa, *Electron dynamics at the initial stage of floating -sheath formation*, ICPP 2004, Nice, France, Book of Abstracts, p.126; http:// hal.ccsd.cnrs.fr/ccsd-00019.

[2] L.D. Landau, E.M. Lifshitz, and L.P. Pitaevskii, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1984), p.19-25.

[3] P.M. Morse and H. Feshbach, *Methods of Theoretical Physics* (Mc Grow-Hill, NY, 1953), pp. 848-857.

[4] D. Tskhakaya and S. Kuhn, *Effect of* $E \times B$ *drift on the plasma flow at the magnetic presheath entrance*, Contrib. Plasma Phys. 42, 302 (2002).

[5] J. P. Verboncoeur, M. V. Alves, V. Vahedi, and C. K. Birdsall, *Simultaneous potential and circuit solution for 1D bounded plasma particle simulation codes*, J. Comput. Phys. 104 (2), 321 (1993). Abstract Submittal Form.