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Numerical Simulation of the Collisionless Diffusion of Particles Across a Magnetic Field at a Plasma Edge

M. Shoucri[1], E. Pohn[2], G. Kamelander[2]

[1]*Centre canadien de fusion magnétique, Varennes, Québec Canada J3X 1S1*

[2]*Oesterreichischer Forschungszentrum Seibersdorf, Austria*

There are many situations at the plasma edge of a tokamak where both ions and electrons are accelerated along the magnetic field lines. This acceleration is generally coupled to a diffusion of the particles across the magnetic field lines. We study this problem using a two-dimensional (2D) gyro-kinetic Vlasov code which includes the finite Larmor radius correction and the polarization drift. The $\vec{E}\, x\, \vec{B}$ drift is charge and mass independent. The ions Larmor radius correction allows the existence of a charge separation between ions and electrons, and the polarization drift, which has opposite signs for ions and electrons, has a tendency to accentuate the charge separation in a time-varying electric field and in the presence of steep gradients. The Kelvin-Helmholtz instability which results from this charge separation can produce a turbulent spectrum [1]. We study the phase-space dynamics of the particles in this turbulent spectrum, for the case of a 2D slab layer, in which z represents the toroidal direction, assumed homogeneous, x represents the poloidal periodic direction and y the radial direction. The magnetic field is located in the x-z plane, close to the toroidal direction z, and makes an angle $\theta = 88.5°$ with the x-axis. The pertinent gyro-kinetic equations for the ions and electrons are given by [1].:

$$\frac{\partial f_{i,e}}{\partial t} + \nabla.\left(\vec{\upsilon}_{\perp e} f_{i,e}\right) + \vec{\upsilon}_{\|}.\nabla_{\|} f_{i,e} \pm \frac{e}{m_{i,e}} \vec{E}_{\|}.\frac{\partial f_{i,e}}{\partial \upsilon_{\|}} = 0 \tag{1}$$

$$\vec{\upsilon}_{\perp i,e} = \vec{\upsilon}_D + \vec{\upsilon}_{p_{i,e}}; \quad \vec{\upsilon}_D = \frac{\vec{E}^* \times \vec{B}}{B^2}; \quad \vec{\upsilon}_{p_{i,e}} = \pm \frac{m_{i,e}}{eB^2}\left[\frac{\partial \vec{E}_\perp^*}{\partial t} + \left(\vec{\upsilon}_D + \vec{\upsilon}_{\|}\right)\nabla \vec{E}_\perp^*\right] \tag{2}$$

Poisson equation: $$\Delta\varphi = -\frac{e}{\in_o}\left(n_i^* - n_e^*\right) \tag{3}$$

The star is an abbreviation for an integral operator: $$a^*(y) = \int_{-\infty}^{\infty} G\,(y - y')\, a\,(y')\, dy' \tag{4}$$

This integral is equivalent to multiplying the coefficient of the mode $e^{i\vec{k}.\vec{r}}$ in Fourier space by a factor $G_k = \exp\left(-k_\perp^2 \rho_{i,e}^2 /2\right), \rho_{i,e} = \upsilon_{ti,e}/\omega_{ci,e}$ .We normalize time to ω_{pe}^{-1}, space to the Debye length λ_{De} , and velocity to the thermal velocity v_{th}. The initial values are:

$$n_{i,e}(y) = N(y)(1+\in \sin k_o x+ \in \sin 2k_o x + \sin 3k_o x) \qquad (5)$$

$$f_e(x,y,z,\upsilon_\|) = \frac{N(y)}{\sqrt{2\pi T_e}}\, e^{-\upsilon_\|^2/2T_e(y)}; \quad f_i(x,y,z,\upsilon_\|) = N(y)\,\frac{(m_i/m_e)^{1/2}}{\sqrt{2\pi T_i}} e^{-(m_i/m_e)\upsilon_\|^2/2T_i(y)}$$

$$T_{eo} = T_{io} = 1, \quad m_i/m_e = 1840, \quad \omega_{ci}/\omega_{pi} = 0.9, \quad \rho_i/\lambda_{De} = 1./0.9$$

$$N(y) = \frac{1}{2}\left[1+\tanh(1.6y)\right]; \quad T_i(y) = T_e(y) = T_{eo}\left[0.2 + 0.4\tanh(1.6y)\right] \quad -6 < y < 6 \qquad (6)$$

$L_x = 28$, $L_y = 12$, $k_o = 2\pi/L_x$. The edge equilibrium $N(y)$ is perturbed at t=0 as indicated in Eq.(5), with $\in = 0.002$. Equations(1-4) are solved using a fractional steps method for the Eulerian Vlasov code [2]. The grid points are N_x N_y N_v = $64 \times 128 \times 128$.The simulation is effected up to t = 50000, and $\Delta t = 1$.

Results.

The system in Eqs(5-6) is initially neutral $n_i = n_e$. However, in view of the finite gyro-radius effect as explained in Eq.(4), there is a charge separation and electric field at the edge.The low frequency waves result from the Kelvin-Helmholtz instabilities which are driven by the $\vec{E}\, x\, \vec{B}$ velocity shear. Fig.(1) shows the time evolution of the first four Fourier modes in the poloidal x direction. The first two modes dominate at the end at essentially the same level. The spectrum is saturating at low level, modulating very slightly the initial density profile. When averaged over the periodic x direction, the different profiles are essentially keeping their initial profiles in the y direction, as shown in Fig.(2) for the potential (solid curve), charge (dotted curve) and electric field (broken curve). Fig.(3) presents a 3D view of the potential, and Fig.(4) presents a 3D view of the of the charge $\left(n_i^* - n_e^*\right)$ at the edge of the plasma. Contour plots for the $\upsilon_\| - x$ phase-space presented in Ref.[1] for the electrons and ions show the distribution functions slightly modulated around $\upsilon_\| = 0$ by the low frequency turbulent spectrum. The low noise level of the Vlasov code allows to use test particles diagnostic technique to study the particles motion in specified regions of the phase-space. These particles follow the characteristics of the Vlasov equation, given by :

$$\frac{dx}{dt} = E_y^* \frac{\sin\theta}{B} + \upsilon_{px} + \upsilon_{\parallel} \cos\theta \,; \qquad \frac{dy}{dt} = -E_x^* \frac{\sin\theta}{B} + \upsilon_{py} \,; \qquad \frac{d\upsilon_{\parallel}}{dt} = \frac{\pm e}{m_{i,e}} E_x^* \cos\theta \,;$$

$\frac{dw}{dt} = -w\nabla.\vec{\upsilon}_p$. Eq. (1) contains a source term $f\nabla.\vec{\upsilon}_p$, since the polarization drift is not divergence free. The quantity *w(t)* is a weight attached to each particle, reflecting the effect of this source term. From these characteristics equations, we see that an invariant exist if $\upsilon_{py} = 0$, which is obtained by dividing the second and the third of these equations:

$$\upsilon_{\parallel} \pm \frac{eB}{m_{e,i} \tan\theta} y = const \tag{7}$$

The invariant in Eq.(7), reflects the constant generalized momentum in the homogeneous direction *z*. Fig.(5) shows how four thousand ion test particles, initially distributed uniformly at grid points at $-0.015 < \upsilon_{\parallel} < 0.015$, and $-2 < y < 1$ are following a straight line in the $(y, \upsilon_{\parallel})$ phase-space, in agreement with Eq.(7). The same behaviour is shown in Fig.(6) for the four thousand electrons, initially located at $-1 < \upsilon_{\parallel} < 1$ and $-2 < y < 1$. This indicates that the polarisation drift is negligible (it is always negligible for the electrons). The mean-square displacements Δy^2 and $\Delta\upsilon_{\parallel}^2$ for these particles are related to the diffusion coefficients in space $D_y = \Delta y^2 / t$, and in velocity space $D_{\upsilon_{\parallel}} = \Delta\upsilon_{\parallel}^2 / t$ [3]. The time evolution of Δy^2 is shown in Fig.(7) for the ions and Fig.(8) for the electrons. The curves for $\Delta\upsilon_{\parallel}^2$ show a similar evolution, since from Eq.(7) $\Delta y^2 = \Delta\upsilon_{\parallel}^2 \tan^2\theta / \omega_{ce,i}$ .A diffusion in velocity space along the magnetic field is coupled to a diffusion in *y* space across the field (see Eq.(7)). In our normalized units, $\Delta y_i = \Delta\upsilon_{i\parallel} \tan\theta\sqrt{m_i / m_e} /(\omega_{ci} / \omega_{pi})$ and $\Delta y_e = \Delta\upsilon_{e\parallel} \tan\theta\sqrt{m_e / m_i} /(\omega_{ci} / \omega_{pi})$,i.e. due to the ratio of masses, the displacement across the magnetic field can be of the same order of magnitude for the ions and for the electrons (see Figs(7,8)), even though $\Delta\upsilon_{i\parallel}^2$ is much les than $\Delta\upsilon_{e\parallel}^2$ (we note also in Figs(5,6) the same displacement in *y* for both populations). We finally note that a similar study was presented for lower hybrid waves at a plasma edge in Ref.[4].

References

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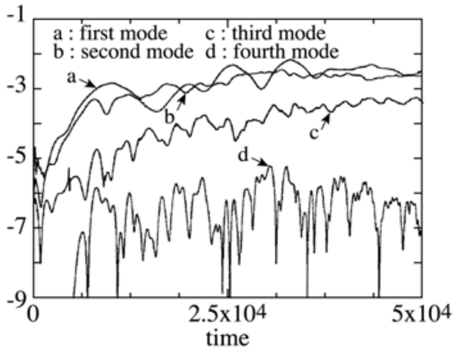

Fig.1

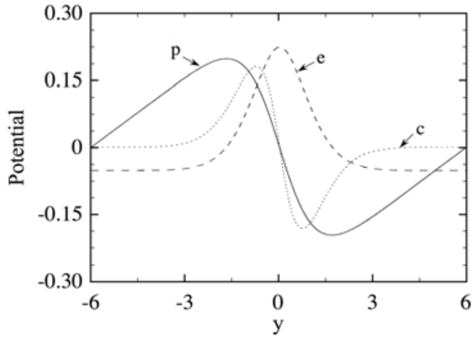

Fig.2

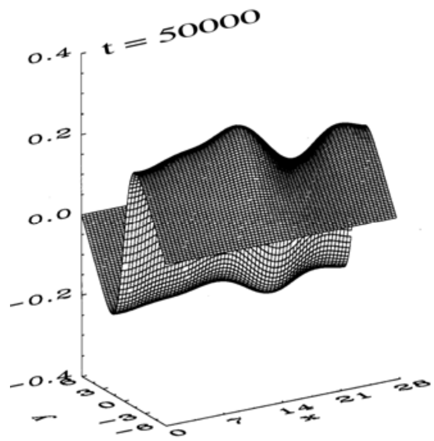

Fig.3

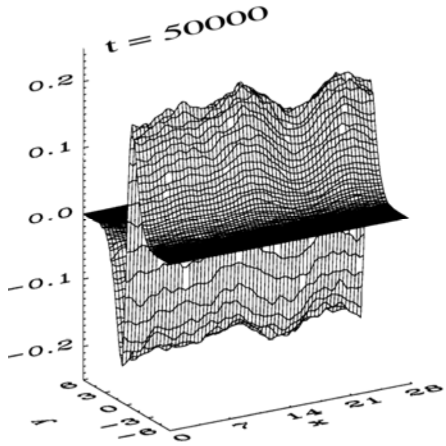

Fig.4

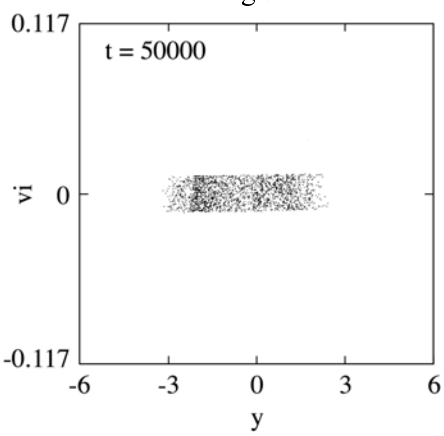

Fig.5

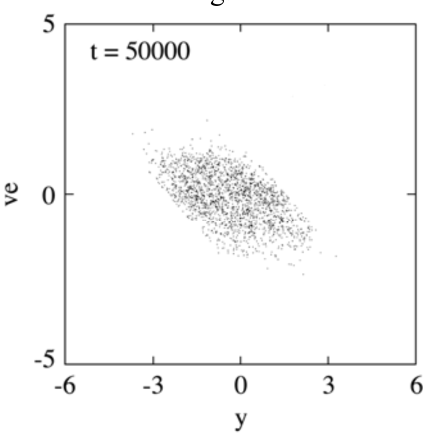

Fig6

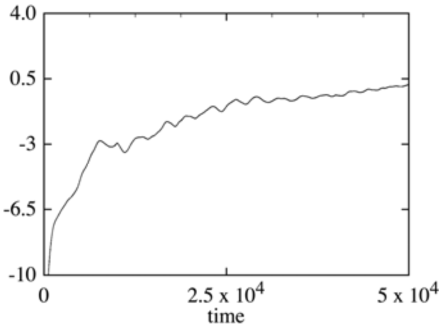

Fig.7

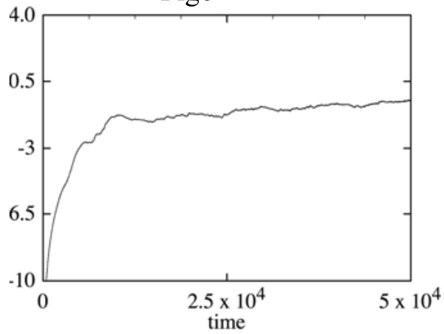

Fig.8