Perturbative studies in DTEM subcritical turbulence for plasma confinement

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1   Introduction

A simple model of drift waves has been investigated under ideas based on self-organized-criticality (SOC). This concept can help us in understanding many of the experimentally observed phenomena that seem to have some of the characteristic properties of SOC systems such as the experimental finding of canonical profiles, the strong transport measured even when the profiles are subcritical on average, the existence of universal indexes in measured broadband fluctuation spectra which has hardly dependence to changes in global parameters or the scaling of transport coefficients with system size (Bohm scaling).

In this work we investigate if SOC should be expected to play any role in this situation by considering a simple model for DTEMs [1] in subcritical state.

2   Model equation

The single-equation model studied in this paper is a paradigm of plasma drift-waves valid for long wavelengths. In previous works, this model has been used to describe the dynamics of a plasma in a supercritical state. Now we use the same basic model to study the dynamics in a subcritical state.

The main difference between this model and the one considered in Ref. [1] is in the evolution of the flux surface averaged density. In that case the averaged density profile was kept fixed to impose that the saturation was achieved only by turbulent processes. Another change in this model is the introduction of a source term of particles $S = S(r,t)$ which is random in time and radius, and does not depend on the angular coordinates. The final difference is the addition of an external diffusion. This term is introduced to study the interplay between the turbulent, supposedly SOC channel and this other diffusive channel.

3   Transport properties

Once the system has evolved from its initial equilibrium to a steady state after being initialized with small random perturbations to the background equilibrium profile, we consider the evolution of the system with an external noise source, as in the sandpile model. As perturbations are added, they trigger instabilities which make fluctuations to interact with profile, flattening it at the corresponding rational surfaces and generating transport.

3.1   Global transport in steady state

It is possible to obtain a measure of the global transport by evaluating the temporal evolution of the following quantity:

$$N(r) = \int_0^r r' dr' \left(\langle n \rangle - \langle n \rangle_{ss} \right) \tag{1}$$

where the angular brackets, $\langle \, \rangle$, denote flux surface averaging and ss stationary state. In Fig. 1(a) we plot the time evolution of N at five different positions. There is some accumulation of density for values larger than 0.68, as we can see in Fig. 1(b) where the value of $N(r)$ at the end of the simulation is plotted. The system reacts to the external source transporting density from the inner part outwards, thus creating a hole inwards and a bump outwards.
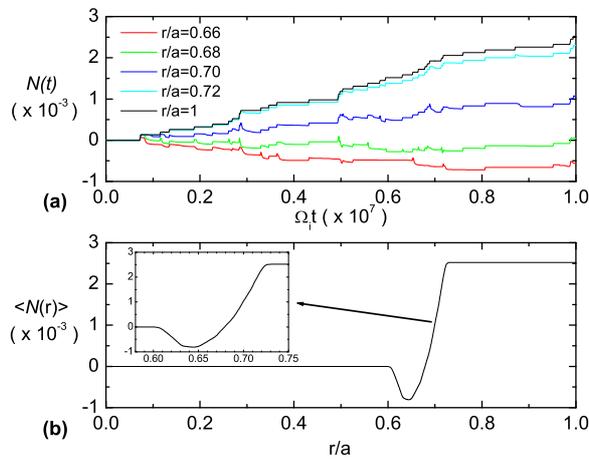

Figure 1: (a) Time evolution of the density accumulated N at different positions in the plasma. (b) Dependence of N with radius. At about $r = 0.68 \, a$ the system begins to accumulate density.

From N we can define an effective flux through the different radial positions in the plasma, $\Phi(r) = \frac{\partial}{\partial t} \left[S_{acc}(r) - N(r) \right]$, where $S_{acc}(r)$ is the part of the source which lies to the left of r.

We can define too an incremental effective diffusivity from the effective flux, $\Phi(r) = D_{eff}(r) \, r \frac{\partial \langle n \rangle}{\partial r} \big|_r$. Fluxes and diffusivities are calculated by time-averaging the signals obtained in steady state. Fluxes begin to increase linearly and reach their maximum values, as in the sand pile model. In this phase the scaling of diffusion coefficients with radius is Bohm-type. After that they begin to decrease towards zero again. We obtain the following incremental diffusion coefficient $D_{eff} \approx 3 \times 10^{-9} \, (r - r_0) \, a^2 \Omega_i$, $r_0 \leq r \leq r_{max}$ where $r_0 \approx 0.6 \, a$ and $r_{max} \approx 0.68 \, a$. For $r > r_{max}$ the flux decreases because it must be null beyond the right edge where rational surfaces are located.

3.2   Power spectrum of avalanche events

Let us study now the statistical properties of the transport. This can be done by inspecting the time trace of a relevant physical quantity, $g(t)$. In our case, the time series are

formed by evaluating at each temporal step the number of unstable sites g (i.e. number of radial grid points for which $L_n < L_n^{crit}$). The magnitude of this record can be interpreted as a measure of the turbulent activity or the total avalanche activity associated to DTEM turbulence. What we want to know is if it is posible to consider our system to be a SOC system. In Figs. 2(a) and (b) $g(t)$ is plotted for the cases $D_0 = 0$ and $D_0 = 10^{-9} \, a^2 \Omega_i$.
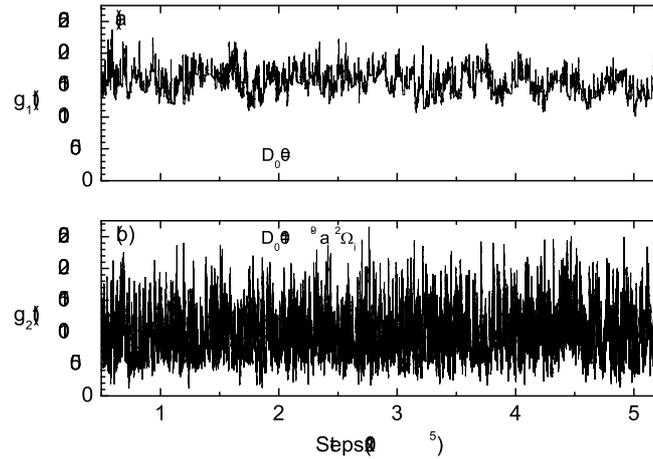

Figure 2: Time histories of $g(t)$: (a) $D_0 = 0$ (SOC case), (b) $D_0 = 10^{-9} \, a^2 \Omega_i$ (diffusive case).

The time averaged number of unstable radial points decrease with the background diffusivity. The roughness of $g(t)$ is a measure of the activity generated by high frequency events. For $D_0 = 0$ (low roughness) there is more avalanche activity (total), and this activity is continuous, it has not fast oscillations. Typical frequencies are in the autosimilar range. For $D_0 = 10^{-9} \, a^2 \Omega_i$ the avalanche activity is bursty (high roughness). This means that high frequencies play a more important role as can be checked by inspecting Fig. 3. The power carried by high frequency events is higher for high background diffusivity [see Fig. 3(b)].

Furthermore, the flat low frequency region extends to higher frequencies. This suggest that as a result of the increased diffusion, not only fast small events are more important now, but larger isolated events become more common too. The continuous smoothing of the local inhomogeneities builds a diffusive profile that allow big events [$\max(g_2(t)) > \max(g_1(t))$] and favours the small ones increasing the roughness of $g(t)$. The power spectrums of $g_1(t)$ and $g_2(t)$ possess well defined regions with power laws, $P(f) \propto f^{-\alpha}$, characterized by different exponents. At very low frequencies the power spectrum is flat. This is apparent from both [Figs. 3(a) and (b)]. The case with high diffusion has a broader flat region. This region is representative of large scale (or low frequency) events. For frequencies in the range ($2 \times 10^{-5} \, \Omega_i/500 < f < 2 \times 10^{-3} \, \Omega_i/500$), the dependence of the spectrum is $P(f) \propto f^{-1}$ for $g_1(t)$. It is interesting that the limit for the maximum frequency at which the overlapping region extents ($1/f$) is just the frequency at which the
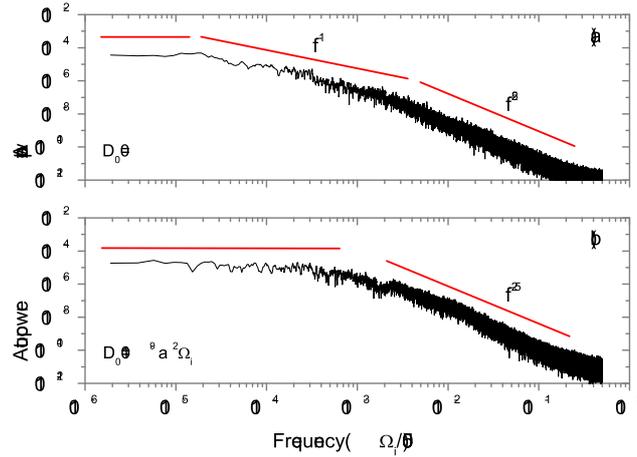
Figure 3: Power spectra of $g(t)$: (a) $D_0 = 0$ (SOC case), (b) $D_0 = 10^{-9}\, a^2\Omega_i$ (diffusive case).

external source is driving the system. For higher frequencies it has not sense to speak of overlapping. For a SOC phenomena to exist, the rates associated to the forcing must be much slower than the rates associated to the transport. This autosimilar region shrinks for increasing ambient diffusivities and practically disappears for values high enough of this parameter, as shown in Fig. 3(b). Finally, at higher frequencies, the spectrum falls off as $f^{-2.5}$ in both cases. This region is associated to small scale, high frequency events. We have to note that for $D_0 = 0$ the three characteristic regions of SOC systems are visible. This is not true for $D_0 = 10^{-9}\, a^2\Omega_i$ where the $1/f$ region practically has disappeared.

4   Conclusions

It has been investigated the main features relative to the fluctuations and transport of a model based on long-wavelength drift wave subcritical turbulence. This model is a very simplified form of turbulence for a magnetic confinement device.

Global transport studies show that incremental diffusivities scale proportional with radius. This is in agreement with Bohm scaling.

Some properties found in the system obey to the concept of self-organized-criticality: The great amount of transport in steady steady even when the averaged density profile is below the prescribed threshold given by the expected instability. The power spectra of the turbulence activity with a marked power law with $1/f$ dependence for more than two decades for the case with no ambient diffusion.

For increasing diffusivities the dynamical mechanism begins to separate from SOC. This fact is clear from the shrinkage of the $1/f$ region in the power spectrum.

References

[1] B. A. Carreras *et al, Phys. Fluids B* **4**, 3115 (1992).