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Analytic approximations of divertor behaviour and application to MAST

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1. Global description of particle balance

In its simplest form, fuelling of tokamak plasmas consists of typically localized direct inputs plus distributed neutral-particle fluxes from the surrounding vessel space, fed by recycling and partly from gas puffing itself. Globally particle balance may therefore be described by rate equations for the numbers of ions in the plasma N_i, atoms $N_{\rm D}$ and molecules $N_{\rm D2}$ in the surrounding volume ("tank"), and equivalent atoms $N_{\rm w}$ adsorbed by walls [1] :-

$$\frac{dN_i}{dt} = -\frac{N_i}{\tau_i^*} + e_{\rm I}\,\Gamma_{\rm I} \quad , \tag{1}$$

$$\frac{dN_{\rm D}}{dt} = -\frac{N_{\rm D}}{\tau_{\rm D}} + f\,\frac{N_{\rm D2}}{\tau_{\rm D2\,p}} + \rho_{{\rm r}i}\left(\frac{N_i}{\tau_i^*} + (1-e_{\rm I})\,\Gamma_{\rm I}\right) + \rho_{\rm rD}\,\Gamma_{\rm D} + (1-w_\Phi)\,2\,(1-e_\Phi)\,\Phi \quad , \tag{2}$$

$$\frac{dN_{\rm D2}}{dt} = -f\,\frac{N_{\rm D2}}{\tau_{\rm D2\,p}} + \frac{\rho_{{\rm d}i}}{2}\left(\frac{N_i}{\tau_i^*} + (1-e_{\rm I})\,\Gamma_{\rm I}\right) + \frac{\rho_{\rm dD}}{2}\,\Gamma_{\rm D} + w_\Phi\Phi - \alpha\,\frac{N_{\rm D2}}{\tau_{\rm D2\,w}} - \frac{a\,\Lambda}{V_{\rm t}}\,N_{\rm D2} \quad , \tag{3}$$

$$\frac{dN_{\rm w}}{dt} = (1-\rho_{{\rm r}i}-\rho_{{\rm d}i})\left(\frac{N_i}{\tau_i^*} + (1-e_{\rm I})\,\Gamma_{\rm I}\right) + (1-\rho_{\rm rD}-\rho_{\rm dD})\,\Gamma_{\rm D} + 2\,\alpha\,\frac{N_{\rm D2}}{\tau_{\rm D2\,w}} \quad , \tag{4}$$

where τ_i^* is the effective ion confinement time allowing for intrinsic refuelling by recycling; $\tau_{\rm D2\,p,w} = 4\,V_{\rm t}/(\,\bar{c}_{\rm D2}\,A_{\rm p,t})$ are plasma / wall pumping times over respective areas $A_{\rm p,t}$ for molecules with thermal speed $\bar{c}_{\rm D2}$ from tank volume $V_{\rm t}$; $\tau_{\rm D} = L_{\rm t}/\,\bar{c}_{\rm D}$ is an equivalent atom loss time for plasma-to-wall distance $L_{\rm t}$; $0 \le \rho_{{\rm r}\,i,{\rm D}} + \rho_{{\rm d}\,i,{\rm D}} \le 1$ are ion / atom reflection / desorption probabilities from the walls; $0 \le \alpha \le 1$ is a corresponding sticking probability for molecules; $\Lambda\ {\rm m^3\cdot s^{-1}}$ is an active pumping speed connected by duct transmission probability $0 \le a \le 1$; $0 \le e_{\rm I} \le 1$ is core plasma fuelling efficiency accounting for prompt recycling of edge ionization; and $0 \le e_\Phi \le 1$ is edge fuelling efficiency of gas input Φ molecules $\cdot\,{\rm s^{-1}}$ applied either at the wall ($w_\Phi = 1$) or immediately at the plasma surface ($w_\Phi = 0$). Ionization rate / atom flux $\Gamma_{\rm I,D}\ {\rm s^{-1}}$ may be approximated :-

$$\Gamma_{\rm I} = (1-w_\Phi)\,2\,e_\Phi\Phi + f\,\frac{N_{\rm D2}}{\tau_{\rm D2\,p}} + \frac{V_{\rm p}}{V_{\rm t}}\,\frac{N_{\rm D}}{\tau_{\rm D}} \quad , \qquad \Gamma_{\rm D} = \left(1-\frac{V_{\rm p}}{V_{\rm t}}\right)\frac{N_{\rm D}}{\tau_{\rm D}} \quad . \tag{5}$$

Here a fraction of atoms which intercepts the plasma is taken equal to the ratio of its volume $V_{\rm p}$ to that of the tank. For each impinging molecule ($N_{\rm D2}$), on average one ion is assumed to be added to the plasma and one atom to the tank [2] ((2), (5)), owing to isotropic dissociation and charge-exchange reactions. Factor $0.3 \le f \le 1$ represents attenuation of the molecular influx due to scattering by these escaping atoms [2].

The foregoing model may be integrated numerically for general conditions [1], but can also be solved exactly for steady states $d/dt = 0$. Since divertor or limiter surfaces seeing direct recycling of ions are likely to be quickly saturated, we assume $\rho_{\mathrm{r}\,i} = \rho_{\mathrm{d}\,i} = 0.5$. For puffing at the wall $w_\Phi = 1$, tank molecular density then becomes :-

$$\frac{N_{\mathrm{D2}}}{V_{\mathrm{t}}} = \frac{1}{2}\frac{V_{\mathrm{p}}}{V_{\mathrm{t}}}\frac{\tau_{\mathrm{D2\,p}}}{\tau_i^*}\frac{1}{f}\frac{1}{e_{\mathrm{I}}}\frac{N_i}{V_{\mathrm{p}}}\frac{2 - V_{\mathrm{p}}/V_{\mathrm{t}} - 2\rho_{\mathrm{r\,D}}\left(1 - V_{\mathrm{p}}/V_{\mathrm{t}}\right)}{1 + V_{\mathrm{p}}/V_{\mathrm{t}} - \rho_{\mathrm{r\,D}}\left(1 - V_{\mathrm{p}}/V_{\mathrm{t}}\right)} \quad , \tag{6}$$

and a corresponding expression can be written for atom density $N_{\mathrm{D}}/V_{\mathrm{t}}$ [1]. Note this result has no explicit dependence on any pumping $\rho_{\mathrm{d\,D}}$, α, $a\Lambda$; for fuelling just from tank gas (N_{D2}) itself, its density is set by the chosen steady plasma value (N_i) and cannot be reduced by any sinks. Alternatively, puffing at the plasma edge $w_\Phi = 0$ implies :-

$$\frac{N_{\mathrm{D2}}}{V_{\mathrm{t}}} = \frac{V_{\mathrm{p}}}{V_{\mathrm{t}}}\frac{\tau_{\mathrm{D2\,p}}}{\tau_i^*}\frac{1}{e_{\mathrm{I}}}\frac{N_i}{V_{\mathrm{p}}} \times \tag{7}$$

$$\frac{e_\Phi\left(1-\zeta_{\mathrm{r}}\right) + \left(2 - e_\Phi\right)\zeta_{\mathrm{d}}}{2f\left[2e_\Phi\left(1-\zeta_{\mathrm{r}}\right) + \left(1 - 2e_\Phi\right)\zeta_{\mathrm{d}}\right] + 2e_\Phi\,\tau_{\mathrm{D2\,p}}\,\Omega\left[2\left(1-\zeta_{\mathrm{r}}\right) - V_{\mathrm{p}}/V_{\mathrm{t}}\right] + \left(3f + 4\tau_{\mathrm{D2\,p}}\,\Omega\right)V_{\mathrm{p}}/V_{\mathrm{t}}} \quad ,$$

where $\zeta_{\mathrm{r}} \equiv \rho_{\mathrm{r\,D}} + (\tfrac{1}{2} - \rho_{\mathrm{r\,D}})V_{\mathrm{p}}/V_{\mathrm{t}}$; $\zeta_{\mathrm{d}} \equiv \rho_{\mathrm{d\,D}} + (\tfrac{1}{2} - \rho_{\mathrm{d\,D}})V_{\mathrm{p}}/V_{\mathrm{t}}$; $\Omega \equiv (\alpha/\tau_{\mathrm{D2\,w}}) + (a\Lambda/V_{\mathrm{t}})$ (s^{-1}). Both (6) and (7) are applicable for any tokamak, but they are illustrated in Fig.1 specifically for MAST, which being characterized by large ratios of tank to plasma volume / surface area $V_{\mathrm{t}}/V_{\mathrm{p}} \approx 9$, $A_{\mathrm{t}}/A_{\mathrm{p}} \approx 5$ particularly emphasizes distributed sources and wall pumping. Varying cryopumping Λ with unconditioned ($\rho_{\mathrm{d\,D}} = 0.5$) or boronized ($\rho_{\mathrm{d\,D}} = 0$) walls, or coverage α by getter panels with just the former, are considered. For gettering, both molecules and atoms are assumed to be adsorbed, implying $\rho_{\mathrm{r\,D}} = 0.5\,(1-\alpha) = \rho_{\mathrm{d\,D}}$. Under edge puffing, this action causes gettering to converge to the boronized cryopumped solution for (impracticable) $\alpha \to 1$, while for wall puffing as $\alpha \to 1$ it actually produces an <u>increase</u> in steady-state tank gas density for a given plasma value (owing to the need to maintain requisite fuelling with fewer atoms).

2. Adaptation for a closed, pumped divertor

Effectiveness of active pumping is greatly improved by increasing compression in front of the duct, a condition usually secured using an enclosed divertor. Such an arrangement can in turn be treated in global-balance terms by writing separate sets of rate equations [1] for tank ("t") and divertor ("d") zones, connected through conductances (inverses of closure) for molecules and atoms $F_{\mathrm{D2}}, F_{\mathrm{D}}\ \mathrm{m}^3\cdot\mathrm{s}^{-1}$. Under certain restrictions, exact steady states can again be determined. Eliminating atoms from the model and supposing recycling $\rho^{\mathrm{d}}_{\mathrm{d}\,i} = 1$ plus cryopumping only within the divertor (ie no wall sinks), a basic "two-chamber" representation may be written :-

$$\frac{dN_i}{dt} = -\frac{N_i}{\tau_i^*} + e_{\mathrm{I}}\,\Gamma_{\mathrm{I}} \quad , \tag{8}$$

$$\frac{dN^{\mathrm{t}}_{\mathrm{D2}}}{dt} = -\frac{1}{2}\frac{N^{\mathrm{t}}_{\mathrm{D2}}}{\tau^{\mathrm{t}}_{\mathrm{D2\,p}}} + \left[1 - e^{\mathrm{t}}_\Phi\,(1 - w^{\mathrm{t}}_\Phi)\right]\Phi^{\mathrm{t}} + F_{\mathrm{D2}}\left(\frac{N^{\mathrm{d}}_{\mathrm{D2}}}{V_{\mathrm{d}}} - \frac{N^{\mathrm{t}}_{\mathrm{D2}}}{V_{\mathrm{t}}}\right) \quad , \tag{9}$$

$$\frac{dN^{\mathrm{d}}_{\mathrm{D2}}}{dt} = -\frac{1}{2}\frac{N^{\mathrm{d}}_{\mathrm{D2}}}{\tau^{\mathrm{d}}_{\mathrm{D2\,p}}} + \frac{1}{2}\left(\frac{N_i}{\tau_i^*} + (1-e_{\mathrm{I}})\Gamma_{\mathrm{I}}\right) + \left[1 - e^{\mathrm{d}}_\Phi(1-w^{\mathrm{d}}_\Phi)\right]\Phi^{\mathrm{d}} - \frac{a^{\mathrm{d}}\Lambda^{\mathrm{d}}}{V_{\mathrm{d}}}N^{\mathrm{d}}_{\mathrm{D2}} - F_{\mathrm{D2}}\left(\frac{N^{\mathrm{d}}_{\mathrm{D2}}}{V_{\mathrm{d}}} - \frac{N^{\mathrm{t}}_{\mathrm{D2}}}{V_{\mathrm{t}}}\right), \tag{10}$$

$$\Gamma_{\mathrm{I}} \;=\; (1-w_{\Phi}^{\mathrm{t}})\,2\,e_{\Phi}^{\mathrm{t}}\,\Phi^{\mathrm{t}} + \frac{N_{\mathrm{D2}}^{\mathrm{t}}}{\tau_{\mathrm{D2\,p}}^{\mathrm{t}}} + (1-w_{\Phi}^{\mathrm{d}})\,2\,e_{\Phi}^{\mathrm{d}}\,\Phi^{\mathrm{d}} + \frac{N_{\mathrm{D2}}^{\mathrm{d}}}{\tau_{\mathrm{D2\,p}}^{\mathrm{d}}} \quad . \tag{11}$$

Four contingencies are defined by puffing either at the wall or plasma edge in either the main vessel space or divertor. In each case, steady-state molecular density in the tank $n_{\mathrm{D2}}^{\mathrm{t}}$ may be derived as a function of normalized conductance $\hat{F} \equiv (\tau_{\mathrm{D2\,p}}^{\mathrm{t}}\,F_{\mathrm{D2}} \big/ V_{\mathrm{t}})$ and cryopumping $\hat{\Lambda} \equiv (\tau_{\mathrm{D2\,p}}^{\mathrm{t}}\,a^{\mathrm{d}}\,\Lambda^{\mathrm{d}} \big/ V_{\mathrm{t}})$, as tabulated below :-

$$\boxed{e_{\mathrm{I}}\,(N_{\mathrm{D2}}^{\mathrm{t}}/V_{\mathrm{t}})/(N_i/V_{\mathrm{p}}) \;=}$$

(*i*) puff at tank wall

$$\frac{2\left[\hat{\Lambda} + \hat{F}\right]}{2(\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{t}})(V_{\mathrm{t}}/V_{\mathrm{p}})\left[\hat{\Lambda} + \hat{F}\right] + (\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{d}})(V_{\mathrm{d}}/V_{\mathrm{p}})\left[1 + 2\hat{F}\right]} \;, \tag{12}$$

(*ii*) puff at divertor wall

$$\frac{2\hat{F}}{2(\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{t}})(V_{\mathrm{t}}/V_{\mathrm{p}})\hat{F} + (\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{d}})(V_{\mathrm{d}}/V_{\mathrm{p}})\left[1 + 2\hat{F}\right]} \;, \tag{13}$$

(*iii*) puff at tank plasma edge (14)

$$\frac{2\left[(1-e_{\Phi}^{\mathrm{t}})\hat{\Lambda} + \hat{F}\right]}{2(\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{t}})(V_{\mathrm{t}}/V_{\mathrm{p}})\left[\hat{\Lambda}+\hat{F}\right] + (\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{d}})(V_{\mathrm{d}}/V_{\mathrm{p}})\left[1+2\hat{F}\right] + 4(\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{t}})(V_{\mathrm{t}}/V_{\mathrm{p}})e_{\Phi}^{\mathrm{t}}\,\hat{\Lambda}\hat{F}} \;,$$

(*iv*) puff at divertor plasma edge (15)

$$\frac{2\hat{F}}{2(\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{t}})(V_{\mathrm{t}}/V_{\mathrm{p}})\left[e_{\Phi}^{\mathrm{d}}\,\hat{\Lambda}+\hat{F}\right] + (\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{d}})(V_{\mathrm{d}}/V_{\mathrm{p}})\left[1+2\hat{F}\right] + 4(\tau_i^*/\tau_{\mathrm{D2\,p}}^{\mathrm{t}})(V_{\mathrm{t}}/V_{\mathrm{p}})e_{\Phi}^{\mathrm{d}}\,\hat{\Lambda}\hat{F}} \;.$$

Accompanying solutions for divertor gas density may also be extracted, but are more readily appreciated through the compression ratio :-

$$\boxed{C_P \;\equiv\; (N_{\mathrm{D2}}^{\mathrm{d}}/V_{\mathrm{d}})\big/(N_{\mathrm{D2}}^{\mathrm{t}}/V_{\mathrm{t}}) \;=}$$

(*i*) $$\frac{1 + 2\hat{F}}{2\left[\hat{\Lambda} + \hat{F}\right]} \;, \tag{16}$$ (*ii*) $$\frac{1 + 2\hat{F}}{2\hat{F}} \;, \tag{17}$$

(*iii*) $$\frac{1 + 2\hat{F}}{2\left[(1-e_{\Phi}^{\mathrm{t}})\hat{\Lambda} + \hat{F}\right]} \;, \tag{18}$$ (*iv*) $$\frac{1 + 2\hat{F}}{2\hat{F}} \;. \tag{19}$$

In the current approximation, wall puffing is therefore identical to edge puffing with zero efficiency $e_{\Phi}^{\mathrm{t,d}} = 0$. Conversely for perfect edge fuelling efficiency $e_{\Phi}^{\mathrm{t,d}} = 1$, whether realized in the tank or divertor, compression is always the same and is independent of pumping. Note that finite closure (F_{D2}) always produces some compression $C_P > 1$ even with $a^{\mathrm{d}}\,\Lambda^{\mathrm{d}} = 0$, but that C_P <u>decreases</u> for stronger divertor pumping when fuelling with imperfect efficiency in the tank. Compression never varies with pumping for puffing anywhere within the divertor.

These results are again general for any tokamak, but are once more illustrated for parameters appropriate to MAST, plus $0 \le a^{\mathrm{d}}\,\Lambda^{\mathrm{d}} \le 10^3\ \mathrm{m^3\cdot s^{-1}}$, $10\ \mathrm{m^3\cdot s^{-1}} \le F_{\mathrm{D2}} \le 10^4\ \mathrm{m^3\cdot s^{-1}}$, in Figs.2 & 3. An important feature revealed is that for tank puffing with $e_{\Phi}^{\mathrm{t}} < 1$, there is a critical conductance $F_{\mathrm{D2}}^{\mathrm{crit}}(e_{\Phi}^{\mathrm{t}})$ below which $n_{\mathrm{D2}}^{\mathrm{t}}$ increases at given n_i as pumping is itself increased, approaching an asymptotic value $e_{\mathrm{I}}\, n_{\mathrm{D2}}^{\mathrm{t}} / n_i \to (\tau_{\mathrm{D2\,p}}^{\mathrm{t}} / \tau_i^{*})(V_{\mathrm{p}} / V_{\mathrm{t}})(1 - e_{\Phi}^{\mathrm{t}}) / (1 + 2 e_{\Phi}^{\mathrm{t}} \hat{F})$. As with gettering in the "single-chamber" model, such initially surprising behaviour arises because tank sources have to rise to maintain required fuelling as divertor sources are diminished. Changes in Fig.2 demonstrate, however, that for example in MAST steady tank gas density for a fixed plasma value could be lowered significantly by a moderately closed, pumped divertor, if it were combined with fuelling in the divertor or in the main vessel with reasonable efficiency ($e_{\Phi}^{\mathrm{t}} > 0$).

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References

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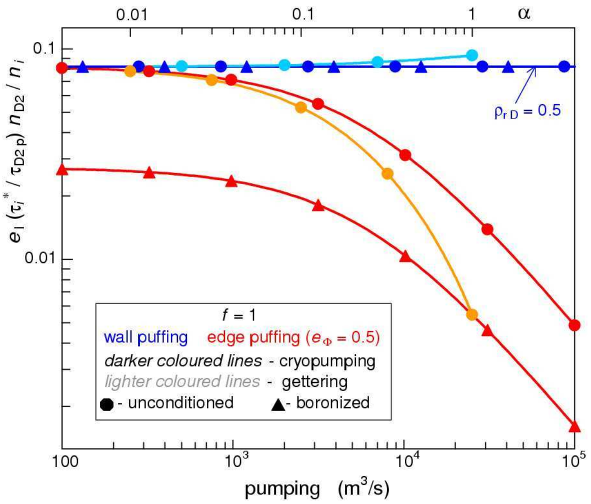

Fig.1 MAST "single-chamber" steady-state tank gas density v. cryo- or getter- pumping.

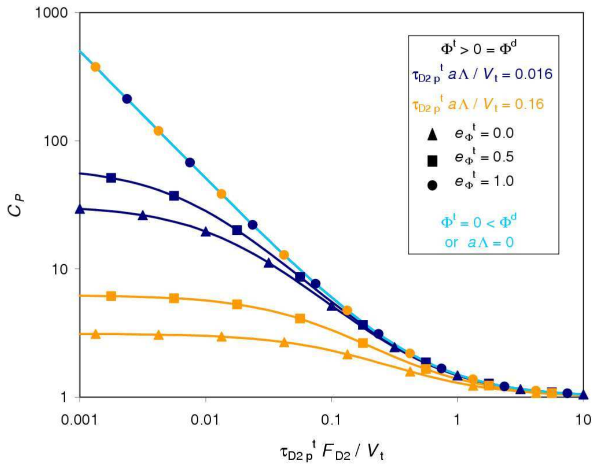

Fig.3 Divertor compression ratio for reduced "two-chamber" steady states in Fig.2 .

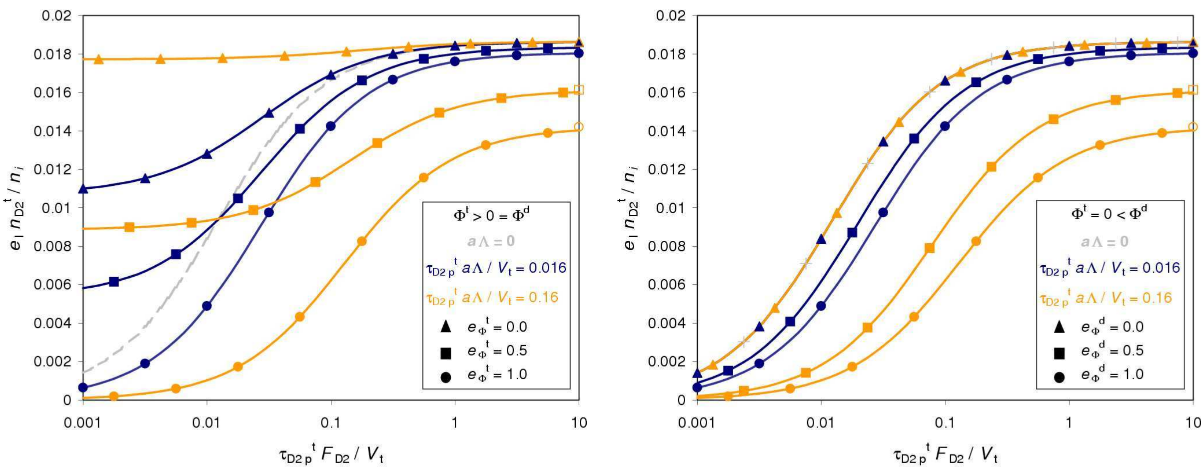

Fig.2 Reduced "two-chamber" solutions for normalized steady-state tank gas density v. divertor conductance (inverse of closure) as functions of pumping and edge fuelling efficiency. *Open symbols*: "single-chamber" limits $F_{\mathrm{D2}} \to \infty$. Puffing in *left*: tank, *right*: divertor.