

Relaxation of a temperature perturbation in collisional plasmas

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Characterization of a heat transport is an important issue for inertial confinement fusion experiments. A nonlocal character of the electron heat transport in laser-produced plasmas is well established both experimentally [1-3] and theoretically [4]. It is well known that the classical local model for the electron heat flux is not applicable if the inhomogeneity scale length, L , is shorter than hundred electron-ion (e-i) mean free paths, λ_{ei} . The nonlocal transport theory has been derived from first principles in the limit of small amplitude perturbations [5]. The nonlocal nonlinear generalization of this theory was tested in comparisons with Fokker-Plank simulations [6] and experimental temperature profile measurements [3]. A good agreement has been found between kinetic simulations and the analytical model of a quasistationary nonlocal transport [5] for the relaxation of localized temperature perturbation with the initial spatial scale, L , longer or on the order of λ_{ei} . For the shorter scale lengths, L , nonstationary effects should be taken into account. This is equivalent to time nonlocality of an electron heat conductivity [7].

Here we describe nonstationary effects in the electron heat transport by solving linearized electron kinetic equation with exact collision operators in the Landau form for the relaxation of the initial temperature perturbations. Our results define limits of the validity of a quasistationary approach which is widely used in nonlocal heat flux models. We have also examined the effect of an external magnetic field on the thermal transport [8]. We have solved the hot spot relaxation problem by describing the heat wave propagation along and across the external magnetic field.

Following Ref. [7] we have found solution to the electron kinetic equation for the initial thermal perturbation given by $\delta T_k(0)$ in k-space,

$$\delta T(x, t) = \frac{3}{8\pi^2} \int_{-\infty}^{+\infty} dk \delta T_k(0) \int_{-\infty}^{+\infty} d\omega e^{ikx - i\omega t} \left(J_T^T - \frac{i\omega J_N^T J_T^N}{k^2 \lambda_{De}^2 \varepsilon} \right), \quad (1)$$

where $\varepsilon = 1 + \frac{(1 + i\omega J_N^N)}{k^2 \lambda_{De}^2}$ is the electron permittivity, $J_B^A = \frac{4\pi}{n_e} \int_0^\infty dv v^2 \Psi^A F_0 S_B$, the functions

Ψ^A are solutions to the equation: $\left(-i\omega + \frac{k^2 v^2}{3\nu_1}\right)\Psi^A = F_0^{-1}C_{ee}[F_0\Psi^A] + S_A$, with sources

$S_N = 1$ and $S_T = 1 - v^2/3v_{Te}^2$ (v_{Te} is the electron thermal velocity), the effective collision frequency ν_1 is defined in terms of the continuous fraction:

$$\nu_1 = -i\omega + \frac{1}{2}l(l+1)\nu_{ei} + \frac{(l+1)^2}{4(l+1)^2 - 1} \frac{k^2 v^2}{\nu_{l+1}}, \quad F_0 \text{ is the Maxwellian distribution, and}$$

$\nu_{ei} = 4\pi Z n_e e^4 \Lambda / m_e^2 v^3$ is the e-i collision frequency. The solution to Eq. (1) for the periodic initial temperature perturbation, $\delta T(x, 0) = \delta T_0 \cos[k_0 x]$, is shown in Fig.1 and is characterized by two different regimes of temperature relaxation – collisionless kinetic and collisional hydrodynamic.

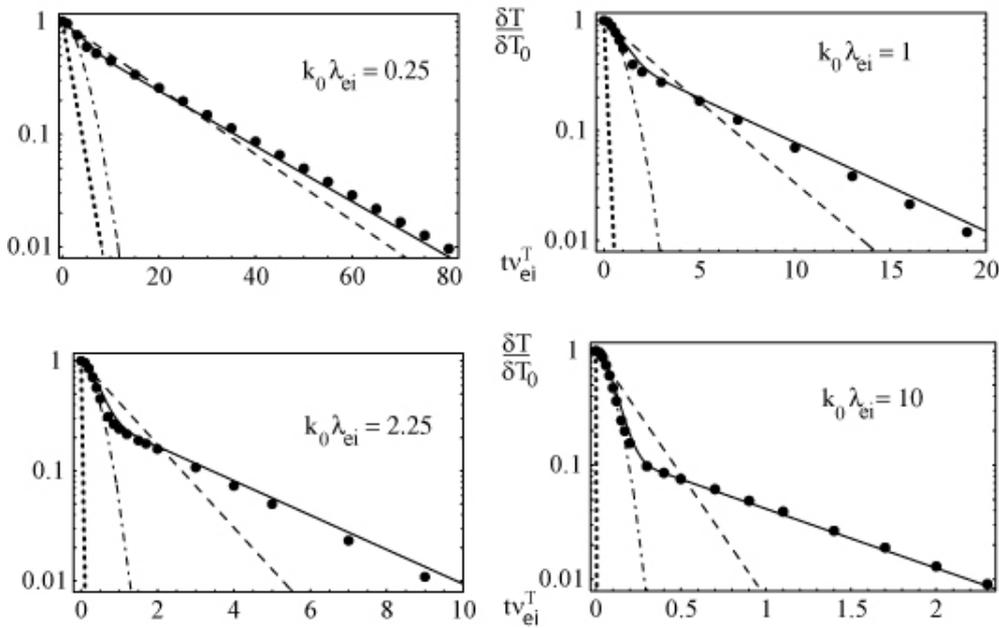


Fig.1 Temperature evolution for the periodic perturbation for different k_0 (big dots) in comparison with fitting (solid lines), quasistationary theory [5] (dashed lines), SH theory (dotted lines), and collisionless theory (dot-dashed lines).

We proposed the following fit for the temperature evolution with an inhibited transport

$$\delta T(x, t) = \left(A \delta T_{hydro}(t) + (1-A) \delta T_{kin}(t) \right) \cos[k_0 x], \quad \text{where } \delta T_{hydro}(t) = \delta T_0 \exp(-t/\tau)$$

$$\tau = 9\pi n_e \left(1 + 12 \left(\sqrt{Z} k_0 \lambda_{ei} \right)^{1.2} \right) / 256 v_{Te} \lambda_{ei} k_0^2, \quad A = \left(1 + (k_0 \lambda_{ei})^{0.8} \right)^{-1}, \quad \text{and}$$

$\delta T_{kin}(t) \approx \delta T_0 \left(2 \exp(-k_0^2 v_{Te}^2 t^2 / 2) + \exp(-3k_0^2 v_{Te}^2 t^2 / 2) \right) / 3$. This can be applied to an arbitrary initial temperature profile, including the localized one, by using inverse

Fourier transform over k_\parallel and treating δT_0 as initial temperature profile in the Fourier space. Our study shows that relaxation of the localized temperature perturbation can be correctly described by nonlocal quasistationary theory for hot spot sizes $L > \lambda_{ei}$.

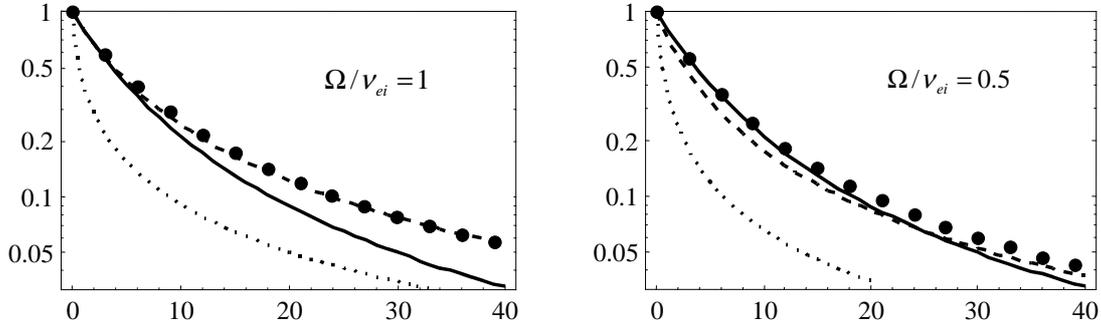


Fig.2 Temperature relaxation in the centre of hot spot (points) in comparison with classical strongly collisional transport theory (dotted lines), nonlocal theory (solid lines) and simplified nonlocal theory (dashed lines) with κ_\perp given by the classical local approach ($L = 3\lambda_{ei}$).

Another important mechanism of electron transport inhibition is due to magnetic field. Here we illustrate its role by considering cylindrical hot spot relaxation in the transversal plane (x, y). The magnetic field is directed along x -axis. Based on the linear theory of an electron transport in a magnetized plasma [8], we describe evolution of the temperature perturbation with the initial Gaussian form, $T(x, y, 0) = T_0 \exp\{-(x^2 + y^2)/L^2\}$, as follows

$$\delta T(x, y, t) = \int \frac{dk_x}{2\pi} \int \frac{dk_y}{2\pi} T_k(0) \exp\{-ik_x x - ik_y y\} \exp\left\{-\frac{2t}{3n_e} (k_x^2 \kappa_\parallel + k_y^2 \kappa_\perp)\right\}$$

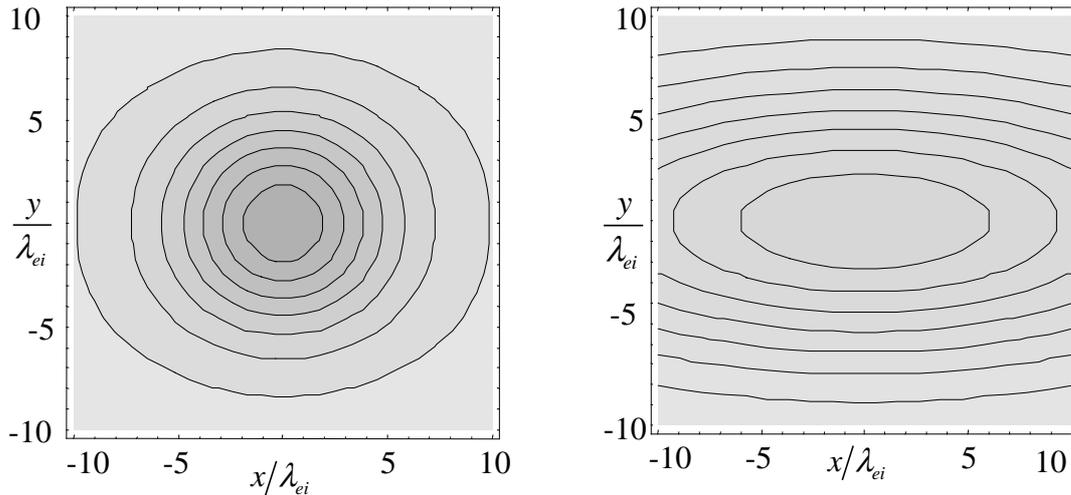


Fig.3 Spatial temperature profile for $t v_{ei} = 10$ (right panel) in comparison with classical theory (left panel) for $L = 3\lambda_{ei}$, $\Omega v_{ei} = 0.5$

Here κ_{\parallel} and κ_{\perp} are heat conductivities in a Fourier-space, along and across magnetic field, correspondingly (cf. Ref. [8]). Figure 2 shows the hot spot temperature evolution in comparison with classical and simplified models, which ignore nonlocal modification of the thermal conductivity across the magnetic field [2]. Interplay between two effects: nonlocal heat flux inhibition along the magnetic field and the reduction of the magnetization parameter makes the spatial temperature asymmetry considerably less pronounced than in the classical case (cf. Fig. 3).

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References

- [1] D. S. Montgomery, O. L. Landen, R. P. Drake *et. al*, Phys. Rev. Lett. **73**, 2055 (1994);
- [2] P. Nicolai, M. Vandenboomgaerde, B. Canaud, and F. Chaigneau, Phys. Plasmas **7**, 4250 (2000).
- [3] G. Gregori, S. H. Glenzer, J. Knight *et. al*, Phys. Rev. Lett. **92**, , 205006 (2004).
- [4] A. R. Bell, R. G. Evans, and D. J. Nicholas, Phys. Rev. Lett. **46**, 243 (1981).
J. F. Luciani, P. Mora, and J. Virmont, Phys. Rev. Lett. **51**, 1664 (1983).
J. R. Albritton, E. A. Williams, I. B. Bernstein, and K. P. Swartz, Phys. Rev. Lett. **57**, 1887 (1986).
A. V. Maximov and V. P. Silin, Phys. Lett. A **173**, 83 (1993).
E. M. Epperlein, Phys. Plasmas **1**, 109 (1994).
- [5] V. Yu. Bychenkov, W. Rozmus, A. V. Brantov, and V. T. Tikhonchuk, Phys. Rev. Lett. **75** , 4405, 1995; A. V. Brantov, V. Yu. Bychenkov, W. Rozmus, and V. T. Tikhonchuk, JETP **83**, 716, 1996.
- [6] A. V. Brantov, V. Yu. Bychenkov, O.V. Batishchev, and W. Rozmus , Comp. Phys. Comm., **164**, 67 (2004) .
- [7] A. V. Brantov, V. Yu. Bychenkov, W. Rozmus, and C.E.Capjack, Phys. Rev. Lett., **93**, 125002 (2004).
- [8] A. V. Brantov, V. Yu. Bychenkov, W. Rozmus, C. E. Capjack, and R. Sydora, Phys. Plasmas **10**, 4633 (2003).