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Magnetic field generation in plasmas due to anisotropic laser heating

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Magnetic fields play an important role in the laser plasma interactions. Their generation mechanisms have been longtime insufficiently investigated because their diagnostics and numerical computations are a complicated task. However it is known that under the conditions previewed for the inertial fusion the anisotropy of the electron distribution function opens a possibility of magnetic field generation. In [1], we have proposed a fluid model where electron pressure is assumed to be a tensor corresponding to an anisotropic distribution function. This model describes electron response within the ten-moments approximation coupled to the equation that takes account of the magnetic field generation.

Let us consider a laser beam propagating in plasma along z-axis and polarized in the x,y-plane. The magnetic field is directed along z axis and it is induced by the plasma motion in the x,y-plane. The code describes the plasma heating and its motion in the hydrodynamic approximation and it accounts for the magnetic field generation due to the anisotropy of laser heating in the direction of the polarization vector. Our goal is to consider the effects of this self-magnetic field on the plasma on the energy transport and the plasma motion. One particular self-consistent effect is the rotation of pressure tensor around the magnetic field axis. We will demonstrate here that anisotropy rotation produces a positive feed-back on the magnetic field generation and induces the Weibel-like instability which cannot be stabilized within the frame of conventional hydrodynamics. We develop a new model for the energy transport related to the gradients of off-diagonal pressure tensor components that accounts for the kinetic effects and allows one to stabilize the short wavelength instability. This model is validated by comparison with the kinetic simulations of the Weibel instability and it allows to use the code for large-scale and long-time modelings of the self-generated magnetic fields in under-dense laser-produced plasmas.

The laser energy density is characterized by the tensor $\overleftrightarrow{W} = \frac{1}{2}\varepsilon_0 \langle \vec{E}_L \otimes \vec{E}_L \rangle$. The trace of this tensor defines the ponderomotive potential $W = \mathrm{tr}\,\overleftrightarrow{W}$ and for the linearly polarized light along the x-axis it has the only component $W_{xx} = W$.

Neglecting inertial terms in electron motion equation [1], one obtains the generalized Ohm's law: $\vec{E} = -\vec{u}_e \times \vec{B} - (2e)^{-1}\vec{\nabla} W - (en_e)^{-1}\,\vec{\nabla}\cdot(n_e \overleftrightarrow{U}) + (en_e)^{-1}\vec{R}_{ie}$, where $\vec{E}$ and $\vec{B}$ are the quasistatic electric and magnetic fields, $\vec{u}_e$ is the average electron velocity, $\overleftrightarrow{U} = n_e^{-1}\overleftrightarrow{P} - \overleftrightarrow{W}$ is the effective electron energy tensor, and $\vec{R}_{ie}$ is the generalized electron-ion friction force.

By calculating the rotation of $\vec{E}$ in this equation and putting the result in Faraday equation $\partial_t\vec{B} = -\vec{\nabla}\times\vec{E}$, one obtains the equation for the axial component of the magnetic field B:

$$\partial_t B + \vec{\nabla}\cdot(B(\vec{V}+\vec{V}_N+\vec{V}_H)) - \mu_0{}^{-1}\vec{\nabla}\times(\sigma_0^{-1}\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times((en_e)^{-1}\vec{\nabla}\cdot(n_e\overleftrightarrow{U})). \qquad (1)$$

Here, $\vec{V}$, $\vec{V}_N$, and $\vec{V}_H$ are the flow, Nerst and Hall velocities, correspondingly, $\sigma_0 = e^2 n_e/m_e\nu_{ie}$ is the plasma conductivity, and ν_{ie} is the electron-ion collision frequency. If $\overleftrightarrow{U}$ is a non-diagonal tensor, the right hand side of (1) does not vanish and it is a source of magnetic field generation. Hence one needs to account for the anisotropy of electron pressure it will be done within the ten-moments approximation.

According to [1], the plasma is assumed to be quasi-neutral and it is described by a set of moment equations – continuity, momentum conservation, and the electron energy transport. The latter one is of particular importance for us:

$$n_e\left[\partial_t\overleftrightarrow{U} + (\vec{u}_e\cdot\vec{\nabla})\overleftrightarrow{U} + (\overleftrightarrow{U}\colon\vec{\nabla}\otimes\vec{u}_e)^S\right] + \vec{\nabla}\cdot\overleftrightarrow{\mathbf{Q}} = 2\nu_T n_e\overleftrightarrow{W} - \frac{1}{3}(\vec{R}_{ei}\otimes\vec{u}_e)^S - \overleftrightarrow{S}_I - \overleftrightarrow{S}_B$$

where $P = \frac{1}{3}\mathrm{tr}\overleftrightarrow{P}$ is the average pressure, $A:B$ denotes the internal scalar product of two tensors of the second rank A and B, $\overleftrightarrow{I}$ is the unitary second rank tensor, and the superscript "S" denotes the symmetrization. The form of the third rank tensor of the thermal transport $\overleftrightarrow{\mathbf{Q}}$ is discussed below. The right hand side of this equation accounts of the inverse Bremsstrahlung and Joule heating, the pressure isotropization, $\overleftrightarrow{S}_I = \nu_p(\overleftrightarrow{P} - \overleftrightarrow{I}P)$, and the rotation of the energy tensor in the self-consistent magnetic field, $\overleftrightarrow{S}_B = (n_e e/m_e)(\overleftrightarrow{U}\times\vec{B})^S$.

Since the gradient of ponderomotive force is directed perpendicularly to the laser beam axis, we limit ourselves to study of two-dimensional plasma evolution in the x,y-plane. Then the tensor $\overleftrightarrow{U}$ has four independent components $U_{xx}, U_{yy}, U_{xy} = U_{yx}, U_{zz}$ and one makes the following decomposition: $\overleftrightarrow{U} = U\overleftrightarrow{I} + \overleftrightarrow{\Pi} + \overleftrightarrow{\Pi}_{\parallel}$ where U is the average energy, $\Pi_{xx} = -\Pi_{yy} = \Pi_{\perp}$, $\Pi_{xy} = \Pi_{yx} = \Pi_{\wedge}$ is the zero-trace stress tensor and $\overleftrightarrow{\Pi}_{\parallel}$ accounts for the anisotropy between the plane x,y and the axis z. Moreover, in this model we consider relatively weak laser beams where the ponderomotive potential is small with respect to the electron energy, $W \ll U$. Then the anisotropic parts are are small, $\overleftrightarrow{\Pi}, \overleftrightarrow{\Pi}_{\parallel} \ll U$. Below we analyze the evolution of the stress tensor, while the anisotropy part $\overleftrightarrow{\Pi}_{\parallel}$ will be ignored because it does not contribute to the magnetic field generation.

The anisotropic hydrodynamic plasma model composed by equations (1) and (2) needs to be closed by appropriate relation between the heat flux tensor $\overleftrightarrow{\mathbf{Q}}$ and the temperature gradients. Due to the hypothesis of a weak anisotropy one may represent the electron heat flux as a sum of two terms, $\overleftrightarrow{\mathbf{Q}} = \overleftrightarrow{\mathbf{Q}}_{iso} + \overleftrightarrow{\mathbf{Q}}_{ani}$. The dominant part of the heat flux, $\overleftrightarrow{\mathbf{Q}}_{iso}$ depends on the gradient of the average electron energy $\vec{\nabla} U$. It can be derived in a form similar to that of Spitzer-Härm and Braginskii [1]: $\overleftrightarrow{\mathbf{Q}}_{iso} = -\frac{2}{5}\kappa_{SH}(\vec{\nabla} U \otimes \overleftrightarrow{I})^S$ where the heat conductivity coefficient accounts also for the magnetization and non-local effects. The second term $\overleftrightarrow{\mathbf{Q}}_{ani}$ is a small correction, which depends on the gradients of the stress tensor $\overleftrightarrow{\Pi}$. However it is a important part of the model because it allows to control the shear and anisotropic effects. The derivation of this new term from the kinetic equation is presented elsewhere. The heat flux vectors in the transport equations for the components $\Pi_\perp$ and $\Pi_\wedge$ have the following forms: $\vec{Q}_{ani,\perp} = -\frac{4}{5}\kappa_{ani}\vec{\nabla}\Pi_\perp - \frac{2}{5}\kappa_{ani}(\vec{\nabla} \times \vec{z})\Pi_\wedge$, $\vec{Q}_{ani,\wedge} = -\frac{4}{5}\kappa_{ani}\vec{\nabla}\Pi_\wedge - \frac{2}{5}\kappa_{ani}(\vec{\nabla} \times \vec{z})\Pi_\perp$, where $\kappa_{ani} = \frac{4}{5}\delta\kappa_{SH}/n_e$ and $\delta \ll 1$ is a constant.

The simulations have been fulfilled for the following parameters: the laser intensity 3×10^{15} W/cm^2; the laser beam of a round Gaussian shape of the radius 10 μm, the laser wavelength $\lambda = 0.35\,\mu$m, the plasma density $n_e = 10^{21}$ cm^{-3}, the electron initial temperature $T_e = 1$ keV, the ion charge $Z = 5$ and the mass $m_i = 6.5\,m_p$ correspond to the fully ionized CH plasma. For these parameters the anisotropy parameter $W/n_eT_e = 0.03$. In figure 1a one can see a typical snapshot of the magnetic field. It has a double quadruple shape - the inner one at the distance of one beam radius, while the external peak almost twice farther from the laser axis. In this case the run is not self-consistent because the term related to the rotation of the stress tensor in the magnetic field $\overleftrightarrow{S}_B$ and the anisotropic part of the heat flux tensor $\overleftrightarrow{\mathbf{Q}}_{ani}$ were ignored.

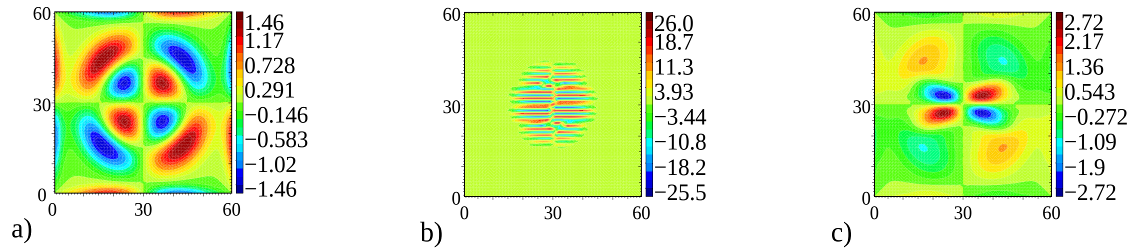

Figure 1: Spatial structure of magnetic field B (in T) in x,y-plane (in μm). a) Time 50 ps, the term $\overleftrightarrow{S}_B$ in the energy equation is suppressed and only the isotropic part of the heat flux tensor is accounted for. In each quadruple, magnetic field rises to 1.46 T. b) Time 15 ps, the term $\overleftrightarrow{S}_B$ is accounted for but the anisotropic part of the heat flux is suppressed. The magnetic field develops a small scale perturbations and explodes, the quadruple shape is lost. c) Time 50 ps, the complete set of equations was solved for the same parameters.

Figure 1b shows magnetic field when the term $\overleftrightarrow{S}_B$ is taken into account. The system looses the stability and develops short wavelength fluctuations of magnetic field growing exponentially in time. This is the Weibel-type instability that is excited because the negative feedback term $\overleftrightarrow{\mathbf{Q}}_{ani}$ has been neglected.

>From equation (2) one can deduce a set of equations for the stress tensor $\partial_t \Pi_{\perp,\wedge} = \pm \omega_{ce} \Pi_{\wedge,\perp}$, where $\omega_{ce} = eB/m_e$. It is obvious that the differential rotation leads to steepening of the anisotropy gradients if the magnetic field is inhomogeneous. In turn, that increases the rate of magnetic field generation – the right hand side of Eq. (1) and consequently the rate of differential rotation.

The perturbation analysis of the coupled equations for the field B (1) and the equation $\partial_t \Pi_\wedge = \omega_{ce} \Pi_\perp$ in the limit of a weak magnetic field demonstrates the instability in the direction perpendicular to the anisotropy axis. In the case where the anisotropic part of the heat flux is neglected in the energy equation, the growth rate γ is proportional to the the perturbation wave number, $\gamma = -\eta k^2 + U_\perp k^2 / m_e \nu_p$, where η is the magnetic resistivity. This formula shows that above some anisotropic level $U_\perp$ all scales become unstable. This fact creates serious difficulties in the numerical solution and it is also in contradiction with the kinetic analysis of the Weibel instability in the semi-collisional regime [2]. The effect of kinetic stabilization of short-wavelength modes can be described in the fluid model by an appropriate diffusion term. This term is $\overleftrightarrow{\mathbf{Q}}_{ani}$, and the stability analysis with this new term indeed gives the short wavelengths cutoff: $\gamma = -\eta k^2 + U_\perp k^2 / m_e (\nu_p + \frac{4}{5} \delta \kappa_{SH} / n_e k^2)$. The value of δ can be chosen in order to match the kinetic theory.

Figure 1c shows the magnetic field obtained for the same parameters at 50 ps with the rotation and diffusion terms taken into account. One can advance the calculation to longer times and to see the effects of nonlinear saturation of the magnetic field generation.

In conclusion, the plasma temperature anisotropy created due the laser heating is an important mechanism of the magnetic field generation under the conditions of present and future laser-plasma interaction experiments. The magnetic fields are generated in short time scale are strong enough to magnetize the heat transport around speckles and correspondingly produce more anisotropic plasma environment. The terms describing the diffusion of the anisotropic parts of the energy tensor are of principal importance for the appropriate account for the non-linear effects and for advancing the numerical calculations in long time scales.

References

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