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Nonlinear collisionless plasma dynamics: the role of small scales and particle correlations

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The continuous Vlasov model is the fundamental physical model used in many space and laboratory plasma regimes. Discrete numerical artificial effects lead intrinsically to non Vlasov plasma regimes. Both numerical and analytical results indicate that the physical mechanism allowing the system to violate the Hamiltonian character of the Vlasov equation and to reach a formally forbidden state can be crucial for the system evolution.

High temperature laboratory plasmas and, even more, rarefied space plasmas are characterized by a typical collisional time scale much longer than the plasma dynamics time scale. As a consequence, these plasmas can be considered in first approximation as collisionless and the dynamics as Hamiltonian. Indeed, the diffusive length scale is many orders of magnitude smaller than any dynamical or kinetic length scale, as for example, in the solar wind where the mean free path is of the order of the dimension of the system (i.e. one astronomical unity) and where non Maxwellian distribution functions are often observed. Nevertheless, on the very long length scales (or low frequencies) a hydrodynamic approach for the modeling of such plasmas has been successfully used in laboratory plasmas and even in the solar wind case where, for example, Alfvén waves have been observed. However, the hydrodynamic approach gives good results only at typical length scales much larger than the kinetic length scales. At such "small" length scales (and high frequencies) the collisionless Vlasov mean field theory is the physical model for the understanding of the processes in which collisions are still neglected, but kinetic effects play now a key role in the plasma dynamics. Such kinetic effects are often crucial to transfer ordered (large scale) energy on small scale fluctuations which, reacting on the particles, push then the system toward an "isotropisation". This process can somewhat replace the role of collisions. However, this picture has been upset by high resolution space observations, where one of the major surprises has been the observation of electrostatic coherent structures on the Debye length scale [1] with corresponding non Maxwellian distribution functions. These structures have been observed in the aurora terrestrial regions and successively also in the solar wind, even in the more quiet regions. Therefore, the theoretical analysis of the generation and evolution of coherent structures and of kinetic processes can be today considered as one of the outstanding problems of plasma physics. As already mentioned, this analysis is based on the Vlasov equation, self consistently coupled to the Poisson equation:

$$\frac{\partial f_a}{\partial t} + v\,\frac{\partial f_a}{\partial x} - \frac{m_e}{m_a}\,\frac{\partial \phi}{\partial x}\,\frac{\partial f_a}{\partial v} = 0; \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \int (f_e - f_p)\; dv; \quad E = -\frac{\partial \phi}{\partial x}; \quad a = e,\, p \tag{2}$$

Here all quantites are normalized to electron characteristic quantities, m_e, ω_{pe}, λ_{De}, $v_{th,e}$, and the electric field to $m_e v_{th,e} \omega_{pe}/e$.

The Vlasov equation, Eq. (1), describes the plasma as a continuous, Hamiltonian collisionless system where the distribution function (hereafter d.f.) is not systematically brought back toward a Maxwellian type equilibrium. This equation, obtained by integrating the Liouville equation for the n-particles Gibbs equation at thermodynamic equilibrium, is based on a mean field theory in which each particle interacts with an average field generated by all plasma particles, while single particles interactions are completely neglected. A fundamental feature of this model is that the d.f. is subjected to strong topological constraints, the invariants, which reduce the degrees of freedom of the system. For example, the d.f. can be transported and roll up in a complex way in the phase space, but different d.f. isolines can never be broken and reconnect. As a result, transitions from *unconnected states* in phase space are forbidden, as for example from a laminar type state (i.e. free streaming) to a vortex type state (i.e. particle trapping).

The Vlasov - Poisson system is mathematically a very complex non linear system of equations, which usually cannot be solved analytically, even in simplified physical problems. As a consequence, the theoretical study of collisionless, kinetic scale, plasma non linear dynamics is today based on (large scale) numerical simulations of the Vlasov equation [2]. Such simulations are by the way a very strong challenge in computational physics which require a careful choice of the numerical schemes to be used depending on the problem of interest and, in general, the use of parallel computers. Nevertheless, there is an intrinsic limit of any numerical approach used to integrate the Vlasov - Poisson system given by the necessity to discretize (in time, space and velocity) the equations on a numerical mesh. This step unavoidably introduces a numerical (diffusive and dispersive) scale length (hereafter ℓ) which, at best, can be considered as comparable to the grid mesh size. The most important physical consequence is that, even by using time advancing sympletic algorithms, due to the discrete sampling of the phase space, the continuous, Hamiltonian character of the equations will be violated as soon as the typical length scale of the dynamically generated fluctuations becomes comparable to ℓ. As an example in Fig. 1 we

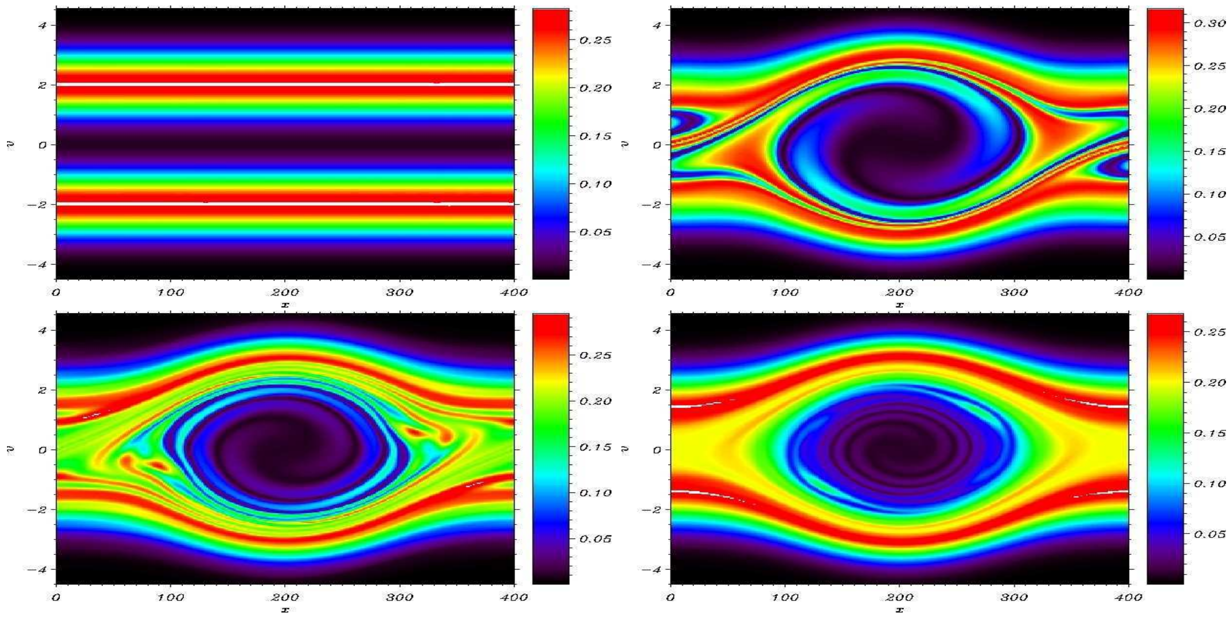

FIG. 1: The phase space electron distribution function at different time instants during the development of the two stream instability. The initial beams have opposite normalized velocity $u_0 = \pm 2$.

show a numerically grid induced *Vlasov forbidden transition* from the initial laminar state, first frame, to a vortex state, last frame, in the case of the well known two stream instability. This situation is similar to that of an ideal plasma where the magnetic field lines frozen-in condition prevents any transition from different (not connected) magnetic energy states.

Recently, by studying the competition between the Landau damping and the particle trapping process (known as the nonlinear Landau damping problem, NLD), we have shown [3] by using different algorithms and/or mesh sizes, that the asymptotic macroscopic Vlasov Poisson state is not independent of ℓ even if such length scale is much shorter than any other non collisional physical length scale of the system, as in our case the Debye length, the thermal velocity, the phase space vortex size, etc. This result is in agreement with a wave particle resonant interaction study concerning the validity of the continuous Vlasov description of a plasma [4].

In order to study the fundamental problem on the possibility that particles correlations, completely neglected in the Vlasov theory, are dynamically generated by the system during evolution, we have introduced in our Vlasov - Poisson code a large number of passive tracers as single charged particles, whose time evolution is determined only by the averaged electric field computed through Vlasov and Poisson equations (on which tracers do not contribute). In this way, we can give a numerical estimation on the formation of correlations between the particles.

As a first step, we have introduced at $t = 0$ $N = N_{tot}$ passive tracers in the two stream simulation shown in Fig. 1. By defining a number of phase space sub-cells with dimensions smaller than the characterist kinetic lengths (λ_D, v_{th}) but larger than the grid size, we calculated the "tracer correlations" by using a spatial averaging technique in order to make a *operative* estimation of the g-plasma parameter, $f_2(z_A, z_B) = f_1(z_A) f_1(z_B)(1+g)$. This technique is described in Refs. [5, 6]. Here we report the results of three different simulations with $N_{tot} = 5\,10^4$, $N_{tot} = 2\,10^5$ and $N_{tot} = 5\,10^5$. We found that, starting from g values linearly varying with the inverse of the passive tracers number, $g(t=0) = 10^{-2};\ 5 \cdot 10^{-3}; 10^{-3}$, the g values calculated nearby the vortex separatrix for a typical correlation length $l_{corr} \sim 0.1 - 0.3\lambda_D$, saturate in all cases at nearly the same value $g(t = t_{fin}) \simeq 0.26$. On the other hand, for larger correlation lengths $l_{corr} \geq \lambda_D$ or far from the separatrix (for example in the vortex or free streaming regions) and for any value of the correlation length, we found $g \ll 1$ at all times.

In summary, when the non linear long time dynamics of a collisionless plasma is investigated numerically, there is a characteristic time (which depends on the numerical scheme and grid used) for which the Vlasov theory is locally violated. By using the passive tracers technique, we found that strong particle correlations are generated around the vortex separatrix where the motion becomes chaotic. These results are a strong indication that a collisional operator, based on a correct description of the microscopic physics, is needed to model small scale plasma dynamics.

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[1] Lacombe et al., Annales Geophysicae **20**, 609 (2002); Mangeney et al., *ibid* **17**, 307 (1999)

[2] A. Mangeney, F. Califano, C. Cavazzoni and P. Travnicek, J. Comput. Phys. **179**, 495 (2002)

[3] L. Galeotti, F. Califano, Phys. Rev. Lett. **95**, 015002 (2005)

[4] M.C. Firpo, Y. Elskens, Phys. Rev. Lett. **84**, 3318 (2000)

[5] L. Galeotti, PhD Thesis, Dip. Fisica, Pisa University http://etd.adm.unipi.it/theses/available/etd-01202006-084723

[6] L. Galeotti et al, in preparation