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Numerical simulation of small-scale fluctuations in space plasmas: compressible turbulence in Hall Magnetohydrodynamics

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Fluctuations on scales of the order of the ion–skin depth $\lambda_i = c/\omega_{pi}$ (ω_{pi} is the plasma frequency) have been commonly observed in a variety of space plasmas, for example in solar wind [1, 2], in the earth magnetosphere [3], and in auroral regions [4]. Recently, near the magnetopause boundary layer (MPBL), Cluster satellites revealed the presence of large–amplitude fluctuations with a well defined anti–correlation between plasma density and magnetic field strength [5, 6]. These structures have been interpreted [5, 6] by the presence of slow–mode coherent waves (solitons) in space plasma, that moves in a direction quasi–perpendicular to the mean magnetic field. Actually the length λ_i in the MPBL is a non–negligible fraction of the boundary layer thickness [7], that is $\Delta/\lambda_i \simeq 1 \div 30$ [5, 7] (Δ is the width of MPBL). So that the usual MHD cannot describe fluctuations on these scales.

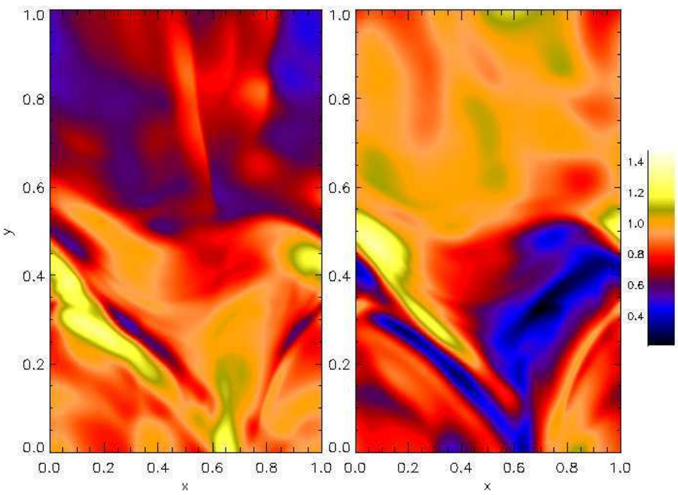

Figure 1: In the left panel is represented the contour plot of density, and on the right the magnetic field intensity. There is a clear evidence of anti–correlation between the two quantities. In the upper region ($y > 0.5$), some shock waves travel in the x direction.

To simulate the small–scale dynamics of the MPBL we numerically solve compressible Hall magnetohydrodynamic (HMHD) equations in a 2.5 D configuration. The HMHD equations, in dimensionless form, reads

$$\begin{aligned}
\frac{\partial n}{\partial t} &= -(\mathbf{V}\cdot\nabla)n - n(\nabla\cdot\mathbf{V}) \\
\frac{\partial \mathbf{V}}{\partial t} &= -(\mathbf{V}\cdot\nabla)\mathbf{V} + \frac{1}{n}\left[(\nabla\times\mathbf{B})\times\mathbf{B} - \frac{\beta}{2}\nabla\cdot\mathbb{P}\right] + \frac{1}{S_\nu}\left[\nabla^2\mathbf{V} + \frac{1}{3}\nabla(\nabla\cdot\mathbf{V})\right] \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla\times\left[\mathbf{V}\times\mathbf{B} - \frac{1}{N_{\lambda_i}\, n}(\nabla\times\mathbf{B})\times\mathbf{B} - \frac{1}{S_\mu}\nabla\times\mathbf{B}\right]
\end{aligned} \tag{1}$$

For simplicity [6] we assume a polytropic closure $\mathbb{P} = n^\gamma \mathbb{I}$ ($\mathbb{I}$ is the unit dyad) with the adiabatic index $\gamma = 5/3$. The parameters S_ν and S_μ are the Reynolds and the Lundquist number respectively. The parameter $N_{\lambda_i} = L_0/\lambda_i$ is the number of times that the large scale length L_0 contains ion skin depth. The plasma parameter β represents the ratio between kinetic and magnetic pressure. In the Eq. (1) n is the mass density, $\mathbf{V}$ and $\mathbf{B}$ the velocity and magnetic field respectively. As usual the magnetic field has been normalized to B_0, the velocity to the Alfvén speed v_A, lengths to L_0 and the time to the Alfvén time $\tau_A = L_0/v_A$.

We simulate a slice of MPBL, that is (x,y) represent coordinates in the equatorial plane where y is the direction across the MPBL. A numerical mesh of 512×528 gridpoints on this plane have been used. The numerical code uses: second–order finite differences to compute spatial derivatives, a second–order Runge–Kutta scheme for time evolution, periodic boundary conditions along the x direction and zero–gradient boundary conditions in the y direction. The divergenceless magnetic field is preserved by a projection–like tecnique [8]. Here we show results obtained by the following set of parameters: $\beta = 1.5$, $N_{\lambda_i} = 25$, $\Delta/\lambda_i = 5$, $S_\nu = S_\mu = 1000$.

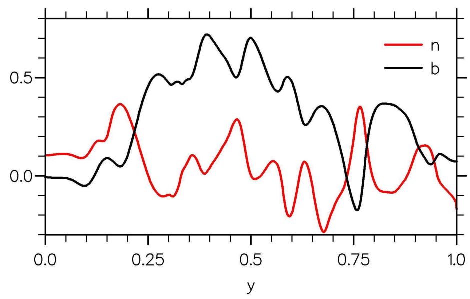

Figure 2: A profile of δn (red line) and δB (black line) obtained performing a cut along y direction for $x = 0.5$ and $t = 18\tau_A$.

The region we want to numerically investigate is highly inhomogeneous, so we use a jump–like functional profile for both the equilibrium density $n_{eq}(y)$, velocity field intensity $V_{x,eq}(y)$ and magnetic field strength. The equilibrium magnetic field $B_{eq}(y)$ has been choosen to satisfy $B_{eq}^2 + \beta n_{eq}^\gamma = const.$, and lies in the (x,z) plane ($B_{z\,eq} = B_{x\,eq} = B_{eq}\sqrt{2}/2$). We perturb the above equilibrium configuration $(n_{eq}, V_{x,eq}, B_{x,eq}, B_{z,eq})$ with a velocity fluctuation $\left[V_y(x,y) = 5\times 10^{-4}\sin(2\pi x)\right]$, and we forced this equilibrium during the simulation.

In the initial stage of simulation there is an exponential growth of Fourier modes indicating the excitation of small-scale fluctuations. At $t \simeq 10\,\tau_A$ fluctuations saturate and a turbulent station-

ary state is reached. In Fig. 1 contour plots for the density and the magnetic field intensity are shown at time $t = 17\ \tau_A$. The plots evidence the presence of an anti–correlation between these two quantities, specially in the high beta region ($y < 0.5$).
In Fig. 2 we report $\delta n(x,y,t) = n(x,y,t) - n_{eq}(y)$ and $\delta B(x,y,t) = |\mathbf{B}(x,y,t)| - |\mathbf{B}_{eq}(y)|$ obtained by performing a cut along a fixed value of x. These structures correspond to small–scales slow magnetosonic fluctuations that are spontaneously generated during the nonlinear evolution, and they can be compared to similar structures observed by Cluster in space plasmas [6].

These strong fluctuations propagate in the direction quasi–perpendicular to the ambient magnetic field, as shown in Fig. 3 and can move toward low-density (high magnetic field) region.
To compare the spectral properties of HMHD with the usual MHD, we drop the Hall term in Eq. (1) and we make a pure compressible MHD simulation. Then we calculate the Fourier coefficients for fields and density, through a Fourier transform in the x direction. In Fig. 4 we report, as a function of m_x (m_x is the wave number in the x direction), the square modulus of Fourier coefficients of fluctuations, namely $\sum_i |\delta u_i(m_x,y,t)|^2$, $\sum_i |\delta b_i(m_x,y,t)|^2$ and $|\delta n(m_x,y,t)|^2$, at a fixed value of y and time. We find a striking difference between the MHD and HMHD, in both cases at large scales the compressive part of fluctuations seems to be scarcely relevant, that is density fluctuations are lower than both velocity and magnetic field fluctuations. At scales $k \simeq k_H$, where the Hall effect take place, the difference is very relevant. In fact, the magnetic intensity decouples from the velocity field, while the density spectrum becomes strongly correlated with the velocity spectrum. The decouplings of the spectra and the enhancement of the relevance of the compressive part corresponds to the spontaneous generation of magnetosonic fluctuations.
To gain some insight into this phenomenon, in Fig. 4 we report the Fourier spectra of the cross–correlations (cross-helicity) between velocity and magnetic field fluctuations. As it can be seen,

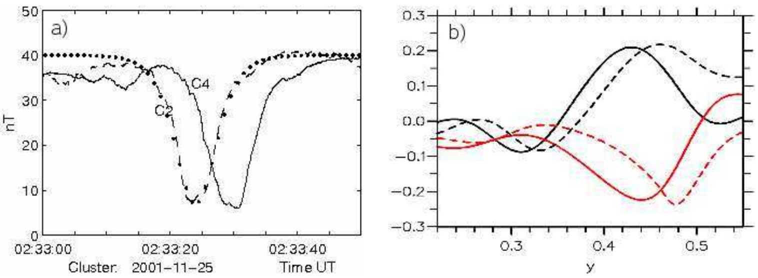

Figure 3: The panel a) represents a magnetic structure detected by two Cluster spacecraft ($C2$, $C4$) with a relative time delay of 6 s which corresponds to a velocity 250 km/s, deduced from the known separation distance between the satellites. In the panel b) we report a single structure coming from numerical simulation, the solid line is taken at $t = 15\tau_A$ and dotted lines at $t = 15.5\tau_A$. Red line represents density and black line represents magnetic field.

in the HMHD case, the strong correlation between $\delta\mathbf{u}$ and $\delta\mathbf{b}$ observed at small wavevectors decays for higher wavevectors k_H^{-1}.

As a conclusion we investigated numerically the time evolution of forced compressible HMHD. We find that high amplitude ($\delta n \simeq 30\%$, $\delta b \simeq 50\%$) and strongly nonlinear fluctuations are spontaneously excited during the nonlinear evolution. These anti–correlated structures can travel in the direction perpendicular to the mean magnetic field, and can moves inside the low beta region [5], as shown in Fig. 3. We conjecture that the observed structures are generated by nonlinear effects, that is they correspond to a magnetosonic turbulence. The energy is carried from larger scale shears to smaller lenghts, where the Hall effect plays a crucial role, breaking the usual features of the nonlinear MHD cascade, i. e. the magnetic field fluctuations strongly decouple from velocity fluctuations.

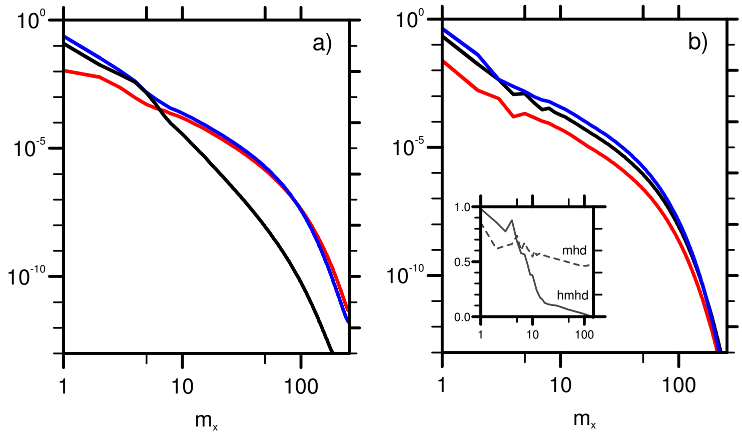

Figure 4: The panel a) represents Fourier spectra of magnetic field (black line), velocity field (blue line) and density (red line) for the HMHD case ($y = 0.8$ and $t = 20\tau_A$). In the panel b) is represented the MHD case. In the inset of panel b) is plotted the cross–helicity spectrum, as a function of m_x, for both Hall MHD (solid line) and MHD (dotted line).

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