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Numerical solution of a reduced model for collisionless magnetic reconnection.

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Magnetic field reconnection is an important process in high temperature magnetically confined plasma. In this process, the magnetic configuration undergoes a topological rearrangement in a relatively short time, during which the magnetic energy is converted into heat and into kinetic flow energy. Typical situations are in tokamak plasma configurations and in solar coronal loops, when strong magnetic fields are present. In the present work, we consider a dissipationless two-dimensional configuration with a strong superimposed homogeneous magnetic field perpendicular to the reconnection plane. In the limit of a small ion gyroradius, this two-dimensional system gives a two-fluid equations model where small scale effects related to the electron temperature and electron inertia are retained, but magnetic curvature effects are neglected.

The pertinent equations.

We consider a 2D configuration with a strong magnetic field in the ignorable z direction, $\vec{B} = B_0 \vec{e}_z + \nabla \psi \times \vec{e}_z$, where B_0 is constant and $\psi(x, y, t)$ is the magnetic flux function. The dimensionless governing equations, normalized to the Alfvén time τ_A and to the equilibrium scale length L_{eq}, are Hamiltonian and can be cast in a Lagrangian invariant form [1,2]:

$$\frac{\partial G_\pm}{\partial t} + [\phi_\pm, G_\pm] = 0; \qquad G_\pm = \psi - d_e^2 \nabla^2 \psi \pm d_e \rho_s \nabla^2 \varphi \qquad (1)$$

The Poisson brackets $[A, B] = \vec{e}_z . \nabla A \times \nabla B$, and the Lagrangian invariants $G_\pm$ are conserved fields advected along the characteristic curves, $x_\pm(t)$:

$$d\vec{x}_\pm(t)/dt = \vec{\upsilon}_\pm(\vec{x}_\pm, t) \ , \qquad \vec{\upsilon}_\pm(\vec{x}_\pm, t) = \vec{e}_z \times \nabla \phi_\pm \qquad (2)$$

where $\phi_\pm = \varphi \pm (\rho_s / d_e)\psi$. d_e is the electron collisionless skin depth and ρ_s is the ion sound gyro radius. The magnetic flux ψ and the plasma stream function φ are given by:

$$\psi - d_e^2 \nabla^2 \psi = (G_+ + G_-)/2 \quad ; \qquad d_e \rho_s \nabla^2 \varphi = (G_+ - G_-)/2 \qquad (3)$$

We apply a method of integration along the characteristics for the numerical solution of Eq(1), similar to what is applied for the numerical solution of the Vlasov equation. To advance Eq.(1) in time, Eq.(2) is solved iteatively to determine the departure point of the characteristics, and the values of $G_\pm$ at these points are calculated by a two-dimensional interpolation using a tensor product of *B*-splines [3,4]. No small scales filtering or dissipation is added to present the code, as is done for instance in [1]. We consider an initial equilibrium $\psi(t=0) = 1/\cosh^2(x) + \delta\psi(x,y)$, and $\delta\psi = -10^{-4}\exp(-x^2/(2d_e^2))\cos y$ is the initial perturbation. $d_e = \rho_s = 0.2$. The equations are integrated numerically in the spatial domain $-2\pi < x < 2\pi$, $0 < y < 2\pi$. The solution of Eq.(1) is followed by a solution of Eq.(3) to determine ψ and φ, and these are used to calculate $\phi_\pm$, to repeat again the integration of Eq.(1). The solution of Eq.(3) is done by Fourier transforming in the periodic y direction, then discretizing the equations in the x direction and solving the resulting tridiagonal system, and then Fourier transforming back. During the evolution of the reconnection process, we see in Fig.(1) in the contours of the magnetic flux a magnetic island generated and growing in the linear phase and early non-linear phase, in which the process exhibit a quasi-explosive behaviour. In the full nonlinear regime, equilibrium is reached, the island growth saturates and remains more or less unchanged. The isocontours of G_+ in Fig.(2) show the formation of a vortex structure. A similar vortex structure is developed also for the invariant G_-, which is advected in the opposite direction with respect to G_+. Asymptotic states for 2D systems showing the formation of vortex structures has been discussed by Knorr [5]. The model preserves parity. If we choose the initial values such that $\psi(-x) = \psi(x)$, and $\varphi(-x) = \varphi(x)$, these relations imply $G_+(-x,y) = G_-(x,y)$, $\phi_+(-x,y) = -\phi_-(x,y)$, which are accurately verified by the code. Fig.(3) shows the stream function φ. Fig.(4) shows a 3D view and a contour plot of the current $J = -\nabla_\perp^2\psi$ at t=40. Note the important peak structure of the current around the x-point, and the small local peaks appearing at small local x-points. The extension of this method to the 3D reduced model [2] for collisionless magnetic reconnection is as follows. The 3D equation :

$$\frac{\partial G_\pm}{\partial t} + [\phi_\pm, G_\pm] = \frac{\partial(\phi_\pm \mp \frac{\rho_s}{d_e} G_\pm)}{\partial z}$$ is splitted:

Step1-Solve for $\Delta t/2$ the equation: $\frac{\partial G_{\pm}}{\partial t} + \left[\phi_{\pm}, G_{\pm}\right] = 0$ using the same method as above.

Step2-Solve for Δt the equation : $\frac{\partial G_{\pm}}{\partial t} = \frac{\partial(\phi_{\pm} \mp \frac{\rho_s}{d_e} G_{\pm})}{\partial z}$; Step3-Repeat Step1-.

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References.
[1] D. Grasso, F. Califano, F. Pegoraro and F. Porcelli, Phys.Rev.Lett. 86,5051(2001).
[2] D. Grasso, D. Borgogno, F. Califano, D. Farina et al. Comp.Phys.Comm. 164,23(2004).
[3] M. Shoucri, H. Gerhauser and K.-H. Finken, Comp.Phys.Comm.154,65(2003).
[4] E. Pohn, M. Shoucri and G. Kamelander, Comp.Phys.Comm. 166,81(2005).
[5] G. Knorr, Plasma Phys. 38,212(1980); J.Plasma Phys. 41,157(1989).

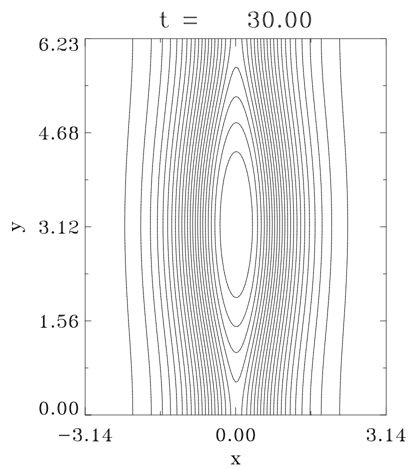

Fig.1 Magnetic flux ψ

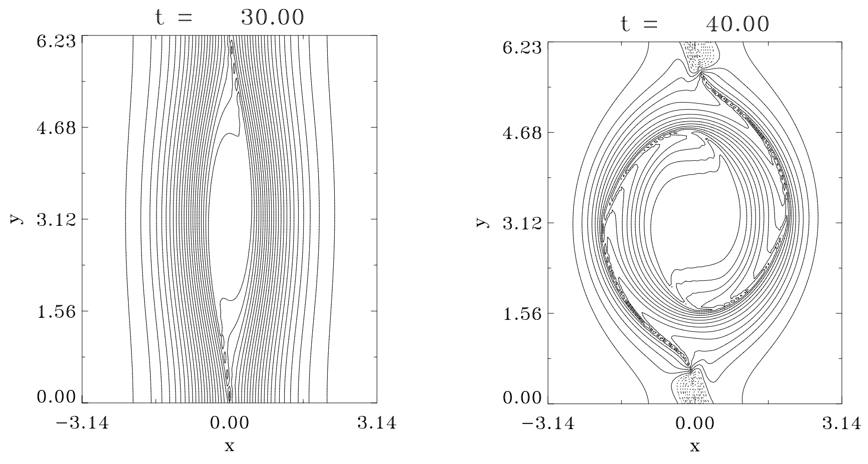

Fig.2 G_{+}

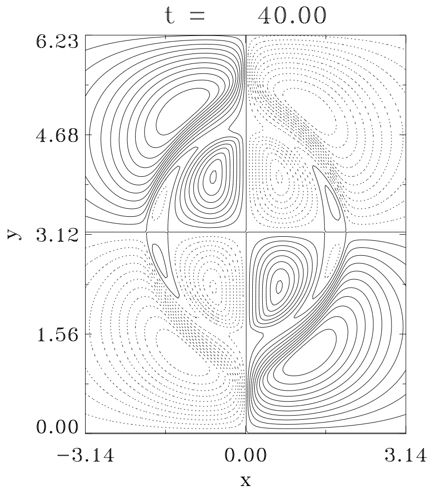

Fig3 Stream function φ

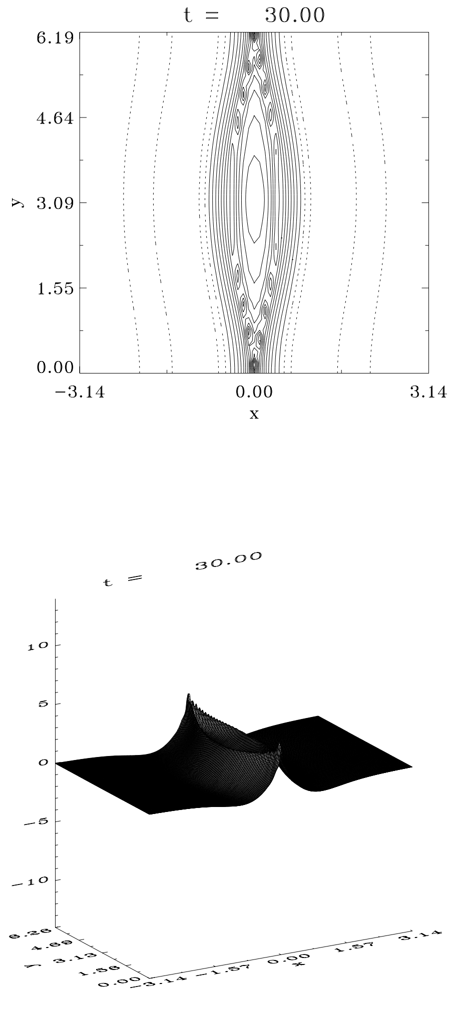

Fig.4 Current J