

Characterisation of edge turbulence in L-mode plasmas in the Mega Amp Spherical Tokamak

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Introduction. We present statistical analysis of L-mode edge plasma fluctuations measured by a reciprocating Langmuir probe in the Mega Amp Spherical Tokamak (MAST) [1]. Two magnetic configurations are explored: unbalanced double null (signal UDN-14222) and connected double null (signal CDN-14219). Other parameters, such as plasma shape and current $I_p \approx 700\text{kA}$, were nearly identical for both signals. Figure shows time series of the ion saturation current, I_{sat} , for both cases. Studied intervals were selected so that the distance from the Langmuir probe to the plasma edge was approximately constant and the same for both shots. The datasets contain 10000 and 30000 samples for signals 14219 and 14222 respectively; sampling rate is 500kHz. These time series are bursty, asymmetric and contain structures on many temporal scales.

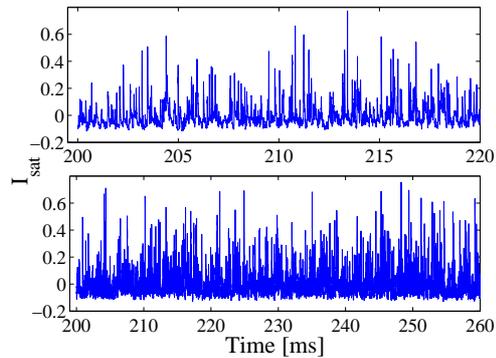


Figure 1: Time series of I_{sat} for UDN-14219 (top) and CDN-14222 (bottom).

There is increasing evidence (see, for example, [2]) that plasma edge turbulence may have statistical properties that are universal and invariant. This invariance is reflected in quantities such as the Hurst exponent and the probability distribution function of the fluctuations in different magnetic confinement systems. It may be due in part to the existence of long-range correlations giving rise to selfsimilar behaviour, and presumably arises from underlying physical processes that are generic. Identification of these processes thus rests on data analysis, comparison with numerical models, comparison with other contemporaneous phenomena (for example, density fluctuations), and the application of new statistical techniques offering additional perspectives.

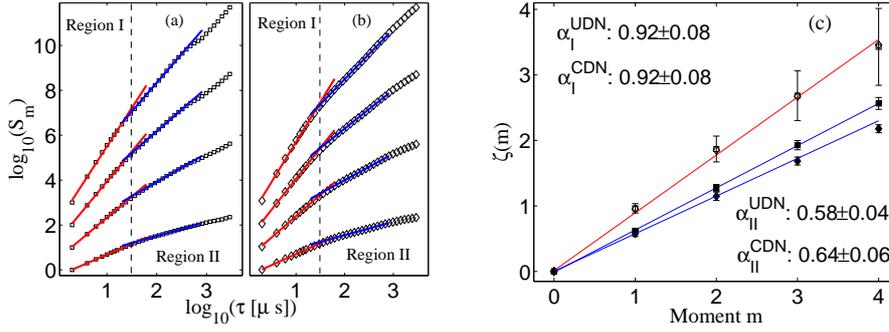


Figure 2: Structure functions of orders $1 \leq m \leq 4$ for (a) signal UDN-14219 and (b) CDN-14222. (c) Scaling exponents $\zeta(m)$ derived from the structure functions of I_{sat} : open squares-UDN in Region I, open diamonds-CDN in Region I, filled squares-UDN in Region II and filled diamonds-CDN in Region II.

The present paper builds on pioneering analyses of edge turbulence in MAST, notably by Antar *et al.* [3] and by Dudson *et al.* [4]. In particular, the latter study investigated a small number of L-mode and dithering H-mode plasma datasets, and found apparently significant differences in Hurst exponent values, in probability density function structure, and in rescaling properties. Here we broaden the analysis by using a fresh set of L-mode data, and apply a variety of model-independent methods ranging from structure functions to probability density function models.

Data analysis. We consider ion saturation signals as stochastic increments given on the temporal scale $\tau_{min} = 2\mu s$, where τ_{min} is the sampling time interval between consecutive measurements. Fluctuations on longer temporal scales are readily obtained by summing the original signal across a window of width τ [4] such that $\delta x(t, \tau) = \sum_{t'=t}^{t+\tau} I_{sat}(t')$. We analyse statistical properties of the higher order moments for these fluctuations using generalised structure functions $S_m(\tau) \equiv \langle |\delta x(t, \tau)|^m \rangle$, where m can be any real number, not necessarily positive. If S_m exhibits scaling with respect to τ then $S_m \propto \tau^{\zeta(m)}$ for some $\zeta(m)$. In this case a log-log plot of S_m versus τ yields a straight line for each m , whose gradient is $\zeta(m)$. Generally, $\zeta(m)$ can be a nonlinear function of order m , however if $\zeta(m) = \alpha m$ (α constant) then the time series is self-similar with single scaling exponent α . We also consider models derived from extreme statistics, namely Fréchet and Gumbel distributions [5], and obtain reasonable fits of these curves to the observed PDFs on different temporal scales.

Results. Previous studies of similar MAST data [4] revealed a dual scaling regime, with the kink in scaling curves occurring for $\tau \approx 60\mu s$. The present structure function analysis confirms this dual scaling. Figure 2(a,b) presents the first four moments of the I_{sat} data for different temporal scales τ for both datasets. It shows that there are two distinct regions of scaling: Region I with a steeper slope extending from $\tau \approx 2\mu s$ to $\tau \approx 60\mu s$ and Region II with a shallower slope

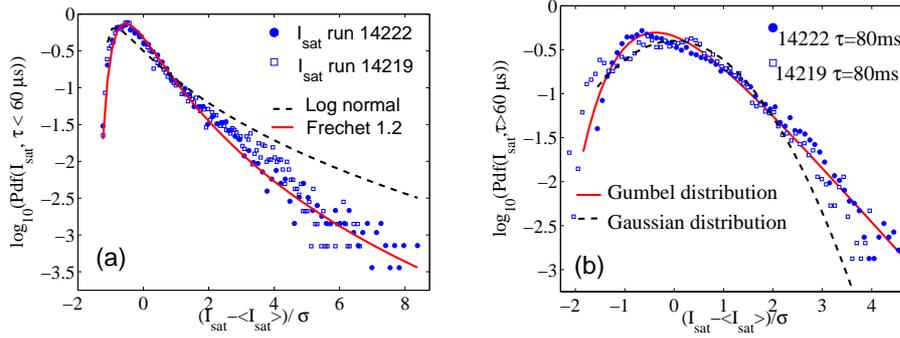


Figure 3: (a) Normalised PDFs for both poloidal geometries and for temporal scales $\tau < 60\mu s$. Lines correspond to log-normal and Fréchet distributions. (b) Normalised PDFs for scales $\tau > 60\mu s$. Lines correspond to normal and Gumbel distributions.

between scales $\tau \approx 60\mu s$ and $\tau \approx 400\mu s$.

Having established that the dual scaling regime applies to both poloidal configurations in MAST, we now examine the functional form of scaling exponents $\zeta(m)$. Figure 2(c) presents these exponents for moments with order m ranging up to $m = 4$. The error bars display standard regression errors of the linear trends fitted to each scaling region. In all cases the scaling exponents can be approximated by the linear relation $\zeta(m) = \alpha m$ with constant parameter α and a constraint $\zeta(0) = 0$. It is known that such scaling may lead to a stochastic model of fluctuations [6]. Both configurations exhibit identical scaling in Region I, suggesting a universal behaviour on temporal scales $\tau < 60\mu s$. The scaling exponents in Region II differ slightly, however, with the UDN configuration exhibiting higher departure from the Gaussian exponent value of 0.5.

Figure 3(a,b) shows PDFs normalised to their standard deviation for both poloidal geometries and for temporal scales $\tau \approx 10\mu s$ and $\tau \approx 80\mu s$ respectively. Thick lines in these figures represent different classes of known distributions with zero mean and standard deviations of 1. For example, log-normal and Fréchet [5], have been tested for temporal scales $\tau < 60\mu s$. These choices are motivated by the general shape of the observed I_{sat} distribution, notably their asymmetry with respect to peak and their heavy tails. For μ and σ derived from the experimental data, the log-normal distribution departs significantly from the observed curve for large fluctuations. Next, we consider a Fréchet distribution defined by $P_{Fr}(y, a) = a \exp(-y^{-a}) / (y^{1+a})$, where a is a single parameter which characterises the turbulence through the shape and scale of its PDF, and is taken here as 1.2. Figure 3(a) shows remarkably good agreement between this P_{Fr} and the experimental PDF. A Fréchet distribution is a limiting case of the probability distribution of maximum samples drawn from an ensemble z_i , with a tail that decays as a power law

$z^{-(a+1)}$. The question thus arises: why is the I_{sat} signal dominated by the largest events? Two general scenarios can be considered. First, it may be intrinsic to the Langmuir probe measurements. Measurements based on the saturation principle favour large events and may operate as a selection process. Second, the underlying plasma turbulence may modulate the I_{sat} signal in such a way that the dominant "patches of activity" contribute most to the observed fluctuations.

The PDFs on temporal scales $\tau > 60\mu\text{s}$ are less asymmetric and do not have pronounced heavy tails. Thick lines in Fig.3(b) show two functional forms of the PDF. A normal distribution does not capture negative fluctuations and departs visibly from the measured PDF for fluctuations larger than 3σ . We therefore consider a distribution from the extreme statistics family, the Gumbel distribution [5] $P_{Gm}(u, a) = k \exp(-u - e^{-u})$, where $u = a(y - b)$. Like a Fréchet distribution, a Gumbel curve is a limit for the PDF of largest events drawn from the underlying ensemble whose PDF decays faster than a power law [5]; a defines shape and scale, and b location. We find that values $k = 1.82$, $b = 0.375$ and $a = 1.3$ give a good fit to the experimental curve, thus providing a few-parameter characterisation of the turbulence PDF.

Conclusions. A broad range of quantitative results and physical inferences have been obtained. The structure function analysis confirms the dual scaling regime [4] of the I_{sat} measurements with temporal scale $\tau \approx 60\mu\text{s}$ marking the transition between different scaling. The common value of the scaling exponents on temporal scales $\tau < 60\mu\text{s}$, for UDN and CDN configurations, suggests a universal behaviour within this region. We have demonstrated that the Fréchet and Gumbel distributions give a good approximation of the observed PDFs for temporal scales $\tau < 60\mu\text{s}$ and $\tau > 60\mu\text{s}$ respectively. Functional forms of these PDFs could be used to model plasma transport coefficients via a Fokker-Planck equation. Such an approach could provide a bridge between statistical analysis and a kinetic description of the plasma.

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