

Local statistical properties of electrostatic fluctuations in the TORPEX experiment

B. Labit, A. Fasoli, A. Diallo, I. Furno, S.H. Müller, G. Plyushchev, M. Podestà and F.M. Poli

*Centre de Recherches en Physique des Plasmas (CRPP), Association EURATOM-Confédération Suisse,
École Polytechnique Fédérale de Lausanne (EPFL), 1015 Lausanne, Switzerland.*

Introduction A statistical description of a complex system characterized by fluctuations and turbulence can be a good approach for a better understanding of it [1]. For instance, Self-Organized Critical systems are characterized by intermittent fluctuations and so by very skewed probability density functions (pdfs) [2]. In plasma turbulence and in particular in the Scrape-Off Layer of fusion plasma, the existence of an universal probability density function that can describe the measured density fluctuations has been emphasized in various machines and experimental conditions ([3] and references therein). The main purpose of this work is to test if the plasma density fluctuations can be described by a known analytical distribution as a first step in the effort of establishing a link between statistical properties, spectral features and spatio-temporal evolution of the fluctuations. We observe that in a specific plasma configuration and for a class of fluctuations, such link can be established.

Experimental setup The results presented in this paper are based on experiments conducted on the toroidal device TORPEX ($R = 1\text{m}$, $a = 0.2\text{m}$). The plasma is confined by a helical magnetic field resulting from a toroidal component $B_T = 800\text{G}$ and a vertical component $B_z = 10\text{G}$. Argon plasmas are produced by microwaves ($P_{RF} = 1\text{kW}$) at $f = 2.45\text{GHz}$, in the electron cyclotron frequency range. Typical densities, temperatures and plasma potentials, measured by ~ 200 electrostatic Langmuir probes, are $n \leq 10^{17}\text{m}^{-3}$, $T_e \sim 5\text{eV}$ and $V_{pl} \leq 20\text{V}$. In these experiments, the neutral gas flux that is injected continuously in the vacuum chamber is varied systematically on a shot-to-shot basis. The resulting Argon pressure range is $p_{gas} \in [1.0, 10] \times 10^{-5}$ mbar. The main diagnostic used here is HEXTIP, a hexagonal array of 86 Langmuir probes that covers almost the whole poloidal cross-section.

Analysis of experimental data To obtain the distribution of the plasma density across the plasma cross-section, the ion saturation current I_{sat} measured by each tip of HEXTIP is binned. Without an *a priori* idea on the pdf shape, a good compromise between resolution in the binned quantity and statistics is given by choosing a number of bins $N_{bin} = \mathcal{O}(\sqrt{N})$ (N is the number of points in the time series). As the binning introduces some noise on the distribution, we choose to deal with a smoothed pdf, with a smoothing parameter fixed to $\sqrt{N}/10$. The smoothed distributions, f^{smooth} , can be separated into two families: single-humped (Fig. 1(b)) and double-humped (Fig. 1(c)). Fig. 1(a) illustrates an example obtained for the lowest gas pres-

Distribution	Analytical expression
Normal	$f_n(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$
Gumbel	$f_{gu}(x; \mu, \sigma) = \frac{\pi}{\sqrt{6}\sigma} \exp[xG - e^{xG}]$ with $x_G \equiv -\frac{\pi}{\sqrt{6}} \frac{x-\mu}{\sigma} - 0.577$
Gamma	$f_{ga}(x; \mu, \sigma) = \frac{A^2}{\mu \Gamma(A^2)} \left(\frac{A^2 x}{\mu}\right)^{A^2-1} \exp\left(-\frac{A^2 x}{\mu}\right)$ with $A \equiv \mu/\sigma$
Log-normal	$f_{lgn}(x; \mu, \sigma) = \frac{1}{x\lambda\sqrt{2\pi}} \exp\left(-\frac{\log(x/(\mu\sqrt{\lambda}))^2}{\log(\lambda^2)}\right)$ with $\lambda = 1+A^{-2}$
Extreme-value	$f_{exv}(x; \mu, \sigma) = \frac{1}{\sigma} \exp\left(\frac{x-\mu}{\sigma}\right) \exp\left[-\exp\left(\frac{x-\mu}{\sigma}\right)\right]$

Table 1: Analytical expressions of the 5 tested single-humped distributions. Parameters μ and σ are the mean value and the standard deviation of the random variable x .

sure. Squares and circles identify double-humped and single-humped distributions, respectively.

A doubled-humped distribution can be due to large amplitude coherent modes.

This is confirmed by the correlation between the presence of a double-humped distribution and a large ratio between the energy contained in the coherent modes to the total energy (colored hexagons). Coherent modes have been identified from the statistical frequency distribution computed from the spectra of the 86 tips of HEXTIP [5]. The range of frequency over which the spectra are integrated to estimate the energy is given by the full-width at half-maximum of the distribution.

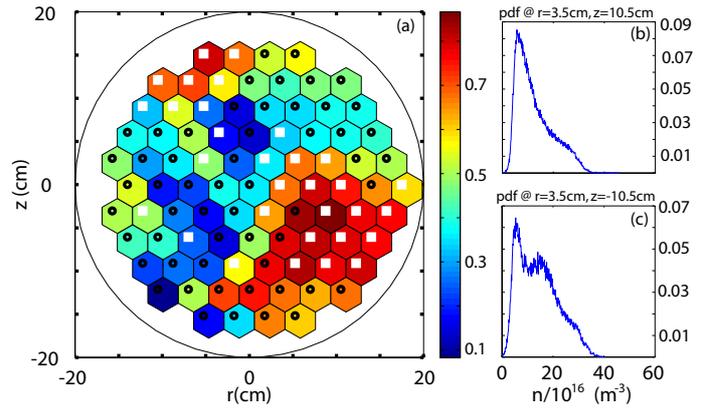


Figure 1: (a) Localization of the double-humped distributions (\square) and single-humped ones (\circ) for the lowest neutral gas pressure. Color-scale indicates the ratio between energy in the coherent modes to the total energy. (b)-(c) Example of both types of distribution.

The range of frequency over which the spectra are integrated to estimate the energy is given by the full-width at half-maximum of the distribution.

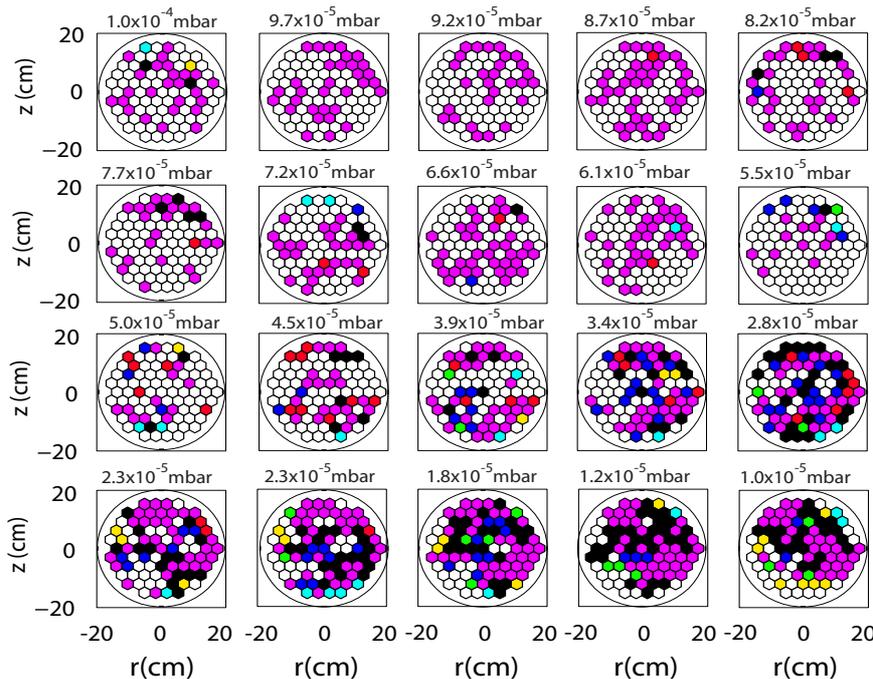


Figure 2: Result of the test of analytical distributions for the pressure scan (p_{gas} decreases from top to bottom and from left to right). The color coding is the following: Normal (blue), Gumbel (cyan), Gamma (green), Log-normal (yellow), Extreme-value (red), Double-humped (magenta), none fit (black), several fit (white)

Having identified the double-humped probability density functions, we now focus only on the single-humped pdfs. Five analytical functions (Table 1) are considered for describing the experimental distributions. All of them have two free parameters, the mean value μ and the variance σ^2 and they are normalized in such a way that $\int_{-\infty}^{+\infty} f(x)dx = 1$. To determine which distribution is the best to fit the experimental data, two different approaches are proposed: to insert the experimental parameters μ_{exp} and σ_{exp} in the analytical expressions or to let them free to evolve in order to minimize the residual. The quality of a fit is estimated by comparing the different residuals: $s = \sum_{i=1}^N [f^{smooth}(x_i) - f(x_i)]^2$, $s_{exp} = \sum_{i=1}^N [f^{smooth}(x_i) - f^{exp}(x_i)]^2$ and $s_{opt} = \sum_{i=1}^N [f^{smooth}(x_i) - f^{opt}(x_i)]^2$ where f the binned distribution from experimental signal, f^{exp} an analytical distribution with μ_{exp} and σ_{exp} and f^{opt} , an analytical distribution with the optimal mean value and variance. A good fit corresponds to $s_{exp,opt} \leq s$.

Results and discussion The analysed plasmas are characterized by fluctuations in the range $5\% \leq \tilde{n}/n \leq 60\%$, depending on the neutral gas pressure and the position in the poloidal cross-section. In the pressure gradient region, low frequency modes ($1 - 8kHz$) are observed and associated with a weak drift-wave turbulence. The results of the present analysis are summarized on Fig. 2 for the case with μ_{opt} and σ_{opt} . For each gas pressure and each position in the cross-section, the color coding indicates which pdf yields the best fit of the experimental distribution. For all the pressure values, double-humped pdf (magenta hexagons) are present,

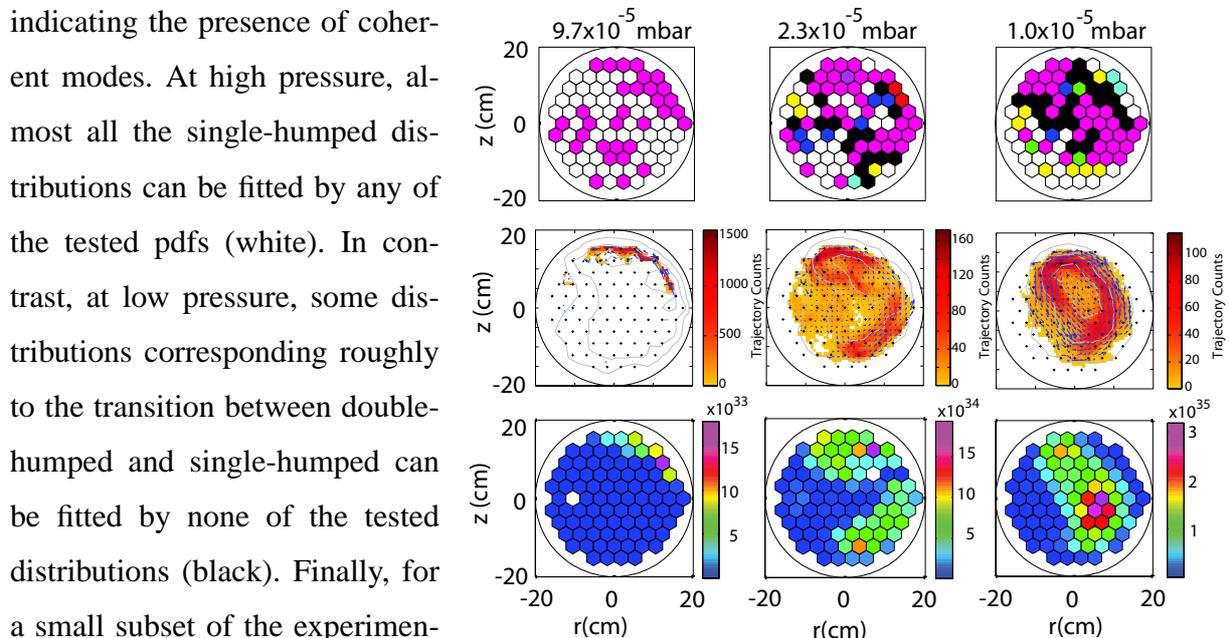


Figure 3: Best fit (top), trajectories and mode amplitude (bottom) for a high (left), an intermediate and a low pressure (right)

indicating the presence of coherent modes. At high pressure, almost all the single-humped distributions can be fitted by any of the tested pdfs (white). In contrast, at low pressure, some distributions corresponding roughly to the transition between double-humped and single-humped can be fitted by none of the tested distributions (black). Finally, for a small subset of the experimental data, we found that only one tested pdf yields a good fit. In this case, one can conclude that none of the analytical functions is predominantly present compared to the others and that no particular spatial localisation or particular pressure is correlated with a given analyti-

cal distribution. On Fig. 3 we represent for three different pressures, the spatial localisation of the best fitting function, the trajectories of the fluctuation structures and the amplitude of the most frequent modes. Structures are defined as spatial regions where the density deviation from average exceeds a threshold value. By following these structures in time, trajectories can be identified [6]. It is clear that the modes with large amplitude and the trajectories are located where the experimental time series are characterized by a double-humped distribution. It seems that the situation is different from what is observed in the SOL of tokamaks. On Fig. 4, the experimental skewness and kurtosis are plotted as a function of the parameter $A \equiv \mu/\sigma$ for different pressure and different radial locations. This plot can be compared with Fig. 3 of Ref. [3] for the TCV tokamak. Although the range of variation of A is much larger in TORPEX than in the SOL of the TCV tokamak, it corresponds to lower fluctuations levels (large A). Despite the experimental skewness and kurtosis are close to the values associated with a Normal distribution, several distributions can fit the experimental data because the analytical pdfs tend to be identical in this range of A . An unambiguous discrimination between distributions is only possible for larger fluctuations ($A \leq 5$) which is rarely the case in the analysed plasmas.

Conclusions For the analysed range of parameters, only a small subset of the density fluctuations time series can be represented by only one single-humped analytical pdf and when this is verified, there is no evidence of an "universal" distribution. The same kind of analysis is under way for density fluctuations characterized by a large turbulent spectrum. The double-humped distributions are localised at the same position as the trajectories and the largest amplitude of the coherent modes. These results represent a first attempt of linking spectral features, spatio-temporal evolution and statistical properties of fluctuations in a toroidal plasma.

This work is partly funded by the Fonds National Suisse de la Recherche Scientifique

References

- [1] U. Frisch, *Turbulence, The Legacy of A. N. Kolmogorov*, Cambridge University Press, (1995)
- [2] P. Bak, *et al*, Phys. Rev. Lett., **59**, 381, (1987)
- [3] J. P. Graves, *et al*, Plasma Phys. Control. Fusion, **47**,L1-L9, (2005)
- [4] A. Fasoli, B. Labit, M. McGrath *et al*, Phys. Plasmas **13**, 055903 (2006)
- [5] F.M. Poli, *et al*, *Experimental characterization of drift-interchange instabilities in a simple toroidal plasma*, sub. to Phys. Plasmas.
- [6] S.H. Müller, *et al*, *Real space statistical characterization of turbulence...*, P1.044, this conference.

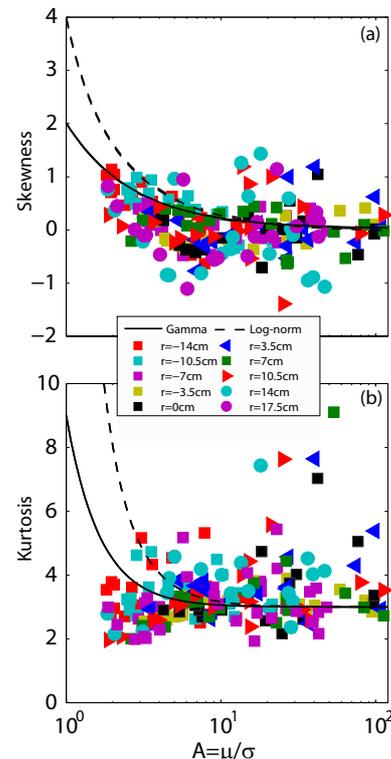


Figure 4: Skewness (a) and Kurtosis (a) as a function of parameter A for the eleven positions at $z = 0\text{cm}$. The variation for a Gamma and a Log-normal distributions are also plotted.