The Role of Magnetic Shear for Zonal Flow Generation and Transport Barriers

J. Anderson[1], H. Nordman[2], Y. Kishimoto[1]

[1] Department of Fundamental Energy Science, Kyoto University, Uji, Gokasho, Kyoto, 611-0011 Japan

[2] Radio and Space Science, Chalmers University of Technology, 412 96 Goteborg, Sweden

The role of magnetic shear for zonal flow generation by ion-temperature-gradient (ITG-) and trapped electron (TE-) mode turbulence is studied analytically using fluid descriptions. The scaling of the zonal flow (ZF) growth rate with magnetic shear is examined and compared with linear growth rates for typical tokamak parameter values. The results indicate that large levels of ZF are obtained in regions of negative magnetic shear, in particular for ZF driven by TE mode turbulence. The strong magnetic shear scaling obtained for TE mode driven zonal flows originates from the bounce average of the electron magnetic drifts.

The study of turbulence suppression and generation in magnetically confined fusion plasma research has attracted much interest recently. It is well established that the interaction between small scale turbulence and zonal flows, which are radially localized flows propagating mainly in the poloidal direction, can reduce the radial transport by shearing the eddies of the driving background turbulence. The purpose of the present paper is to extend previous analytical studies [1]-[2] to investigate the effects of magnetic shear on the zonal flow generation by ion-temperature-gradient (ITG-) and trapped electron (TE-) mode turbulence, respectively. The ITG- and TE-mode driven cases will be considered separately and the corresponding ZF generation by ITG and TE mode turbulence will be calculated and compared. For the ITG mode description, a reduced version of the standard Weiland model [3] is utilized. In this formulation the ITG model includes the ion continuity and the ion temperature equations, neglecting the influence of parallel ion momentum and electromagnetic effects, while the electrons are assumed to be Boltzmann distributed. The TEM model is based on the bounce averaged electron continuity and electron temperature equations for the trapped electron fluid [4]. The ion and trapped electron mode fluid models are fairly symmetric except for the FLR terms which are included in the ion fluid model and the bounce average which is performed on the electron quantities. The zonal flow evolution is treated by the vorticity equation. The generation of ZF by ITG- and TE-modes are studied

separately for $\eta_i \gg \eta_e$ (ITG dominated case) and $\eta_e \gg \eta_i$ (TEM dominated case) respectively. In these limits it is assumed that the electron and ion responses are decoupled and hence the effects of ion perturbations on the TE mode are neglected (and vice versa). The previous work on zonal flow generation by ITG and TE modes is here extended to include the magnetic shear dependence through the magnetic drift frequencies and the perpendicular wave vector. The effect of magnetic shear enters through the averaged magnetic drift frequency for the electrons and ions. The ion fluid model is averaged using an approximate eigenmode function resulting in an averaged magnetic drift frequency: $\psi = \frac{1}{3\pi}(1 + \cos\theta)$ and $|\theta| < \pi$, giving [5]

$$\langle \omega_{Di} \rangle \propto \varepsilon_n g_i = \varepsilon_n (\frac{2}{3} + \frac{5}{9}s - \varepsilon)$$

$$k_\perp^2 \propto (1 - 2.5s^s + \frac{\pi^2}{3}s^2).$$

Here s is the magnetic shear $\varepsilon_n = \frac{2L_n}{R}$ and $\varepsilon = \frac{a}{R}$ is the inverse aspect ratio parameter. The electron fluid equations are bounce averaged and the electron magnetic drift is replaced by the precession frequency of trapped electrons $\omega_{De} = \omega_{De0} g_e$ where $\omega_{De0} = \frac{2k_\theta \rho_s c_s}{R}$ and

$g_e = \frac{1}{4} + \frac{2}{3}s$ [4]. The original formulation of the Weiland model for deeply trapped electrons is recovered for s = 9/8 in this model. The zonal flow dispersion relations are found using the vorticity equation and combining this with the wave kinetic equation as an additional relation between the background turbulence and the zonal flow. In the case of zonal flows (Ω) generated from ITG mode turbulence the dispersion relation is [1]

$$(\Omega - q_x v_{gx})^2 = -q_x^2 (1 + \frac{1}{\tau} + \frac{\delta}{\tau})|\tilde{\phi}_k|^2$$

and in the TEM generation case we obtain [2]

$$\Omega = iq_x k_y \sqrt{|\tilde{\phi}_k|^2}.$$

Here, Ω is the zonal flow growth rate, k_x and k_y is the wave numbers for the background turbulence, q_x is the wavenumber of the zonal flow, v_{gx} is the group velocity for the drift wave. The δ is a wave number independent factor given by

$$\delta = \frac{\Delta_k k_y}{\Delta_k^2 + \gamma_k^2}(\eta_i - \frac{2}{3}(1+\frac{1}{\tau})\varepsilon_n g_i)$$

$$\Delta = \frac{k_y}{2}(1 - \varepsilon_n g_i + \frac{4}{3\tau}\varepsilon_n g_i)$$

Hence, the zonal flow growth rate scales as $\Omega \propto \left|\tilde{\phi}_k\right|$. Proceeding from here, several different

assumptions are possible on the saturation level for the background turbulence in absence of

zonal flows. In the present work only the mixing length saturation level ($\tilde{T} = \frac{1}{k_x L_{Tj}}$) is

considered. The dispersion relations for the zonal flow (Ω) is solved numerically. Special

attention is given to the magnetic shear dependency of the zonal flows generated from ITG

mode and TEM turbulence, respectively. In Figure 1 (below), the magnetic shear scaling of

the zonal flow growth rate driven by ITG modes (normalized to the linear ITG growth rate) is

displayed. The parameters are $k_x = k_y = 0.3$, $q_x = 0.3$, $\tau = 1.0$ and $\varepsilon = 0$. In Figure 1a the

other parameters are $\eta_e = 0$ (suppressing the TE mode), $\eta_i = 4.0$, $f_t = 0$ (fraction of trapped

electrons) and ε_n is given by $\varepsilon_n = 1.5$ (rings), $\varepsilon_n = 1.0$ (asterisk) and $\varepsilon_n = 0.5$ (diamond).

The corresponding result for the TEM driven case is shown in Figure 2 (below) for $\eta_i = 0$

(suppressing the ITG mode), $\eta_e = 4.0$ and $f_t = 0.5$. The parameter ε_n is given by $\varepsilon_n = 2.0$

(plus), $\varepsilon_n = 1.5$ (rings) and $\varepsilon_n = 1.0$ (asterisk). For sufficiently large negative shear the

linear TEM is stabilized due to the change of sign of the precession frequency. For the zonal

flows generated by TEM background turbulence there is a significant increase in growth rate

in the weak and negative shear regions where $\gamma_{ZF} \gg \gamma_{TEM}$. For zonal flows generated by ITG

turbulence on the other hand, only a modest increase in the corresponding region is found.

For positive magnetic shear, the ZF scaling with magnetic shear is rather weak. In general we

obtain $\gamma_{ZFITG} > \gamma_{ZFTEM}$ in this region. We have verified that the ZF growth scales weakly with

the other parameters. In summary, the results indicate that the effects of magnetic shear for

ZF generation are significantly different for ITG- and TE-mode driven cases. This difference

is related to the different shear dependence of the ion and electron magnetic drift frequency

respectively. Hence, for the TE mode dominated case, a weak/negative magnetic shear

results in a large zonal flow growth rate ($\gamma_{ZF} \gg \gamma_{TEM}$). This may result in a strong

stabilization of the turbulence. For the ITG dominated case on the other hand, the results

indicate that the effects of magnetic shear on the zonal flow growth rate is rather weak. Hence,

additional effects of negative magnetic shear, not included in the present study, may be needed in order to stabilize the turbulence. This observation may have some relevance for the observed difference in the dynamics of electron and ion transport barriers in tokamak plasmas. More work, including a treatment of the coupled ITG/TE mode system using more realistic models, is needed in order to confirm this. This is left for future work.

Figure 1

Figure 2
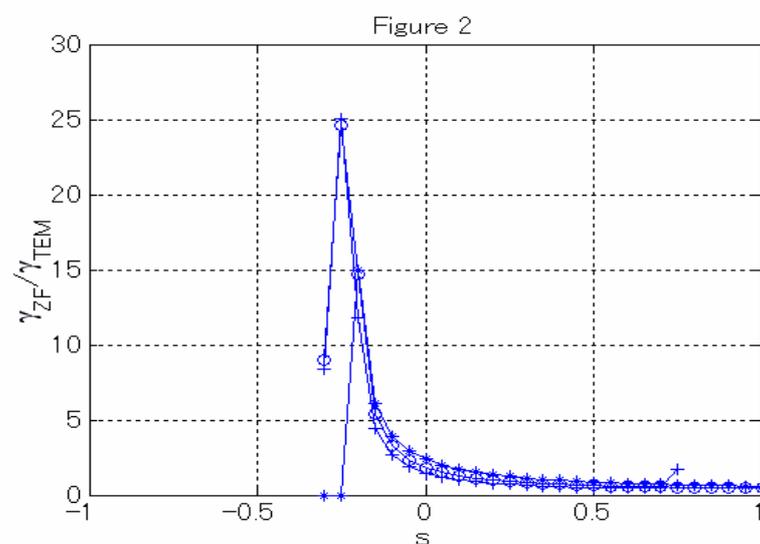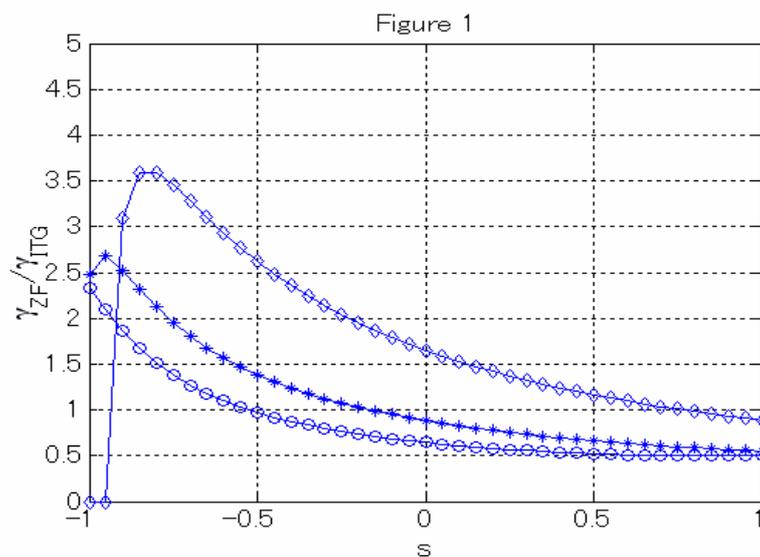
1. Anderson, J., Nordman, H., Singh, R. *et al* 2002 Phys. Plasmas **9**, 4500.

2. Anderson, J., Nordman, H., Singh, R. *et al* 2006 Plasma Phys. Contr. Fusion **48**, 651.

3. Weiland, J. 2000 Collective Modes in Inhomogeneous Plasmas, Kinetic and Advanced Fluid Theory (IOP Publishing Bristol) 115.

4. Garbet, X., Garzotti, L., Mantica, P. *et al* 2003 Phys. Rev. Lett. **91**, 035001-1.

5. Nordman, H., Jhowry, B. and Weiland, J. 1993 Phys. Fluids B **5**, 3465.