

## Lattice kinetic schemes in toroidal geometry

G. Breyiannis, D. Valougeorgis

*Association EURATOM - Hellenic Republic*

*Department of Mechanical and Industrial Engineering*

*University of Thessaly, Volos 38334, Greece*

### Abstract

Lattice kinetic schemes has been consistently developed for the last 15 years as a tool to tackle complicated problems from a mesoscopic perspective. The system is described by a velocity distribution function, which follows a BGK kinetic type equation and is evolved under specific constrains in order to ensure a desired macroscopic behavior. This mesoscale scheme has been recently extended to MHD configurations with noticeable success.

In the present work the in-house 3D MHD lattice kinetic code is properly modified to simulate dissipative flows in a toroidal geometry. The evolution of the MHD field is followed in time via the aforementioned lattice kinetic solver and numerical results are reported for space dependent and overall quantities.

### Formulation

The numerical implementation of the method is based on the discretized Boltzmann equation with a BGK formulation of the collision term (LBGK), which takes the form

$$\partial_t f_i + \xi_i \cdot \nabla f_i = -\frac{1}{\tau} (f_i - f_i^{(0)}) \quad (1)$$

where  $f_i = f(\mathbf{x}, \xi_i, t)$ , with  $\mathbf{x}$  the spatial vector,  $\xi_i$  the microscopic velocity set chosen,  $t$  the time and  $\tau$  the relaxation time, all in dimensionless quantities. For the isothermal case, the equilibrium distribution function is given by a low Mach number series expansion of the Maxwellian as

$$f_i^{(0)} = \rho w_i \left[ 1 + \frac{\xi_i \cdot \mathbf{u}}{\theta} + \frac{(\xi_i \cdot \mathbf{u})^2}{2\theta^2} - \frac{\mathbf{u} \cdot \mathbf{u}}{2\theta} \right], \quad (2)$$

with the weighting factors  $w_i$  depending on the lattice and  $\theta = c_s^2$ . In 3D space, there are several lattice models that can be used for the hydrodynamic part of the problem, utilizing 15, 19 or 27 discrete velocity vectors. We chose to perform our simulations with the 19-velocity model (Figure 1a) incorporating  $c_s = c/\sqrt{3}$  with  $c = \delta x/\delta t$  being the lattice speed and  $w_i$  the corresponding weights [1,2].

The macroscopic quantities can be computed by moments of  $f$ , i.e.  $\rho = \sum_i f_i$  and  $\rho \mathbf{u} = \sum_i \xi_i f_i$ , where  $\rho$  and  $\mathbf{u}$  are the density and velocity vector respectively and the viscosity of the fluid is given by  $\nu = \tau c_s^2$ .

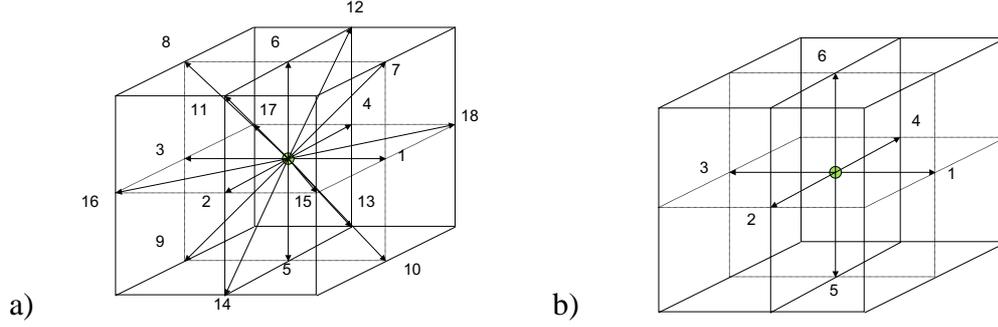


Figure 1: Discrete velocity lattices a) hydrodynamic (3DQ19), b) magnetic (3DM7).

A corresponding formulation can be used for tracking in time the induction equation [2]. We use a vector distribution function with its zeroth moment providing the magnetic field vector  $\mathbf{B} = \sum_{j=0}^M \mathbf{g}_j$ .

The evolution of  $\mathbf{g}_j$  obeys a BGK-type kinetic equation

$$\partial_t \mathbf{g}_j + \Xi \cdot \nabla \mathbf{g}_j = -\frac{1}{\tau_m} (\mathbf{g}_j - \mathbf{g}_j^{(0)}) \quad (3)$$

where  $\mathbf{g}_j^{(0)}$  are the corresponding equilibrium distribution functions given by

$$\mathbf{g}_{j\beta}^{(0)} = W_j [B_\beta + \Theta^{-1} \Xi_{j\alpha} (u_\alpha B_\beta - B_\alpha u_\beta)] \quad (4)$$

with  $\alpha, \beta$  denoting the spatial directions and  $\Xi$  the corresponding discrete velocity vector (not necessarily the same as  $\xi$ ). The magnetic field lattice used is depicted in Figure 1b. The relaxation time  $\tau_m$  allows us to set the magnetic resistivity as  $\eta = \Theta \tau_m$ , independently from the fluid's viscosity, which is related to  $\tau$ .

The first moment of  $\mathbf{g}$  gives the electric tensor as

$$\Lambda_{\alpha\beta}^{(0)} = \sum_{j=0}^M \Xi_{j\alpha} \mathbf{g}_{j\beta}^{(0)} = u_\alpha B_\beta - B_\alpha u_\beta. \quad (5)$$

Note that consistent expressions for  $\nabla \cdot \mathbf{B}$  and  $\nabla \times \mathbf{B}$  can be obtained [3]. Finally, the incorporation of the Lorenz force can be implemented in two ways either by an appropriate expansion of the  $f^{(0)}$  [3] or by adding a forcing term in the Boltzmann equation [2].

### Toroidal implementation

Applying the methodology in toroidal fields poses certain difficulties. Note that Eq.( 1) is formulated in terms of a cartesian system. The primary reason for this being the constraints imposed on the discrete lattice in terms of isotropy and galilean invariance. In cartesian configuration the torus obeys the equation

$$a^2 = (R_0 - \sqrt{x^2 + y^2})^2 + z^2 \quad (6)$$

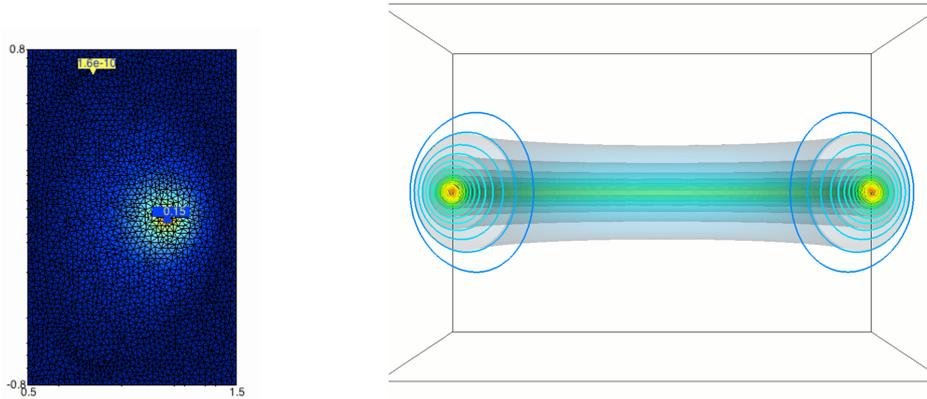


Figure 2: Solution of the Shafranov Eq.(left) and 3D initial field for the LK3D code (right).

where  $-1 < x, y, z < 1$  are the independent normalized variables of the domain, with  $a$  and  $R_0$  denoting the minor and major radius of the torus respectively.

The computational domain is consisting of a cube containing the torus, while the boundary of the torus is where the appropriate boundary conditions are imposed. Thus, we chose to store the nodes of the surface of the torus in a dynamic list in order to facilitate the application of the boundary conditions. In particular, typical no-slip boundary conditions are applied for the velocity  $\mathbf{u}$ , on the torus surface, while the corresponding ones for the magnetic field  $\mathbf{B}$  can be either conductive or insulating [2] depending on the problem at hand. For the runs presented here, no slip boundary conditions are employed for both the hydrodynamic and magnetic field. The code is written in F90/95 and runs in a parallel cluster using mpich.

The initialization of the present cartesian fields, given that the majority of the available stability solvers are written in cylindrical or toroidal frame, is cumbersome. For example, at the left hand side of Figure 2 the solution of the Shafranov Equation for a dirac-like current obtained by an elliptic 2D code [4] is depicted. We use a 2D scatter points interpolation to compute the necessary values on the nodes and thus initialize the LK3D code (right hand side of Figure 2). The main advantage of this approach is that any shape and distribution either experimental or numerical can be incorporated into the code. The poloidal cross-section has  $a/R_0 = .423$ , ellipticity  $\kappa = 1.68$  and triangularity  $\delta = 0.3$ .

The simulations are initiated by introducing initial magnetic and/or velocity fields. The evolution of the MHD field is followed in time by the lattice kinetic solver and all relevant quantities can be computed by appropriate moments of the distribution functions. In Figure 3, the energy and enstrophy of the system versus time is given for the magnetic field that corresponds to

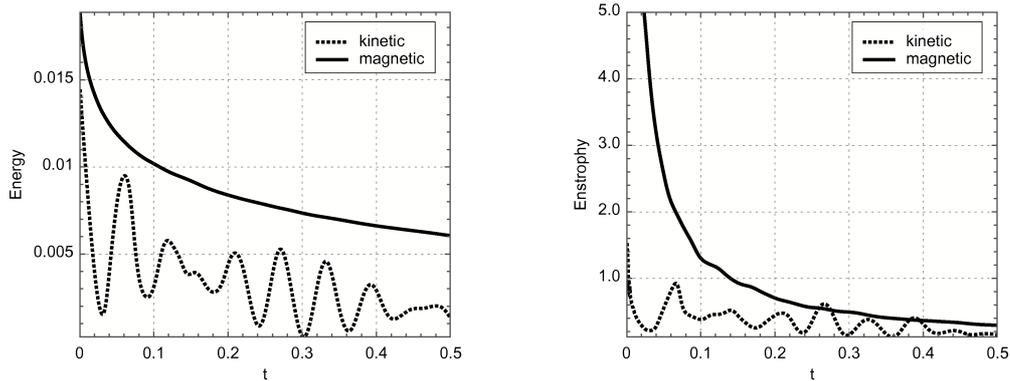


Figure 3: Temporal evolution of energies (left) and enstrophies (right) for  $\nu = \eta = 0.05$ .

Figure 2 and a velocity perturbation of the form

$$u = u_0[\cos(\theta)\cos(\phi), \sin(\theta), \cos(\theta)\sin(\phi)] \quad (7)$$

where  $\theta, \phi$  are the poloidal and toroidal angles respectively. The viscosity and resistivity were both set to 0.05.

### Concluding remarks

We have presented preliminary results of the LK3D code, modified to simulate MHD problems in an axisymmetric toroidal geometry. The present formulation retains the ability to tackle any generalized toroidal domain applicable to tokamaks. A systematic benchmarking procedure is under way within the EFDA's ITM project.

### Acknowledgments

This work has been partially supported by the Association: Euratom - Hellenic Republic.

### References

- [1] G. Breyiannis and D. Valougeorgis, Lattice kinetic simulations in 3-D magnetohydrodynamics, *Phys. Rev. E*, **69**, 065702 (2004).
- [2] G. Breyiannis and D. Valougeorgis, Lattice kinetic simulations of 3-D MHD turbulence, *Computers & Fluids*, article in press.
- [3] P. J. Dellar, Lattice kinetic schemes for magnetohydrodynamics, *J. Comput. Phys.*, **179**, 95 (2002).
- [4] J.C. Mollis and A. Pletzer, *Princeton Plasma Physics Laboratory*, [w3.pppl.gov/pletzer/papers-reports/python9paper/ellipt2d.html](http://w3.pppl.gov/pletzer/papers-reports/python9paper/ellipt2d.html).