

## Dynamical effects of sheared equilibrium flows on neoclassical tearing modes

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### Introduction

The dynamical evolution of neoclassical tearing modes (NTMs) in the presence of sheared equilibrium flows is a topic of much current interest particularly in the context of confinement limits for long pulse experiments in superconducting tokamaks. In an earlier detailed numerical investigation of this problem [1], employing a fully toroidal code based on a set of generalized reduced MHD equations [2], we had observed a number of salient effects. In particular we had found that differential flow provided a strong stabilizing influence on the NTMs leading to lower saturated island widths and reduced growth rates. The effect of velocity shear depended on the sign of the shear at the mode rational surface, with negative shear providing a stabilizing effect and positive shear acting in a destabilizing fashion. The dependence of the stability on the sign of the velocity shear could be partly explained on the basis of a Rutherford model calculation that was carried out in [3]. Specifically the island evolution equation had the following form,

$$G_1 \frac{\partial W}{\partial t} = D_R^{neo} \left[ \frac{\Delta'_c}{4} + G_2 \frac{\sqrt{\epsilon} \beta_\theta \frac{L_q}{L_p}}{W} \frac{W^2}{W^2 + W_\chi^2} - G_3 \frac{D_I}{\sqrt{W^2 + 0.65 W_\chi^2}} + \frac{L_s^2}{k_\theta^2 v_A^2} \left( G_4 \frac{(\omega - \omega_E)(\omega - \omega_E - \omega_*)}{W^3} + G_5 \frac{\omega_E'^2}{W} \right) - G_6 \frac{L_s}{k_\theta v_A} \frac{\bar{v}_{\parallel 0}}{v_A} \frac{\omega_E'}{W} \right] \quad (1)$$

where the terms  $\omega_E$ ,  $\omega_E'$  arise from the poloidal flow velocity and shear in the poloidal flow velocity respectively.  $v_{\parallel 0}$  is the parallel flow velocity, the  $G_n$ s are numerical coefficients and the rest of the notation is standard. The term proportional to  $G_6$  clearly shows the kind of sign dependence on the flow shear that has been observed in numerical simulations. The above model equation however takes into account only the modifications in the inner layer dynamics of the tearing mode, ignoring important flow induced effects that can arise in the outer layer solution and the concomitant changes in  $\Delta'$ . In this paper we examine these effects by deriving a generalized Newcomb equation that includes sheared flow and numerically solving this equation for the outer layer solutions. Specifically we try to assess the relative changes in  $\Delta'$  that can arise from changes in the velocity profiles, both with and without finite  $\beta$  effects.

### Newcomb equation in the presence of flow

For a quantitative determination of  $\Delta'$  in a realistic geometry we have extended the standard cylindrical derivation of the Newcomb equation to include a uniform equilibrium flow along the  $z$  axis and a sheared poloidal flow along the  $\theta$  direction. Omitting details of the derivation, we give below the resultant final equation:

$$H \frac{d^2 \psi}{dr^2} + \left( \frac{dH}{dr} + h_f \right) \frac{d\psi}{dr} - \left[ \frac{g}{F^2} + \frac{g_f}{F^2} + \frac{1}{F} \frac{d}{dr} \left( H \frac{dF}{dr} \right) \right] \psi = 0 \quad (2)$$

where,

$$F = \mathbf{k} \cdot \mathbf{B} = k + (m/r) B_\theta,$$

$$G = \mathbf{k} \cdot \mathbf{V} = kV_z + (m/r) V_\theta,$$

$$H = \frac{r^3}{k^2 r^2 + m^2},$$

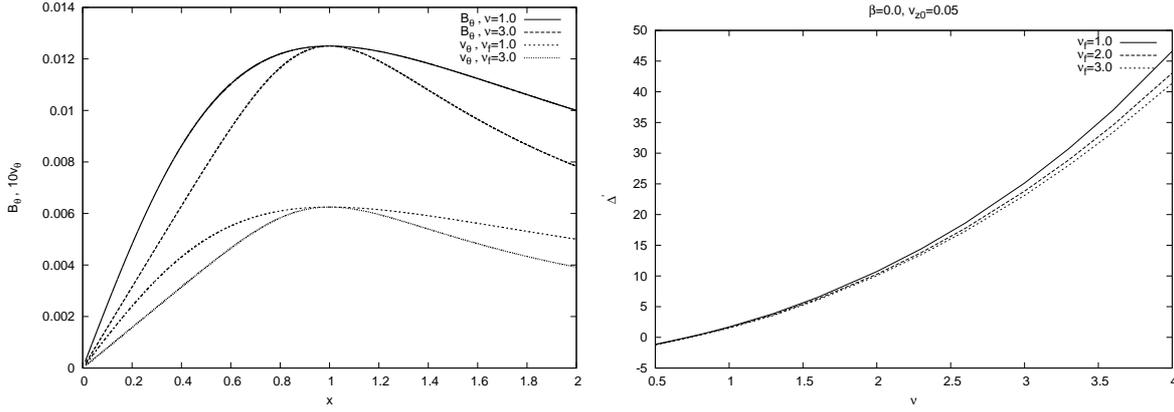
$$g = \frac{(\alpha m^2 - 1) r F^2}{\alpha (k^2 r^2 + m^2)} + \frac{k^2 r^2}{\alpha (k^2 r^2 + m^2)} \left( \alpha r F^2 + F \frac{2(kr - mB_\theta)}{k^2 r^2 + m^2} + \beta \frac{dP}{dr} \right),$$

$$h_f = \frac{2HG}{\alpha F} \left( \frac{G}{F} \frac{1}{F} \frac{dF}{dr} - \frac{1}{F} \frac{dG}{dr} \right),$$

$$g_f = \frac{2HG}{\alpha F} \frac{dF}{dr} \left( \frac{G}{F} \frac{dF}{dr} - \frac{dG}{dr} \right) + \frac{2V_\theta}{\alpha} \frac{dV_\theta}{dr} - \frac{G}{\alpha} \frac{d}{dr} \left( \frac{2mHV_\theta}{r^2} \right) + \left( \frac{m^2}{k^2 r^2 + m^2} - \frac{2}{\alpha} \right) \frac{2V_\theta^2}{r\alpha} \\ + \frac{Gr}{\alpha} \frac{d}{dr} \left( \frac{HG}{r^2} \right) + \left( \frac{4}{\alpha^2} \frac{k^2 r^2}{k^2 r^2 + m^2} \frac{G B_\theta}{F r} - \frac{2mH}{r^2 \alpha^2} \left( \frac{dG}{dr} - \frac{G}{r} \right) \right) \left( V_\theta - \frac{G}{F} B_\theta \right) \\ - \left( \frac{2mHG}{r^2 \alpha^2 F} \left( \frac{dG}{dr} + \frac{2mV_\theta}{r^2} - \frac{G}{r} \right) \right) \left( B_\theta - \frac{G}{F} V_\theta \right) + \frac{4}{r\alpha^2} \frac{G}{F} V_\theta B_\theta + \frac{GH}{r\alpha} \left( \frac{\partial G}{\partial r} + \frac{2mV_\theta}{r^2} \right)$$

$$\text{and } \alpha = 1 - \frac{G^2}{F^2} \quad (3)$$

In the above  $B$  has been normalized by  $B_z$ ,  $V$  the equilibrium flow velocity by the Alfvén velocity  $V_A$ ,  $P$  by  $P_0$  and  $\beta = 2\mu_0 P_0 / B_z^2$ . For  $G = 0$  and  $\beta = 0$  eq.(2) reduces to the standard outer layer equation that has been analyzed in the paper by Furth, Rutherford and Selberg [4]. More recently, Nishimura [5] extended the results of [4] to include finite  $\beta$  effects and showed that finite  $\beta$  can have a stabilizing effect on  $\Delta'$ . The effect of equilibrium sheared flows on  $\Delta'$  has been examined in the past by Chen and Morrison [6] but only in a simple slab geometry. For finite  $G$  and in the limit of a slab geometry ( $r \rightarrow \infty$ ,  $d/dr \rightarrow d/dx$ ) our eq.(2) reduces to the set of equations that have been discussed by Chen and Morrison [6]. Note that in the slab limit the finite  $\beta$  contribution disappears. Thus eq.(2) is the most generalized description of the outer layer dynamics which takes into account finite  $\beta$  contributions, cylindrical curvature effects as well as sheared flow effects.

Figure 1: (a) Profiles of  $B_\theta$  and  $V_\theta$  (b) Change of  $\Delta'$  with  $v$ 

### Evaluation of $\Delta'$

We have solved eq.(2) numerically to determine  $\Delta'$  for a variety of profiles of  $B_\theta$  and velocity  $V_\theta$ . Some typical profiles are shown in the left panel of Fig. 1. To determine  $\Delta'$  we have adopted a shooting method involving integration away from the singular layer towards the boundaries. The following analytic expressions representing asymptotic solutions for  $\psi$  near the resonant surface have been used to launch the numerical solutions.

$$\psi = A_l |s|^{h+1} - B_l |s|^{-h} ; \text{ for } x < x_s \quad (4)$$

$$\psi = A_r |s|^{h+1} + B_r |s|^{-h} ; \text{ for } x > x_s \quad (5)$$

with  $s = x - x_s$ ,  $h = -\frac{1}{2} + \frac{1}{2}\sqrt{1 - 4D_s}$  and

$$D_s = -\frac{q_s^2}{q_s'^2 \alpha x_s} \left[ \beta \frac{dp}{dx} + \frac{2x_b^2 x}{\hat{H} k^2 a^2} V_\theta \frac{dV_\theta}{dx} + \frac{2x_b^2}{\hat{H} k^2 a^2} \left( \frac{m^2}{k^2 a^2 (x/x_b)^2 + m^2} - \frac{2}{\alpha} \right) V_\theta^2 + \left( \frac{4}{\alpha} \frac{G}{F} \frac{B_\theta}{x} - \frac{2mx_b}{\alpha x k^2 a} \frac{dG}{dx} \right) (V_\theta - \frac{G}{F} B_\theta) - \frac{2mx_b}{\alpha x k^2 a} \frac{G}{F} \left( \frac{dG}{dx} + \frac{2mx_b V_\theta}{ax^2} \right) (B_\theta - \frac{G}{F} V_\theta) + \frac{4x_b^2}{\alpha \hat{H} k^2 a^2} \frac{G}{F} V_\theta B_\theta \right]_{x=x_s}$$

Here  $x = r/r_s$ ,  $x_b = a/r_s$  and  $\hat{H} = \frac{x^3}{k^2 a^2 (x/x_b)^2 + m^2}$ .

We iterate the constants A and B until the solution satisfies the boundary conditions [5]. The value of  $\Delta'$  is then obtained as,  $\Delta' = \frac{A_r}{B_r} - \frac{A_l}{B_l}$ .

### Numerical Results and Discussion

We now present some numerical results of  $\Delta'$  calculated for a typical set of profiles given as,

$$b(x) = \frac{ka}{q_0} \frac{x/x_b}{(1+x^{2\nu})^{1/\nu}} ; q(x) = q_0 (1+x^{2\nu})^{1/\nu}$$

$$v_\theta(x) = \frac{kav_{z0}}{q_{v0}} \frac{x/x_b}{(1+x^{2\nu_f})^{1/\nu_f}} ; q_v(x) = q_{v0} (1+x^{2\nu_f})^{1/\nu_f} ; p(x) = 1 - (x/x_b)^2$$

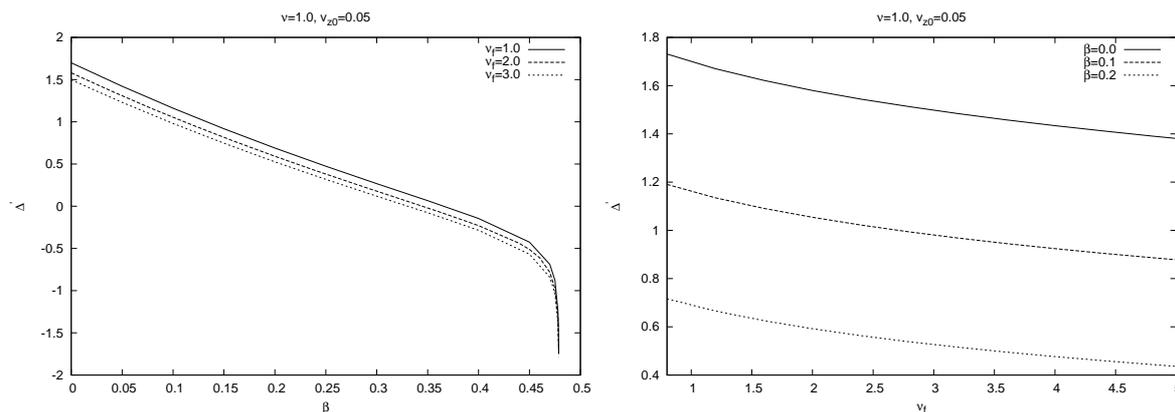


Figure 2: Change of  $\Delta'$  with  $\beta$  for different  $V_\theta$  profiles

Here  $\nu$  and  $\nu_f$  are indices that control the flatness of the magnetic field and flow velocity field profiles respectively. In Fig. 1(b) we show how  $\Delta'$  changes with the magnetic field flatness parameter  $\nu$  for a zero  $\beta$  plasma. Fig. 2 shows the effect of finite  $\beta$  on  $\Delta'$  with velocity profiles of different  $\nu_f$ . The figures show that  $\Delta'$  increases with  $\nu$  suggesting that as we increase the peakedness of the magnetic field profile there is a destabilization effect. Finite  $\beta$  decreases the  $\Delta'$ , so increases the stability but its stabilizing influence can be affected by the value of  $\nu_f$ . We can see that  $\Delta'$  decreases with increase of  $\nu_f$  for a given  $\nu$  and  $\beta$ . So the velocity profile has a stabilizing effect on the  $(m = 2, n = 1)$  tearing mode as we increase its peakedness. However the effect of  $\nu_f$  is relatively small compared to that of  $\nu$ . These preliminary results suggest that the combination of the magnetic and velocity profile variations along with finite  $\beta$  effects can profoundly influence the magnitude of  $\Delta'$  and consequently the stability of the tearing mode. A more systematic and quantitative assessment of these influences in order to provide a better analytic understanding of the results of the NEAR code [1] is currently under progress and will be reported elsewhere.

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