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On the Drag on an Object Immersed in a Flowing Plasma

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1 Introduction

The motivation for the present investigation was to determine the drag force on a dust particle in a flowing plasma, but the method adopted may be applicable to other objects in motion in a plasma. In low pressure plasmas in the laboratory there is always an appreciable flow of ions towards the plasma-sheath boundary. During their travel towards the surrounding space charge sheath, the ions acquire a velocity near to the Bohm velocity, which is equal to the ion acoustic velocity [1]. In the present work use is made of a "control surface", as employed in the subject of fluid mechanics [2], together with the concept of the Maxwell stress in the electric field [3].

2 The Bohm Sheath

The assumptions made in the usual model of the sheath:

- ionization in the sheath is negligible,
- distribution of ion energies is ignored,
- electric field at the plasma edge is taken to be very small.
- Boltzmann relation holds for the electron density.

Poisson's equation takes the following form

$$\varepsilon_0 \frac{d^2V}{dx^2} = -n_s e\left\{\left(1-\frac{V}{V_0}\right)^{-\frac{1}{2}} - e^{\frac{eV}{kT_e}}\right\} \qquad (1.1)$$

where eV_0 represents the kinetic energy of ions on entry to the sheath. This equation can be integrated once analytically. A second integration must be carried out numerically. The first integration gives the following equation

$$2n_s eV_0\left(1-\frac{V}{V_0}\right)^{\frac{1}{2}} + n_s kT_e e^{\frac{eV}{kT_e}} - \frac{1}{2}\varepsilon_0\left(\frac{dV}{dx}\right)^2 = 2n_s eV_0 + n_s kT_e \qquad (1.2)$$

This result is of considerable interest, however, since it can be interpreted physically as a momentum/stress balance [4]. At the wall the force is due to ion impact, the (greatly reduced) electron pressure, and the electric force on the surface charge; the last of these acts like a "negative pressure".
The momentum/stress equation is valid at all points in the sheath.

3 A Basic Momentum-Stress Theorem

Let us now consider the case in point, that of an object, e.g. a dust particle, immersed in a flowing plasma. The theory adopted assumes that the positive ions behave as a cold gas; the directed motion is assumed to dominate over any random motion. The electrons are assumed to be in Boltzmann equilibrium

$$n_e = n_0 e^{\frac{eV}{kT_e}} \tag{1.3}$$

Collisions with neutral atoms or molecules are neglected. The momentum equation for the ions can be written

$$n_i M\left(\boldsymbol{v}\cdot\nabla\boldsymbol{v}\right)\boldsymbol{v} = n_i e\boldsymbol{E} \tag{1.4}$$

Differentiation of equation (1.2) gives

$$\nabla p_e = -n_e e\boldsymbol{E} \tag{1.5}$$

Adding equations (1.4) and (1.5) and substituting from Gauss' Law we obtain

$$n_i M\left(\boldsymbol{v}\cdot\nabla\boldsymbol{v}\right)\boldsymbol{v} + \nabla p_e - \varepsilon_0\left(\nabla\cdot\boldsymbol{E}\right)\boldsymbol{E} = 0 \tag{1.6}$$

A volume integration of equation (1.6) reads

$$\iiint n_i M\left(\boldsymbol{v}\cdot\nabla\right)\boldsymbol{v}\,d\tau + \iiint \nabla p_e\, d\tau - \iiint \varepsilon_0\left(\nabla\cdot\boldsymbol{E}\right)\boldsymbol{E}\,d\tau = 0 \tag{1.7}$$

The volume integrals can be converted into surface integrals using Gauss's divergence theorem, and applied to the case of an immersed object, e.g. a dust particle.

Referring to Fig.1, to the volume between S_1 and S_2, where S_1 is a surface near the object and S_2 is any larger radius, yields the result that

$$\iint_{S_1}\left[\rho\left(\boldsymbol{v}\cdot\boldsymbol{n}\right)\boldsymbol{v} + p_e\boldsymbol{n} - \underline{\boldsymbol{T}}\cdot\boldsymbol{n}\right]dS + \iint_{S_2}\left[\rho\left(\boldsymbol{v}\cdot\boldsymbol{n}\right)\boldsymbol{v} + p_e\boldsymbol{n} - \underline{\boldsymbol{T}}\cdot\boldsymbol{n}\right]dS = 0 \tag{1.8}$$

where

$$\iint \underline{\boldsymbol{T}}\cdot\boldsymbol{n}\,dS = \iint\left[\varepsilon_0\left(\boldsymbol{E}\cdot\boldsymbol{n}\right)\boldsymbol{E} - \frac{1}{2}\varepsilon_0 E^2\boldsymbol{n}\right]dS \tag{1.9}$$

the electric field can be described as exerting a tension $\varepsilon_0 E^2$, together with an Isotropic pressure of $\frac{1}{2}\varepsilon_0 E^2$. The first integral, however is equal to the force on the enclosed object so that

$$\boldsymbol{D} = -\iint_{S_2}\left[\rho\left(\boldsymbol{v}\cdot\boldsymbol{n}\right)\boldsymbol{v} + p_e\boldsymbol{n} - \underline{\boldsymbol{T}}\cdot\boldsymbol{n}\right]dS \tag{1.10}$$

Evaluation of the integral over the surface S_2 can therefore be used to determine the force on the immersed object.

4 Application to the case of weak flows

At large distances from the object (Fig 2) the (normalized) velocity potential for weak flows is given by

$$\Phi = \nu r\cos\theta + \frac{J}{r} \tag{1.11}$$

where $4\pi J$ is the normalized flux to the object [6]. $\nu = \frac{U}{c_s}$ and $c_s = \left(\frac{kT_e}{M}\right)^{\frac{1}{2}}$ is the ion acoustic speed. The ion velocity is given by the gradient of the velocity potential. The rate of flow of momentum leaving the spherical surface is given by

$$M = \int_0^{\pi} 2\pi R^2\rho v_r\sin\theta\left(v_r\cos\theta - v_\theta\sin\theta\right)d\theta = -\frac{16\pi\nu J}{3} \tag{1.12}$$

converting to physical values we have

$$\iint \rho(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v}dS = -\frac{4}{3}MI\boldsymbol{U} \tag{1.13}$$

where $\boldsymbol{U}$ is the stream velocity and I is the flux of ions to the immersed object.
The integral of the electron pressure over the surface of the large spherical surface is given by the following expression,

$$\iint p_e\boldsymbol{n}dS = \int_0^\pi p_e 2\pi R^2 \sin\theta\cos\theta d\theta \boldsymbol{u}_z \tag{1.14}$$

At large distances, for weak flows only [6],

$$\phi = \frac{J\nu\cos\theta}{r^2} \tag{1.15}$$

evaluating eqn. (1.14) using the Boltzmann relation,

$$\boldsymbol{P} = \frac{4\pi J\nu}{3}\boldsymbol{u}_z \tag{1.16}$$

reverting to physical quantities we have

$$\iint p_e\boldsymbol{n}dS = \frac{MUI}{3}\boldsymbol{u}_z \tag{1.17}$$

The electric field at large radii is given by the taking the gradient of eqn. (1.15),

The Maxwell stress could be integrated over the surface of a large spherical surface, but since $E^2 \sim O\left(\frac{1}{r^6}\right)$ the integral tends to zero as r tends to infinity.
It is interesting to note that there is a flux of ions across a large spherical surface, but no flux of the electric field .

Substituting for the first two terms in the surface integral S_2

$$\boldsymbol{D} = \iint_{S_1}\left[\rho(\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{v} + p_e\boldsymbol{n} - \underline{\boldsymbol{T}}\cdot\boldsymbol{n}\right]dS = MUI\boldsymbol{u}_z \tag{1.18}$$

Thus the force on the immersed object is equal to the initial momentum associated with those ions which are destined to be captured by the object.
The same result can also be shown to hold for the case of strong flows

5 Application to Experimental work on Dusty Plasmas

The set of basic equations (1.3), (1.4) and (1.6), together with the continuity equation for the positive ions, remains to be solved [5]. Thus our principal result, eqn. (1.18), gives the total drag force on a dust particle in terms of the ion current, which is an unknown quantity.
The charge Q on a dust particle can be determined in various ways, for example by observing the period of oscillation in the background electric field [6]. An estimate of the floating potential acquired by the dust particle can then be made using the "vacuum capacitance" formula

$$Q \simeq 4\pi\varepsilon_0 r_d V_f \tag{1.19}$$

Hence the electron flux to the dust particle can be calculated. This flux is equal to the positive ion flux, so using our principal result, eq (18), we obtain

$$D = 4\pi r_d^2 n_\infty e^{\frac{eV_f}{kT_e}} MU\sqrt{\frac{kT_e}{2\pi m}} \tag{1.20}$$

The force on the dust particle is due to ion impact, a (reduced) electron pressure and an electric force. The theory presented in this paper does not give these individual components, but only their sum, the total force.

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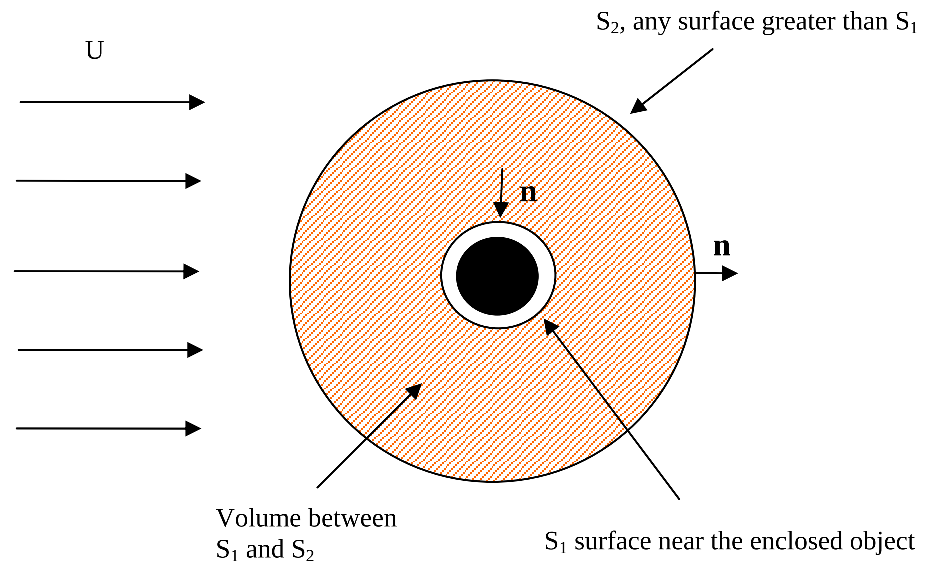

Figure 1: Diagram to illustrate the momentum-stress calculation

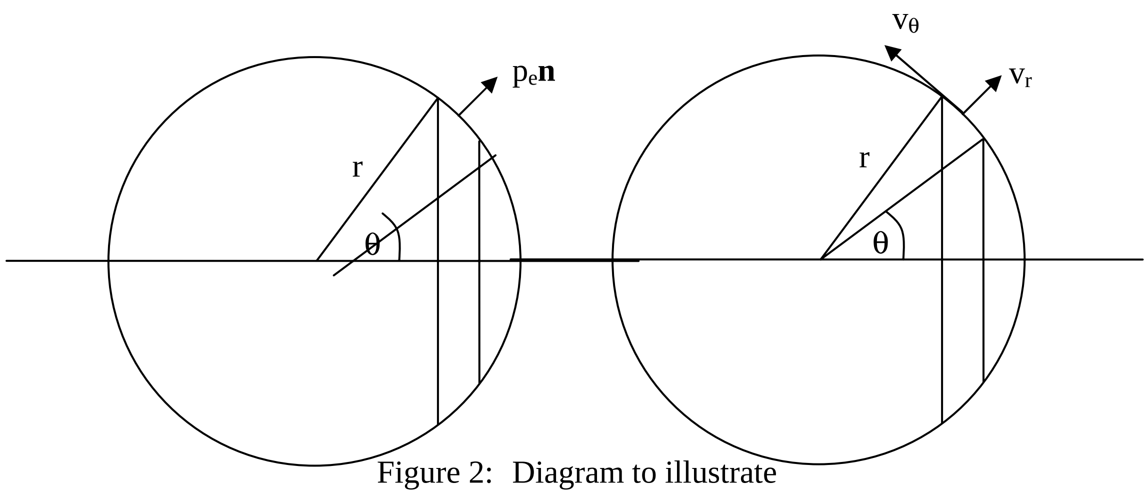

Figure 2: Diagram to illustrate the momentum-stress calculation for weak flows