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Self-consistent model of magnetic electron drift mode turbulence

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Introduction

Since the end of the 1970s, experiments showed that quasi-steady magnetic fields are created in laser-produced plasma, and the physical mechanisms for the self-generated magnetic fields have been examined based on the linear theory of the magnetic electron drift mode involving collisions and also the collisionless regime. Here we focus on the non-linear mechanism of generation of large scale magnetic structures, so called meso-scale magnetic fields, by magnetic electron drift wave turbulence and the mutual interaction of structures in different regions of the wave spectrum.

Basic equations

Magnetic electron drift modes are low frequency motions fed by inhomogeneities in electron density and temperature. These modes have typical frequencies of the order of κv_{Te}, where κ is the inverse characteristic length of the background inhomogeneity and v_{Te} the thermal electron velocity. The time scale of interest is in-between the inverse ion and the inverse electron plasma frequency, $\omega_{pi} \ll \omega \ll \omega_{pe}$. Hence we consider an unpolarised electron fluid and immobile ions. So, the ions play a passive role as a neutralising background and the dominant role in dynamics is played by electron species. Therefore density perturbations can be neglected, i.e. n equals its equilibrium value n_0. The equilibrium temperature is denoted T_0. We assume here $\nabla n_0 = \partial_x n_0 \hat{\mathbf{x}}$ and $\nabla T_0 = \partial_x T_0 \hat{\mathbf{x}}$. Further assumptions are that $\kappa_n, \kappa_T \ll k$, $|\nabla n_0 \times (\nabla \times B)| \ll |n_0 \nabla^2 B|$, and we also consider a quasi-two-dimensional case, where the perturbed magnetic field defines the z-axis. The inhomogeneity parameters are defined as follows: $\kappa_{n.T} \equiv |\nabla \ln(n_0, T_0)|$.

With all of these assumptions, the momentum together with Maxwell's and the energy equation [3] yield the model equations for magnetic electron drift mode turbulence,

$$\frac{\partial}{\partial t}\left(B - \lambda^2 \nabla^2 B\right) + \beta \frac{\partial T}{\partial y} = \frac{e\lambda^4}{m}\left\{B, \nabla^2 B\right\} \tag{1a}$$

$$\frac{\partial}{\partial t} T + \alpha \frac{\partial B}{\partial y} = -\frac{e\lambda^2}{m}\left\{B, T\right\} \tag{1b}$$

with $\alpha \equiv \lambda^2 eT_0/m\,(2/3\kappa_n - \kappa_T)\,, \beta \equiv \kappa_n/e$ and $\lambda \equiv c/\omega_c$ the skin depth. The curly brackets on the RHS denote the Poisson brackets.

First, we linearise the model equations and assume $(B,T) \sim \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$. The resulting linear dispersion relation is

$$\omega_k = k_y\sqrt{\frac{\alpha\beta}{1+k^2\lambda^2}}, \tag{2}$$

There is a purely growing solution for $\kappa_T > (2/3)\kappa_n$ and the growth rate vanishes for modes with $k_y = 0$. Note that the purely growing solution can explain the strong magnetic fields measured in laser plasma experiments, but it does not allow for a description of large scale magnetic structures. Another useful result from the Fourier decomposition in the model equations is that one can find a relation between the magnetic field Fourier amplitudes B_k and those of the temperature T_k, namely $T_k = \alpha k_y/\omega_k B_k$.

Coupled dynamics

The main goal of this article is to describe the appearance of large scale symmetric magnetic structures out of unorganised small scale electron drift mode turbulence. For this sake, the linear approximation clearly is not sufficient and we have to take into account the non-linear terms on the RHS of Eqs. (1). Our filtered model of large scale structures and small scale turbulence consists of a decomposition of the total spectrum into two parts, $(B,T) = \sum_k (B_k,T_k)\exp[i\mathbf{kr}] + (B_q,T_q)\exp[i\mathbf{qr}] + cc.$, with $k \ll q$, such that k denotes the small, q the large scales of the spectrum.

In order to find the evolution equations of T_q and B_q, we average the model equations (1) over the short fast scales. Furthermore, we are interested in two limits of the large scale structures, the zonal fields (ZF) with $\mathbf{q} = q\hat{\mathbf{x}}$ and the magnetic streamers (MS) with $\mathbf{q} = q\hat{\mathbf{y}}$. The evolution equations for ZF are decoupled:

$$\frac{\partial B_q}{\partial t} = \frac{e\lambda^2}{m}\frac{q^2\lambda^2}{1+q^2\lambda^2}\sum_k k_x k_y B_k B_{-k}, \qquad \frac{\partial T_q}{\partial t} = 0, \tag{3}$$

whereas the equations for MS are not:

$$\frac{\partial B_q}{\partial t} + \frac{i\beta q}{1+q^2\lambda^2}T_q = -\frac{e\lambda^2}{m}\frac{q^2\lambda^2}{1+q^2\lambda^2}\sum_k k_x k_y B_k B_{-k} \qquad \frac{\partial T_q}{\partial t} + i\alpha q B_q = 0, \tag{4}$$

The common term on the RHS is the magnetic Reynolds stress, and it shows us clearly that indeed large scale structures are excited via non-linear interaction of small scale turbulence.

A convenient way of closing the system (i.e. find the evolution equations for the small scale turbulence) is the description of the propagation of waves in a turbulent medium with slowly varying parameters through a wave kinetic equation for the wave action density N_k [1]. The standard definition of the wave action density or wave spectrum is $N_k = E/\omega_k$. This is, however, not applicable here since the energy is not only contained in the small scale turbulence but also in the large scale structures [2, 4]. The appropriate expression for the wave spectrum can be found to be $N_k = 4\alpha/\beta(1+k^2\lambda^2)|B_k|^2$, and the wave kinetic equation takes the form

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_k^{NL}}{\partial \mathbf{k}}\frac{\partial N_k}{\partial \mathbf{r}} - \frac{\partial \omega_k^{NL}}{\partial \mathbf{r}}\frac{\partial N_k}{\partial \mathbf{k}} = 2\gamma_k N_k - St(N_k). \tag{5}$$

The linear frequency is not valid anymore, but it is Doppler-shifted by the presence of the large scale "flows" and reads explicitly $\omega_k^{NL} \equiv \omega_k^{Re} + \Delta$, with $\Delta = (1+2k^2\lambda^2)/(1+k^2\lambda^2)\mathbf{k}\cdot\mathbf{v}_B^{(q)} - 1/\sqrt{1+k^2\lambda^2}\mathbf{k}\cdot\mathbf{v}_T^{(q)}$ and $\mathbf{v}_B \equiv -e\lambda^2/(4m)(\nabla B\times\hat{\mathbf{z}}), \mathbf{v}_T \equiv -e\lambda^2/(4m)\sqrt{\beta/\alpha}(\nabla T\times\hat{\mathbf{z}})$.

The wave kinetic equation (5), describing the evolution of the small scale turbulence in the presence of large scale structures, and Eqs. (3,4), describing the evolution of the large scale structures in the presence of small scale turbulence, represent a self-consistent description of the system large scales-small scales.

Quasi-linear analysis

As an application of the newly derived self-consistent description, let us investigate the effect of the large scale structures on the underlying small scale turbulence. For this sake, we decompose the wave spectrum into an average and a perturbed part, $N_k = N_k^0 + \tilde{N}_k$ and assume $St\{N_k\} = \Delta\omega_k N_k^2$, such that for a stationary equilibrium the average wave spectrum $N_k^0 = 2\gamma_k/\Delta\omega_k$. When we average (5) over the fast/small scales and linearise the resulting equation, we get

$$\frac{\partial N_k^0}{\partial t} - \overline{\frac{\partial}{\partial \mathbf{r}}(\mathbf{k}\cdot\mathbf{v}_g)\frac{\partial \tilde{N}_k}{\partial \mathbf{k}}} = 0. \tag{6}$$

If we furthermore assume $\tilde{N}_k \sim \exp(-i\Omega t + i\mathbf{q}\mathbf{r})$, the latter equation transforms into a diffusion equation in k-space

$$\frac{\partial N_0}{\partial t} - \frac{\partial}{\partial k_{x,y}}\left[D_{k_{x,y}}\frac{\partial N_0}{\partial k_{x,y}}\right] = 0, \tag{7}$$

for ZF,MS respectively. The corresponding diffusion coefficients are then

$$D_{k_{x,y}} \equiv k_{y,x}^2 q_{x,y}^4 \frac{e^2\lambda^4}{16m^2}\left(|B|^2 + \frac{\beta}{\alpha}|T|^2\right)R(\Omega,p), \tag{8}$$

where $R(\Omega,p) = i/(\Omega - \mathbf{p}\cdot\mathbf{v}_g)$ is the response function. We can understand this last equation as a diffusion in k-space, towards larger k ($D>0$), i.e. smaller scales. So the large scale structures regulate the underlying turbulence in that they shear them to smaller and smaller scales, until they finally disappear due to important dissipation at very small scales. Note that by shearing the turbulence, the large scale fields destroy their support and thus are diminished as well, so that the turbulence can grow again and thus generate new large scale fields and the circle closes on itself, which suggests a predator-prey-like behaviour of the system.

Conclusions

We have derived the evolution equations for large scale magnetic structures and seen that small scale random magnetic turbulence drives them via magnetic Reynolds stress. As a second step, the wave kinetic equation for a suitable wave action invariant was derived and thus a self-consistent description of the system large scale fields-small scale turbulence could be stated. Finally, the effect of the large scale structures on the underlying turbulence was elucidated and a predator-prey-like behaviour was found.

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