

Nonlinear evolution of magnetic islands in a turbulent plasma

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Introduction

The interaction of a background turbulence of microscopic fluctuations with the dynamics of $m = 1$ or $m \geq 2$ resistive tearing modes is a fundamental issue in reconnection physics with important implications for various fusion devices as well as in the modeling of astrophysical phenomena such as solar flares or magnetospheric substorms. In this study, in order to examine the mechanism of the reconnection process in the presence of a dynamical background of micro-turbulence in the velocity field, we carry out detailed numerical simulations of a model two dimensional reduced MHD system.

Model system

The reduced MHD equations we study are

$$\frac{\partial \psi}{\partial t} + [\phi, \psi] = \Psi'_0(x) \frac{\partial \phi}{\partial y} + \eta j, \quad (1)$$

$$\frac{\partial \omega}{\partial t} + [\phi, \omega] = [\psi, j] + \Psi'_0(x) \frac{\partial j}{\partial y} - J'_0(x) \frac{\partial \psi}{\partial y} + \mu \nabla_{\perp}^2 \omega, \quad (2)$$

where ψ and ϕ are the magnetic flux and electrostatic potential respectively, $\omega = \nabla_{\perp}^2 \phi$ is the vorticity, $j = \nabla_{\perp}^2 \psi$ is the current density, η is the resistivity and μ is the viscosity. Note that ϕ corresponds to the stream function in fluid mechanics. The subscript 0 refers to equilibrium quantities, and $[f, g]$ are the Poisson brackets. A slab geometry has been assumed with x corresponding to the radial direction and y to the poloidal direction. Accordingly periodic boundary conditions are imposed in the y direction and all perturbations in the x direction are taken to vanish at the boundaries. The equations are normalized by $\tau_A = L/v_A$ for time, LB_0 for ψ , Lv_A for ϕ and L/v_A for ω , where τ_A and v_A are the Alfvén time and velocity respectively, and L is a typical length in the perpendicular direction. The equilibrium magnetic field is chosen to have the form,

$$\mathbf{B}_0(x) = \tanh\left(\frac{x - L_x/2}{a}\right) \hat{\mathbf{y}} \quad (3)$$

where L_x is the box size in the x direction and the parameter a controls the width of the profile and correspondingly the value of Δ' (the tearing mode instability parameter). This form of $\mathbf{B}_0(x)$ provides to a $q = 1$ rational surface at $x = L_x/2$, and is more representative of tokamak configurations.

Equations (1) and (2) are solved numerically with a predictor–corrector scheme in the simulation box of size $L_x = 2\pi$ and $L_y = 2\pi$. All variables are expressed in the real space with grid points of $N_x = 512$ for the x direction, and in the Fourier space with mode number of $N_y = 512$ for the y direction. The basic code used for solving Eqs. (1) and (2) has been benchmarked against standard linear and nonlinear tearing mode evolution results. In particular, the visco-resistive scalings with μ and η are found to agree quite well with the analytic results by Ref. [1].

Results and discussions

As a preliminary step towards addressing the fully self-consistent problem of the interaction of microturbulence with tearing modes we have carried out a simpler “forcing” simulation study. The “forcing” is in the form of initial perturbations to represent an initial turbulent field as

$$\begin{pmatrix} \psi(t=0) \\ \phi(t=0) \end{pmatrix} = \varepsilon \sum_{\ell=0}^{N_x} \sum_{m=0}^{N_y} \sin(k_x(\ell)x) \left[\begin{pmatrix} \alpha_\psi \\ \alpha_\phi \end{pmatrix} \cos(k_y(m)y) + \begin{pmatrix} \beta_\psi \\ \beta_\phi \end{pmatrix} \sin(k_y(m)y) \right], \quad (4)$$

where

$$\alpha_\psi, \beta_\psi = \cos(2\pi N_{\text{random}}); \quad \alpha_\phi, \beta_\phi = \sin(2\pi N_{\text{random}}); \quad -1 < N_{\text{random}} < 1$$

For our present set of simulations we have chosen the parameters as $\mu = 10^{-3}$, $\eta = 10^{-3}$, and

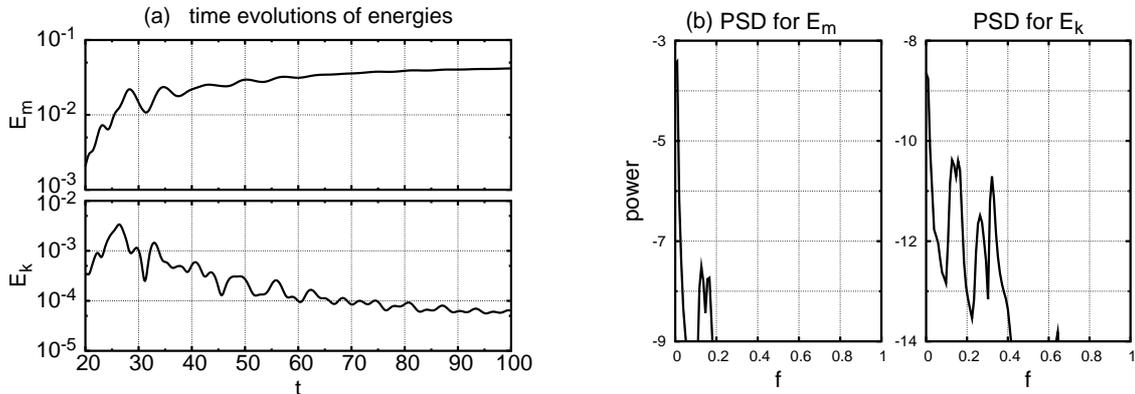


Figure 1: Time evolutions (a) and power spectrum densities (b) of kinetic and magnetic energies. The frequency of kinetic energy has peaks twice the magnetic one.

$$a = 0.05 (\Delta' \simeq 40).$$

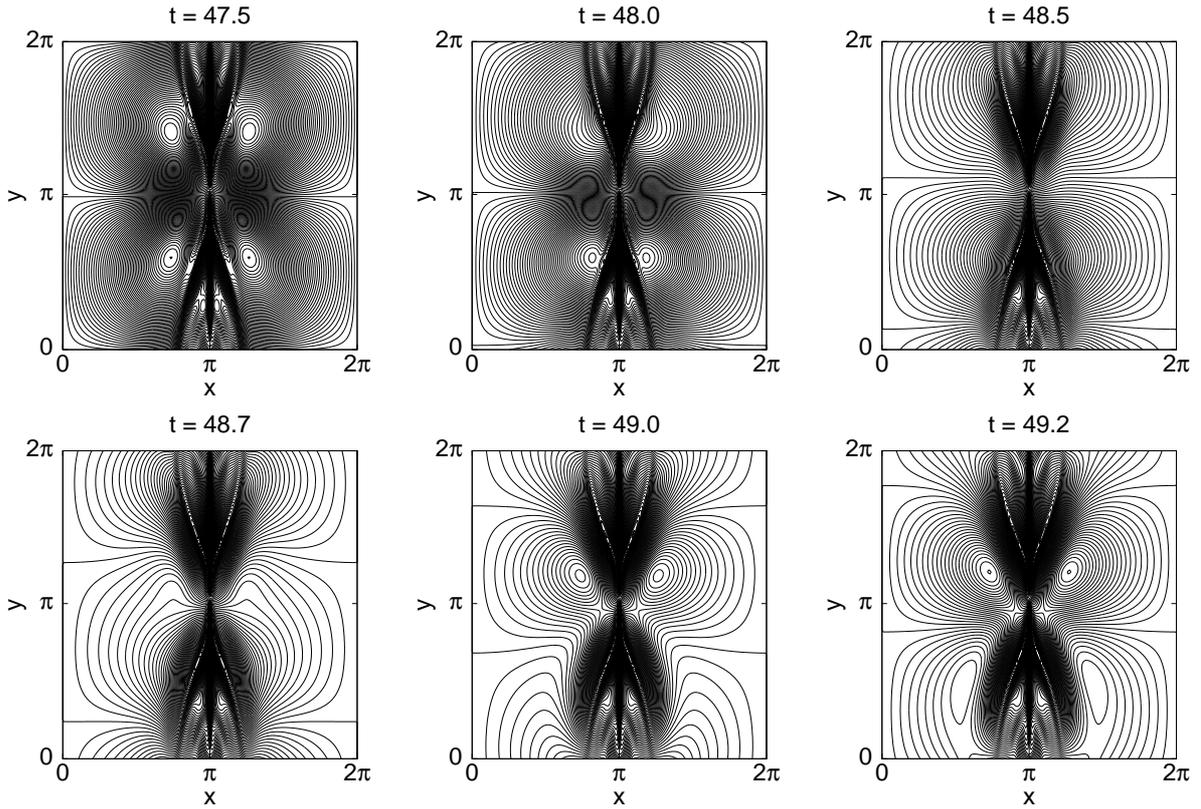


Figure 2: Time sequence of snapshots of stream function ϕ during a half period of the energy oscillations. Here $\mu = 10^{-3}$ and $\eta = 10^{-3}$.

We find that when Δ' is large enough, the kinetic and magnetic energies oscillate in time (Fig. 1) such that the frequency of the kinetic energy twice that of the magnetic one. The magnitude of the frequency depends on the value of resistivity η . The time evolutions of kinetic and magnetic energies and their power spectrum densities are shown in Fig. 1. The magnetic configuration is such that the $m = 1$ magnetic island pulsates following these oscillations, while at the same time, intermittent and robust quadrupole-like structures are seen in the velocity field around the island. Figure 2 shows the time sequence of snapshots of the stream function ϕ during a half period of the energy oscillations. The external quadrupole seen at $t = 47.5$ is absorbed into the island region and disappears at $t = 48.5$. Soon after that another quadrupole with, however, its polarity reversed is generated from $t = 48.7$. The kinetic and magnetic energies undergo an exchange amongst themselves through the periodic sequence of the interactions between the island and external quadrupole regions.

When the viscosity becomes large, $\mu = 10^{-2}$, the dynamics of the velocity field shows different behaviour. External quadrupoles are too weak to be separated from the island and reversals of polarity do not occur. The time sequence of velocity field configuration during a period of energy oscillations is depicted in Fig. 3. A butterfly like periodic pattern appears around the

island region. The interchange between kinetic and magnetic energies is much weaker, which leads to smaller amplitude of energy oscillations.

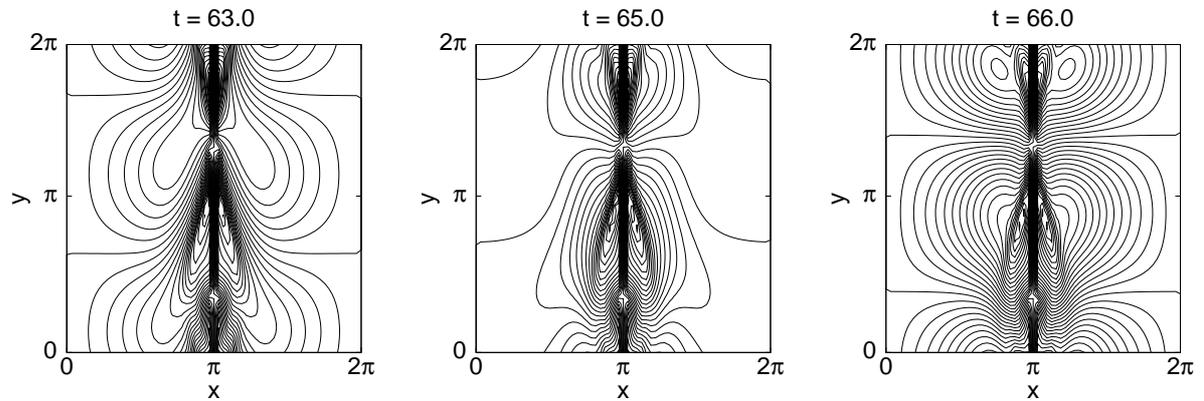


Figure 3: Time sequence of snapshots of stream function ϕ during a period of the energy oscillations. Here $\mu = 10^{-2}$ and $\eta = 10^{-3}$.

In conclusion, the growth of $m = 1$ magnetic island and the velocity field configuration are modified by the oscillatory behaviour for large Δ' . The kinetic energy is transferred into the magnetic energy through the sequence of quadrupole absorption into island region and regeneration. Viscosity can strongly influence this exchange process because it allows a modification of the topology of the vorticity, and may therefore have a fundamental role in the interaction between the velocity field and the evolving magnetic island.

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