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A model for resistive wall modes control

C.V. Atanasiu[1], I.G. Miron[1] and V.D. Pustovitov[2]

[1] *National Institute for Laser Plasma and Radiation Physics, Association EURATOM-MEdC, Bucharest, Romania*

[2] *Russian Research Centre "Kurchatov Institute", Moscow, Russia*

Introduction

The "advanced tokamak" concept is attractive if the ideal external kink beta limit is raised substantially. This is possible only if the resistive wall mode (RWM) is stabilized by either passive (close fitting conducting wall) and active (feedback coils) stabilization, or/and rapid plasma rotation. A plausible stabilization mechanism is a combined effect of plasma rotational inertia and dissipation due to interaction with the sound wave continuum at a toroidally coupled resonant surface lying within the plasma [1, 2]. Fitzpatrick and Aydemir [3, 4] have developed a model with plasma dissipation provided by the edge plasma viscosity. There is, at present, no clear consensus of opinions as to what are the necessary ingredients for the rotational stabilization of this mode.

Our analysis, based essentially on Fitzpatrick's resistive shell mode model [3, 4], considers a large aspect ratio, low beta, circular flux surface tokamak equilibrium; linearised, incompressible, non-ideal (i.e. including the effects of plasma inertia, resistivity and viscosity) equations of reduced MHD (at the plasma edge); a general m/n current-driven external-kink mode in the presence of edge plasma rotation and a thin resistive nonuniform vacuum vessel (made by cylindrical pieces of different materials) with feedback and detector coils placed around the tokamak plasma. The influence of different parameters on the RWM growth rate has been investigated.

1-D equations

In a "cylindrical" tokamak, periodic in the z direction, with a large aspect ratio and low β, the linear stability equations are obtained by considering the linearized force balance equation (with viscous force), the linearized Ohm's law and assuming for the perturbed quantities a dependence of the form $\exp[i(m\theta - nz/R_0) + \gamma t]$. γ is the complex growth rate in the laboratory frame. The edge plasma rotation ($\omega = m\Omega_\theta - n\Omega_z$) has also been taken into account. Finally, considering the jump conditions at the wall and across the feedback coils, the feedback current, the inductive voltages generated in the feedback coils and detector loops, the circuit equation, and the applied feedback voltage, related to the voltage measured by the detector loop, then the full dispersion relation reads like

$$\sum_{j,k}\left(\gamma^2 q^{mn}_{jk2}+\gamma q^{mn}_{jk1}+q^{mn}_{jk0}\right)\left(\gamma^4\tilde{A}^{mn}_{jk4}+\gamma^3\tilde{A}^{mn}_{jk3}+\gamma^2\tilde{A}^{mn}_{jk2}+\gamma\tilde{A}^{mn}_{jk1}+\tilde{A}^{mn}_{jk0}\right)\frac{1}{k}K^{jk}_{wf}(Q^{jk}_{wf})^2\Psi^{jk}_{w}=0.$$

This represents a homogeneous system of equations with Ψ^{jk}_{w} the unknowns harmonics of the perturbed flux function at the wall level, with the complex coefficients $\tilde{A}^{mn}_{jk0} \div \tilde{A}^{mn}_{jk4}$, K^{jk}_{wf}, Q^{jk}_{wf}, $q_{jk0} \div q_{jk2} = f(j,k,a,R_0,q_a,q_0,B_{za},\tau_A,\tau_R,\tau_V,r_f,r_w,r_d,d_{w1},d_{w2},\Delta^{\theta}_{f,d},\Delta^{\phi}_{f,d},M,N,\eta_w,\eta_f,G_p,G_d,\Omega_{\phi,\theta})$. $m,j=m_1 \div m_2$ are poloidal indexes, while $n,k=n1 \div n2$ are toroidal indexes, a and R_0 are the minor and the equivalent major radius, q_a and q_0 are the safety factors at the plasma boundary and at the plasma center respectively, B_{za} is the toroidal magnetic field, τ_A, τ_R, τ_V are the Alfvén, resistive, and viscous time scales at the edge plasma, respectively; the subscripts f,w,d of r represent the feedback coil radius, the wall radius and the detector radius; $d_{w1,2}$ are the thicknesses of the wall (cylindrical segments) and $\Delta^{\theta;\phi}_{f,d}$ are the geometrical parameters of the feedback and detector coils; M and N are the numbers of feedback coils in poloidal and toroidal directions, respectively; $\eta_{w,f}$ are the electrical resistivities of the wall (aluminium and stainless steel) and of the feedback coils, respectively; $G_{p,d}$ are the proportional and the derivative gains, respectively, $\Omega_{\phi,\theta}$ are the toroidal and poloidal angular plasma velocities, respectively.

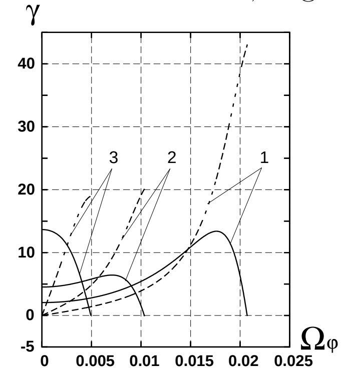
The complex growth rate of the RWM is found by solving either a $2(K+1)^{th}$ degree complex polynomial dispersion equation, with K the product of the numbers of poloidal and toroidal modes considered, or, for large K, by finding the zeros of a complex analytical function representing the vanishing determinant of the above system of equations [5]. Note that we succeeded to identify two spurious roots (of the second order polynomial), decreasing therefore the polynomial degree.

Numerical results

As a “standard” model, we have considered the following tokamak plasma parameters: $a=1$ m; $R_0=6$ m; $q_a=2.9$; $q_0=1.3$; $B_{za}=2.1$ T; $m_0/n_0=3/1$; anomalous plasma viscosity at the edge $\mu_\perp=9\times10^{-11}$ kg/m/s; plasma density at a: $\rho_a=9\times10^{-14}$ kg/m^3; $r_w=1.15$ m; $d_{w1}=.001$ m; $d_{w2}=.001$ m; $r_f=1.25$ m; $d_f=0.005$ m; $r_d=1.20$ m; number of the toroidally disposed pairs of wall pieces (Aluminium - subscript “1”, without feedback coils, and Stainless steel - subscript “2”, with feedback coils) $N=5$; $M=4$; $G_d=31$; $G_p=5.5$; number of sideband harmonics $m=1\div7$, $n=1\div2$. In the following, the plasma edge rotation Ω_ϕ has been normalised to the Alfvén time $\tau_A=9.6086^{-10}s$, while the growth rate γ has been reported to the resistive wall time τ_w.

Fig. 1 shows the growth rate of the 3/1 RWM calculated as a function of the plasma toroidal rotation for various wall radii in the absence of feedback. A wall close to the plasma gives rise to a RWM with a rotation frequency ($\gamma_i = \Im(\gamma)$) closer to zero, and this mode is stabilized at a relatively high plasma rotation. We found that initially the plasma rotation has a destabilizing effect on the RWM and that only when $\gamma_i > \gamma_r = \Re(\gamma)$, does the stabilization of the mode start. The growth rate dependence on plasma rotation for different wall materials, in the presence of feedback, is given in Fig. 2.

Fig. 1 $\gamma(\Omega_\phi)$ dependence without feedback and for different wall radii: (1) $r_w = 1.1$ m, (2) $r_w = 1.2$ m and (3) $r_w = 1.3$ m. The solid lines represent the real part of γ, the growth rate, while the doted lines the imaginary part, the rotation of the mode.



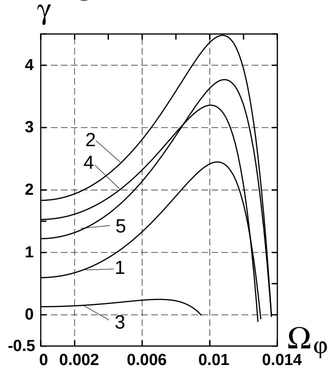
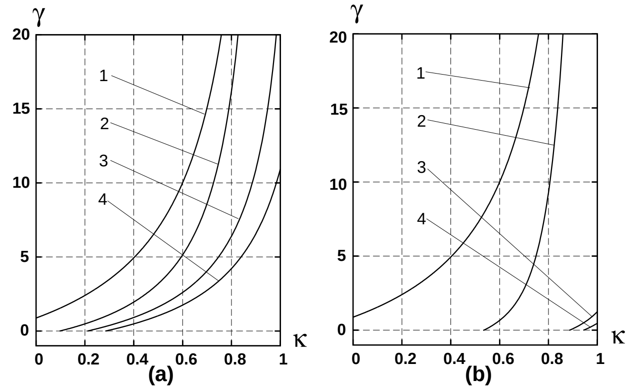


Fig. 2 $\gamma_r(\Omega_\phi)$ dependence with feedback and for different wall resistivities. (1) $\eta_1 = \eta_{Al},\ \eta_2 = \eta_{ss},\ G_p = 5.5,\ G_d = 31$, (2) $\eta_1 = \eta_{Al},\ \eta_2 = \eta_{Al},\ G_p = 5.5,\ G_d = 31$, (3) $\eta_1 = \eta_{ss},\ \eta_2 = \eta_{ss},\ G_p = 5.5,\ G_d = 31$, (4) $\eta_1 = \eta_{ss},\ \eta_2 = \eta_{Al},\ G_p = 5.5,\ G_d = 31$, (5) $\eta_1 = \eta_{Al},\ \eta_2 = \eta_{ss},\ G_p = 2.75,\ G_d = 15.5$. $\eta_{Al} = 0.465 \times 10^{-07}\ \Omega$m and $\eta_{ss} = 0.9 \times 10^{-06}\ \Omega$m.

The growth rate dependence on the stability parameter $\kappa = (r_w^{2m} - a^{2m})/(r_c^{2m} - a^{2m})$ [4] is given in Fig. 3, with and without feedback, and for different locations of wall, feedback and detectors coils. The critical radius r_c defines the radius up to which the wall converts the ideal external kink mode into a non-rotating RWM ($r_c = 1.381$ m). Fig. 4 shows the stability parameter κ dependence on toroidal plasma rotation, for different perpendicular plasma edge viscosities $\mu_\perp$. In Figs. 5-6, the growth rate dependences on G_d and G_p are given for different wall materials. The growth rate dependence on the location of the resistive wall r_w, with fixed positions of the detector and feedback coils, is presented in Fig.7. The destabilizing effect by approaching the wall radius to the critical Newcomb radius with no rotation has been found again.

Conclusions

To study the physics of resistive wall mode stability, the Fitzpatrick model has been considered and generalised. We sought to find the necessary rotation which, combined with an appropriate plasma-wall distance and active feedback, can decouple the kink modes from the eddy currents in the shell, leading to the stabilization of the RWMs. An destabilizing factor of the RWMs that we have investigated is the coupling between a



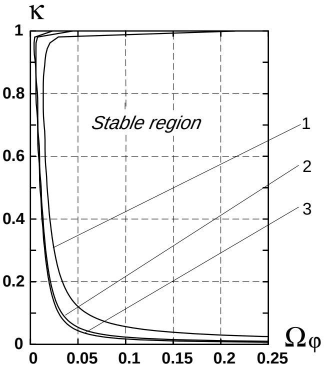
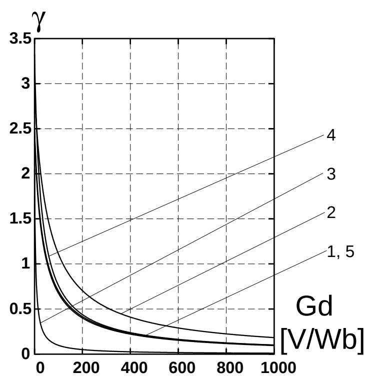
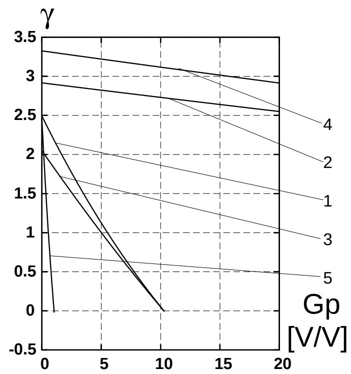


Fig. 3 $\gamma_r(\kappa)$ dependence. $G_p = 5.5$, $G_d = 31$, $\Omega_\phi = -2.883^{-05}$ and $d_{w1} = d_{w2} = .001$ m. **(a)** (1) without feedback; (2) $r_f - r_w = .1$ m and $r_d - r_w = .05$ m; (3) $r_f - r_w = .1$ m and $r_w - r_d = .05$ m; (4) $r_w - r_f = r_w - r_d .05$ m. **(b)** the same parameters, but with $d_{w1} = d_{w2} = .0001$ m. **Fig. 4** $\kappa(\Omega_\phi)$ dependence for different perpendicular plasma edge viscosities $\mu_\perp$. (1) $\mu_\perp = 9 \times 10^{-15}$ kg/m/s; (2) $\mu_\perp = 4.5 \times 10^{-14}$ kg/m/s; (3) $\mu_\perp = 9 \times 10^{-14}$ kg/m/s.





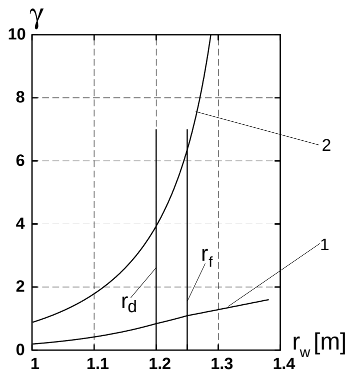


Fig. 5 $\gamma_r(G_d)$ dependence for $G_p = 0$. (1) $\eta_1 = \eta_{Al}$, $\eta_2 = \eta_{ss}$, (2) $\eta_1 = \eta_{Al}$, $\eta_2 = \eta_{Al}$, (3) $\eta_1 = \eta_{ss}$, $\eta_2 = \eta_{ss}$, (4) $\eta_1 = \eta_{ss}$, $\eta_2 = \eta_{Al}$, (5) $\eta_1 = \eta_{Al}$, $\eta_2 = \eta_{ss} \times 10$. $\Omega_\phi = 2.883^{-05}$. **Fig.6** $\gamma_r(G_p)$ dependence for $Gd = 0$. (1) $\eta_1 = \eta_{Al}$, $\eta_2 = \eta_{ss}$, (2) $\eta_1 = \eta_{Al}$, $\eta_2 = \eta_{Al}$, (3) $\eta_1 = \eta_{ss}$, $\eta_2 = \eta_{ss}$, (4) $\eta_1 = \eta_{ss}$, $\eta_2 = \eta_{Al}$, (5) $\eta_1 = \eta_{Al}$, $\eta_2 = \eta_{ss} \cdot 10$. $\Omega_\phi = 2.883^{-05}$. **Fig.7** $\gamma_r(r_w)$ dependence with r_d =const. and r_f =const. (vertical solid lines). (1) $G_d = 31$ and $G_p = 5.5$; (2) without feedback. $\Omega_\phi = 2.883^{-05}$.

RWM and its adjacent modes, due to discrete geometry of both feedback coils and wall. Another result we have obtained was that the angular extent of the feedback coils is important in order to stabilize the RWM. Finally, by translating the stability limits of this current driven mode into two limiting values of β, the dependencies of the growth rate on $C_\beta == (\beta_N - \beta_N^{no-wall})/(\beta_N^{ideal} - \beta_N^{no-wall})$ for different configurations and plasma parameters have been investigated.

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