*34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, D-1.007 (2007)*

Stability of Ion Temperature Gradient Driven Modes in the Presence of a Magnetic Island in Tokamaks

H. R. Wilson[1], D.J. Applegate[1] and J.W. Connor[2]

[1]*Department of Physics, University of York, Heslington, York YO10 5DD UK*

[2]*EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, Oxon OX14 3DB UK*

Abstract The stability of ion temperature gradient driven modes in a sheared slab model of a tokamak plasma containing a chain of narrow magnetic islands is presented. The modification to the pressure and flow profiles in the vicinity of the islands is evaluated self-consistently taking account of the different responses of the electrons and ions to the electromagnetic field. Small amplitude, time-dependent perturbations about this equilibrium are described by linearised gyro-kinetic equations. It is found that the magnetic islands provide a significant stabilising effect on the instability, as well as localising the mode in the vicinity of the island X-point. This would be expected to suppress transport in the vicinity of the rational surface about which the island chain is centred, although there could be a (reduced) remnant electron heat transport that remains, driven by the magnetic islands. This theory may be of relevance for triggering internal transport barriers and NTM threshold physics.

Introduction

There has been much progress in understanding the properties and consequences of small scale micro-instabilities and the turbulence they can drive in tokamak plasmas. Generally, existing theories are based on the assumption that the equilibrium magnetic flux surfaces form a set of nested tori. Theory predicts, however, that under certain situations, magnetic islands can form in the vicinity of rational surfaces. These can be macroscopic structures, such as neoclassical tearing modes (NTMs), or small scale features such as drift tearing or micro-tearing modes. These small scale islands, which typically have a width comparable to the ion Larmor radius, ρ_i, and would be difficult to detect experimentally, are the ones of interest here. In particular, we explore how they affect electrostatic micro-instabilities, working with a reduced model for the ion temperature gradient (ITG) mode as an example. Our approach complements a recent study of a “minimal self-consistent model” of such a system [1].

Theoretical model

Starting with a standard, sheared slab model of the tokamak plasma, we introduce a magnetic island chain through a perturbed flux, ψ, to provide a magnetic field:

$$B = B_0 \nabla z - \nabla\psi \times \nabla z \qquad\qquad \psi = -\frac{B_0 x^2}{2L_s} + \tilde{\psi}\cos K_y y \tag{1}$$

which provides an island chain along the y-direction, of half-width $w = \left(4L_s\tilde{\psi}/B_0\right)^{1/2}$ at the flux surface where x=0. We restrict consideration to long-thin islands satisfying $K_y w$<<1 and work in the rest frame of the island chain. We consider linear perturbations about this equilibrium with characteristic wavenumbers in the x and y-directions of $k_x \sim k_y \sim w^{-1} \sim \rho_i^{-1}$. While full FLR effects can be retained, as will be described in a future publication, here we simplify the system by treating FLR effects perturbatively. We calculate the responses of the electrons and ions to the imposed magnetic perturbation (of the island) and a self-consistent electrostatic potential, Φ, assumed to take the form:

$$\frac{e\Phi}{T_e} = \frac{e\Phi_0}{T_e} + \overline{\Phi} + \varphi e^{-i\omega t} \tag{2}$$

The first contribution is linear in x and represents the equilibrium radial electric field in the island's rest frame; the second term is a time-independent potential, localised about the island caused by the different responses of the electrons and ions to the island; the third term is the electrostatic perturbation associated with the instability, with complex mode frequency ω. A singly charged ion species, of charge e is assumed, and T_e is the electron temperature.

To solve for Φ, we adopt the gyro-kinetic formalism with the standard drift wave ordering $k_{\|}v_{th,e} >> \omega >> k_{\|}v_{th,i}$, where $k_{\|}$ is the wavenumber parallel to the magnetic field and $v_{th,j}$ is the thermal speed of species j. Full non-linearities are retained, except for terms that are non-linear in φ alone. Solving for the ion and electron responses, and imposing quasi-neutrality, we independently balance time-dependent and time-independent terms. The time-independent terms determine $\overline{\Phi}$, the precise form of which depends on the details of the particular cross-field transport model. We do not get involved with this here, but instead adopt the following model, which is consistent with the equations and boundary conditions:

$$\overline{\Phi} = -\frac{\omega_E w}{L_n}\left(\frac{x}{w} - h(\chi)\right) \qquad h(\chi) = \sigma\frac{\left(\sqrt{\chi}-1\right)}{\sqrt{2}}\Theta(\chi-1) \qquad \chi = \frac{2x^2}{w} - \cos K_y y \tag{3}$$

Here $\sigma = x/|x|$, Θ is the Heaviside function and χ is a flux function, defined such that $\chi>(<)1$ is outside (inside) the island. The drift frequency, ω_E, is associated with Φ_0, and represents the island propagation frequency in the ExB rest frame normalised to the electron diamagnetic frequency associated with the island. This form for $\overline{\Phi}$ ensures that the full time-independent piece of Φ is constant on the perturbed island flux surfaces. The density profile is also modified by the island, and is related to the profile for the potential. It takes the form:

$$n_e = n_i = n = n_0\left[1 + \frac{w}{L_n}h(\chi)\right] \qquad \frac{1}{L_n} = \frac{1}{n}\frac{dn}{dr} \tag{4}$$

where n_0 is the density at the rational surface. As shown in Fig 1, this form is continuous at the island separatrix, but does have a discontinuous derivative there (caused by the neglect of diffusion across flux surfaces). The temperature, and therefore pressure, would have a similar form. Equations (3) and (4) show that there are two main effects of the island on the plasma profiles: a strongly sheared *E*x*B* flow and a reduction in the gradient drives close to the rational surface.

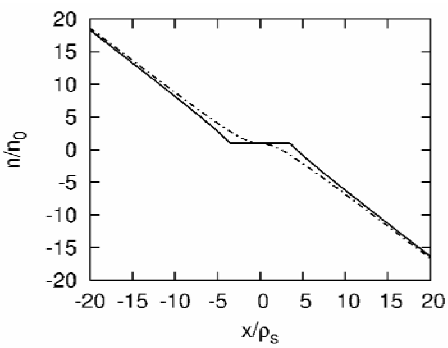


Fig 1: Density profile across the island O-point (full) and X-point (dashed)

We now equate the time-dependent pieces of the quasi-neutrality equation. This yields the following equation for the ITG mode in the presence of the magnetic island, which is valid in the limit that the ratio of the density to temperature gradient length scales, η_i>>1.

$$\frac{\rho_s^2}{\tau}\frac{\partial^2\varphi}{\partial x^2}+\left[\frac{L_n^2\omega_{*e}^2}{L_s^2\tau\rho_s^2(\Omega-\alpha_d S)^2}x^2-\left[\frac{(\Omega-\alpha_d S)-(\omega_{*e}-\alpha_n S/\omega_E)}{(\Omega-\alpha_d S)\tau+(1+\eta_i)(\omega_{*e}-\alpha_n S/\omega_E)}+b\tau^{-1}\right]\right]\varphi=0 \qquad (5)$$

Here, $b=(k_y\rho_s)^2$, ω_{*e} is the electron diamagnetic drift frequency, $\tau=T_e/T_i$, ρ_s is the ion Larmor radius evaluated with the electron temperature, $\Omega=\omega-\omega_{*e}\omega_E$ is the Doppler-shifted mode frequency and S represents the profile modifications:

$$S(x,y)=\omega_{*e}\omega_E\left(1-w\frac{\partial h}{\partial x}\right) \qquad (6)$$

Note that there are two effects of the modified profiles. Those in Eq (5) that are labelled with the parameter α_d represent the effects of a Doppler shift due to the flow shear about the island. Those labelled with α_n represent the effects of the pressure profile modifications. When $\alpha_n=\alpha_d=0$ Eq (5) describes both electron drift modes and ITG modes in the absence of the island. The case $\alpha_n=\alpha_d=1$ describes the modes' stability in the presence of an island.

We focus on the ITG mode here, setting $\eta_i=10$, $\tau=2$, $\rho_s/w=0.28$, $L_s/L_n=-15$, $\omega_E=-0.5$ and $K_y\rho_s=0.031$, for example. With $b=0.18$, we first demonstrate that the effect of the island is indeed to stabilise the ITG mode. Fig 2 shows that the dominant effect is due to the pressure profile modifications and, at least for this case, the shear flow is actually slightly *destabilising*. Equation (5) provides a local eigenvalue, $\Omega=\Omega_0(y,k_y)$, where the y-dependence is slow, arising from the variation in the y-direction due to the island geometry (the y-direction points from one X-point to the next; recall we are considering long, thin islands). The largest growth rate is in the vicinity of the island X-point, which is not surprising as there does remain a pressure gradient there to drive the instability. Nevertheless, there is also a maximum of Ω_0 with respect to y in the vicinity of the O-point. In addition, Ω_0 has a maximum at $k_y\rho_s=0.424$. These

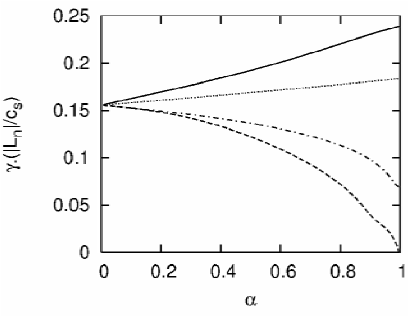
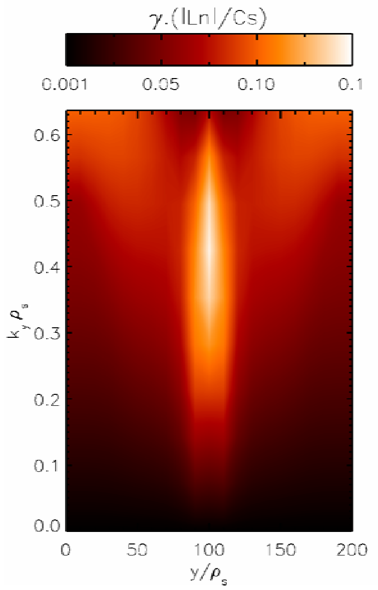


Fig 2: Effect of magnetic island on ITG growth rate fixing α_n=0 and varying $\alpha=\alpha_d$ (across X-point/O-point full/dotted), and varying $\alpha=\alpha_d=\alpha_n$ (across X-point/O-point) dot-dashed/dashed).



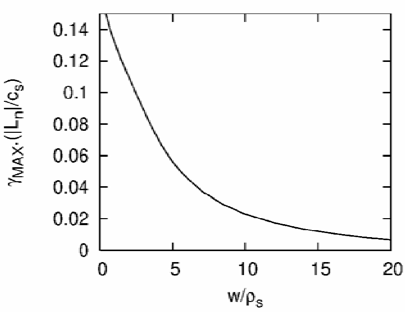


Fig 4:(Above) Maximum growth rate plotted as a function of island width w/ρ_s

Fig 3:(Left)Contour plot of $Im(\Omega_0(y,k_y))$ for the parameter set given in the text.

facts are illustrated in Fig 3, which shows a contour plot of $\Omega_0(y,k_y)$ for the parameter set given above (y=0 and 200 are at the O-point, y=100 is the X-point). In Fig 4, we consider the effects of varying the island width. As expected, larger islands have a greater stabilising influence due to their larger reduction of the pressure gradient. Nevertheless, even quite small islands, $w\sim 4\rho_s$, still have a significant stabilising effect.

This local analysis provides a local complex mode frequency, $\Omega_0(y,k_y)$. A WKB analysis is performed to determine the mode structure in the y-direction and also how the true mode frequency is related to Ω_0. The results are: (1) the mode is localised around the most unstable position (ie at the X-point, where $y=y_X$) within a width $\Delta y/L\sim(k_yL)^{-1/2}<<1$ ($L=2\pi/K_y$ is the island length); (2) the value of k_y must be chosen so that $\Omega_0(y,k_y)$ is stationary with respect to k_y (ie $k_y=k_{y0}$); (3) $\Omega_0(y_X,k_{y0})$ then represents the complex mode frequency of that localised mode.

In summary, we have shown that a magnetic island chain can both suppress the growth rate of ITG modes, and restrict the area of flux surface over which it is unstable to the region around the island X-points. This would suppress transport in both the ion and electron channels, but leave a residual electron thermal transport due to the islands. We propose this as a possible mechanism that contributes to the creation of transport barriers. It will also have implications for threshold models for NTMs.

References

[1] C.J. McDevitt and P.H. Diamond, Phys. Plasmas 13, 032302 (2006)

Acknowledgements: This work was funded by the United Kingdom Engineering and Physical Sciences Research Council and in part by the European Communities under the contract of Association between EURATOM and UKAEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.