

Non-locality and perturbative transport

D.del-Castillo-Negrete¹, P.Mantica², V.Naulin³, J.J. Rasmussen³ and JET EFDA contributors[#]

¹*Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-8071*

²*Istituto di Fisica del Plasma, Associazione Euratom-ENEA-CNR, Milano, Italy*

³*Association EURATOM - Risoe National Laboratory Technical University of Denmark*

Perturbative transport experiments aim at following the transient response of the plasma to externally applied small perturbations, e.g. plasma edge cooling and heating power modulation. They provide valuable time dependent transport information in a relatively controlled setting that can be used for validating and testing transport models [1]. Perturbative experiments have been carried out in the JET tokamak to study electron heat transport [2]. One puzzling piece of evidence is that, whilst heat waves from ICRH power modulation can be adequately explained by local transport models based on the critical gradient concept, cold pulses from the edge reach the plasma core much faster than expected from the modulation observations, hinting to non-local transport processes.

Previous attempts to describe these experiments using local models have found problematic to reconcile the fast propagation of the cold pulses with the comparatively slower propagation of the heat modulation waves [3], whilst models featuring turbulence spreading got somehow more promising results [4]. The goal of this paper is to show that these experimental observations can be quantitatively described using a recently proposed non-local transport model based on the use of fractional diffusion operators [5]. The starting point is the heat transport equation

$$\partial_t [3/2 n_e T_e] = -\partial_x [q_d + q_{nl}] + S \quad (1)$$

where n_e and T_e are the electron density and temperature, x is a normalized radial coordinate, S is the source, and the flux, $q_d + q_{nl}$, consists of a local diffusive channel, $q_d = -n_e \chi_d \partial_x T_e$, and a nonlocal transport channel q_{nl} .

Non-locality is believed to play an important role in non-diffusive plasma transport in general and in the fast propagation of pulses in particular [6]. Within the standard diffusive transport model some level of non-locality can be incorporated through non-trivial nonlinear dependences of the effective diffusivity χ_d on the transported fields and their gradients. However, here we explore a more direct approach based on the use of transport

[#] See the Appendix of M.L. Watkins et al., Fusion Energy 2006 (Proc. 21st Int. Conf. Chengdu, 2006) IAEA, (2006)

operators that replace the local flux-gradient relation by a non-local integro-differential relation of the form $q_{nl}(x) = -\chi_{nl} n_e \partial_x \int K(x-y) T_e(y) dy$ where the function $K(x-y)$ accounts for the non-local contribution of the temperature at point y to the flux at point x . The decay of the function K measures the degree of non-locality. As expected, in the case of a Dirac delta function, $K = \delta(x-y)$, the local diffusive model is recovered. The type of non-local model is determined by the specific form of the function K . Here, following Ref.[5] we consider algebraic decaying function of the form $K \sim 1/(x-y)^{(\alpha-1)}$, and write the non-local flux as

$$q_{nl}(x,t) = -\chi_{nl} n_e \partial_x \left[l \int_0^{\dagger} \frac{T_e(y,t)}{(x-y)^{\alpha-1}} dy + r \int_x^{\dagger} \frac{T_e(y,t)}{(y-x)^{\alpha-1}} dy \right] \quad (2)$$

where $1 < \alpha < 2$, l and r are constants, and χ_{nl} is the non-local diffusivity. The first term on the right hand side of Eq.(2) represents the non-local contribution of the flux at x from the plasma between the core located at 0 and point x , whereas the second term on the right hand side of Eq.(2) represents the contribution from the plasma between x and the edge located at $x=l$. The relative weight of these two terms is determined by l and r . Here we assume $l=r$. In principle one could use a different function $K(x-y)$ to define the nonlocal operator. However, there are strong physical, analytical, and computational reasons to choose algebraic decaying functions. In Fourier space, Eq.(2) takes the form $\hat{q}_{nl} = -\chi_{nl} n_e [l(-ik)^{\alpha-1} - r(ik)^{\alpha-1}] \hat{T}(k)$. As expected, in the limit $\alpha=2$, $l=r$, q_{nl} reduces to the local flux in Eq.(2). The limit $\alpha \rightarrow 1$ is less trivial but it can be shown [5] that in this case (3) reduces to the non-local flux for Landau-fluid closures. Thus, depending on the value of α , the proposed non-local flux interpolates between the local diffusive flux and a free streaming flux. For general α , the scaling $\hat{q}_{nl} \sim k^{\alpha-1}$ motivates the interpretation of the operator on the right hand side of Eq.(2) as a fractional derivative of order $\alpha-1$, i.e. $q_{nl} \sim \partial_x^{\alpha-1} T$. There is also a very appealing interpretation of Eq.(2) in the context of self-similar, non-Brownian stochastic process without a characteristic transport scale. This motivates the use of the non-local model in Eq.(2) to describe scale-free, self-similar turbulent transport in plasmas as discussed in Refs.[7] for the case of resistive pressure-gradient driven turbulence.

In the results presented here, we assume $\alpha=1.25$, $\chi_d = (0.75 + 6x) \text{m}^2/\text{sec}$ and $\chi_{nl} = 2 \text{m}^\alpha/\text{sec}$ for $x > 0.1$ and zero elsewhere. Details on the numerical method can be

found in Ref.[5]. The source $S = P_0(x) + P_m(x, t)$ on the right hand side of Eq.(1) is taken from the experiment and it consists of a steady state including Ohmic heating, NBI heating and off-axis RF heating, and a time modulated RF source with frequency $\nu = 14.5\text{Hz}$. The density, $n_e = 2.6 \times 10^{19} \text{ part/m}^3$ is assumed constant in time and uniform in space. As shown in Fig.1 for these parameter values, the fractional transport model is able to reproduce the profiles of the amplitude and the phase of the dominant Fourier harmonics of the electron temperature corresponding to the propagation of the heat wave excited by the ICRH modulation. Similar agreement has also been obtained with other models including critical gradient models and turbulence spreading models.

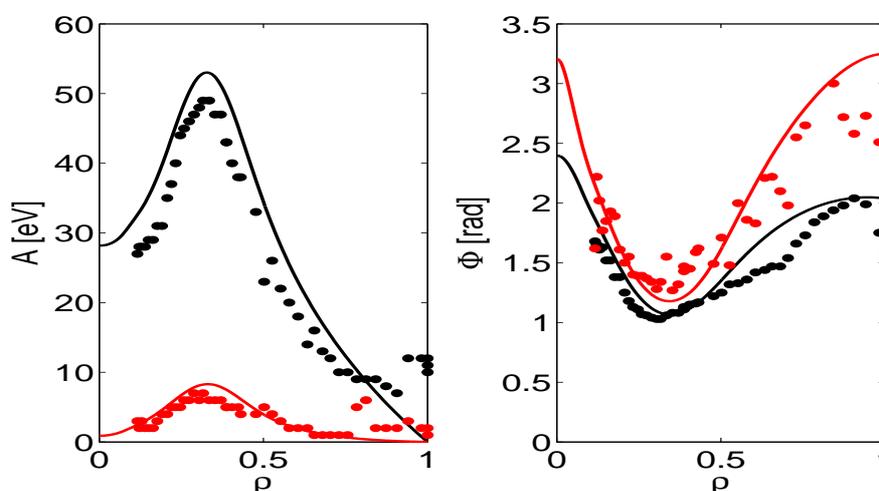


Fig.1 Experimental (dots) and fractional model (lines) profiles of A and Φ corresponding to the 1st (black) and the 3rd (red) Fourier harmonics of the electron temperature.

However, the critical issue is to be able to reproduce for these *same* parameter values and conditions the fast propagation of the pulse, something that previous models has not been able to accomplish [3,4]. Figure 2 shows that the fractional model can successfully accommodate the propagation of pulses with speeds comparable to those observed in the experiment while still retaining the slower propagation of the modulation heat pulses.

Summarizing. Recent JET experiments show an asymmetry between the propagation of perturbations due to heat modulation and cold pulses. For $\rho > 0.3$ waves and pulses propagate fast. For $\rho \leq 0.3$ the heat wave slows down and is damped, but cold pulses still travel fast. Local transport models are not able to reproduce these results. We have shown that a recently proposed non-local transport model based on the use of fractional diffusion operators is able to reproduce these results. In particular the fractional diffusion model

reproduces the radial dependence of the amplitude and phase of the modulation experiments. Most importantly, for the same model parameters, it describes the observed fast propagation of the pulses.

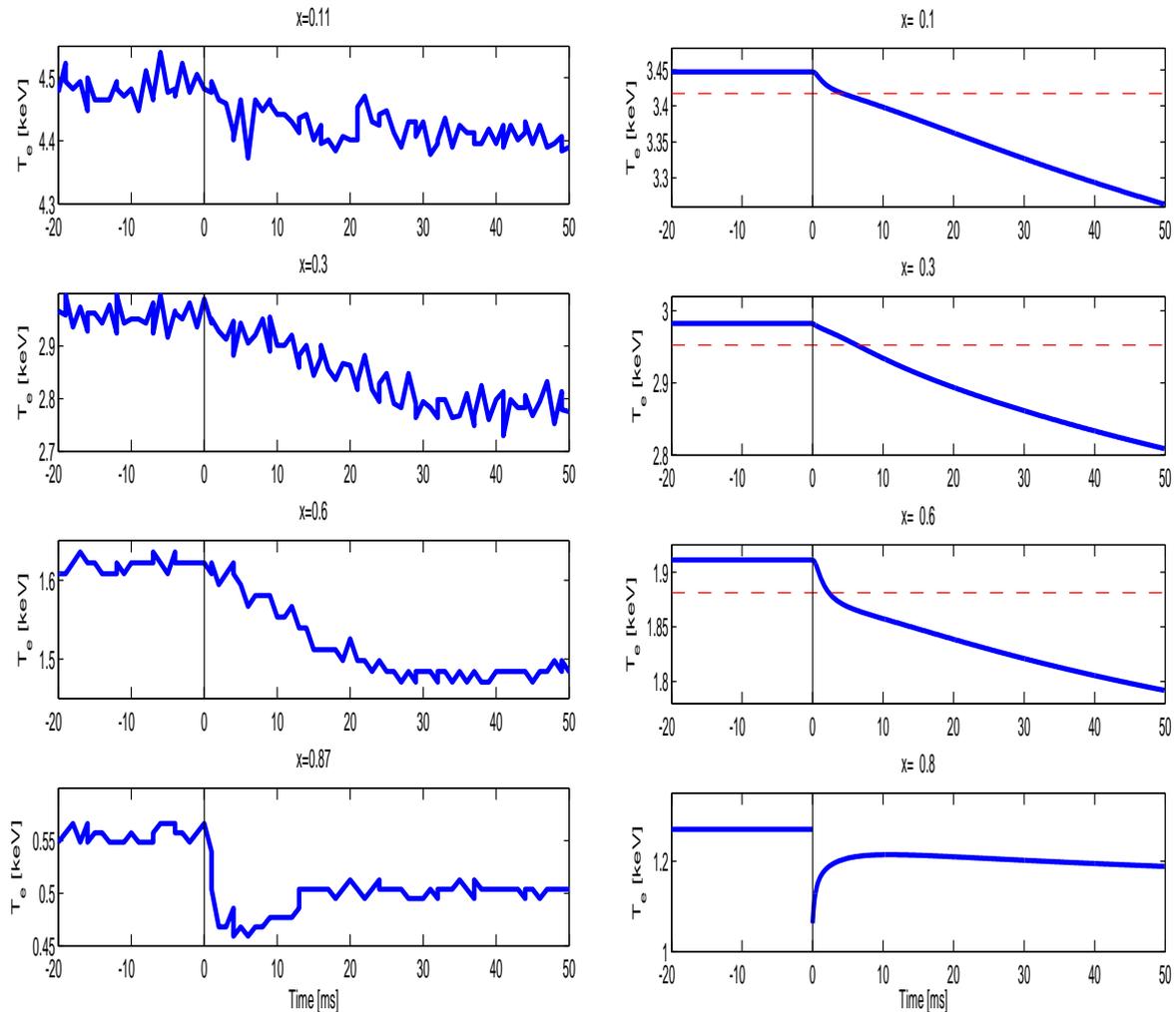


Fig.2 Comparison between the temperature traces in the experiment (left panel) and the model (right panel). Consistent with the experiment, the model exhibits a drop of 30 eV (corresponding to the dashed red line) in about ~4ms.

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- [1] P. Mantica and F. Ryter, C.R. Physique **7**, 634-649 (2007).
- [2] P.Mantica et al., Proc.19th Intern. Conf. on Fusion Energy, Lyon [IAEA,Vienna,2002] EX/P1-04.
- [3] P.Mantica et al., "Fast core response to edge cooling in JET:experiments and modelling", oral presentation at 11th EU-US Transport Task Force Workshop, Marseille, France, 2006, <http://www-fusion-magnetique.cea.fr/ttf2006/prog/drafts/025.pdf>
- [4] J.J. Rasmussen et al., 33rd EPS Conf. on Plasma Phys. Rome, June 2006 ECA Vol.**30I**, P-1.076 (2006); Naulin V, Nielsen AH, Rasmussen JJ Phys. Plasmas **12** (122306 (2005).
- [5] D. del-Castillo-Negrete, Phys. Plasmas **13**, 082308 (2006).
- [6] J.D. Callen, and M.W. Kissick. Plasma Phys. Controlled Fusion **39**, B173 (1997).
- [7] D. del-Castillo-Negrete et al., Phys. Rev. Lett. **94**, 065003 (2005); Phys. Plasmas **11**, 3854 (2004).