*34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, O-4.031 (2007)*

Theoretical Resolution of Magnetic Reconnection in High Energy Plasmas*

B. Coppi

Massachusetts Institute of Technology, Cambridge, USA

1. Introduction

Magnetic reconnection in well confined, high temperature plasmas is involved in the onset and evolution of macroscopic modes. Among these the so called drift-tearing mode [1] is particularly, important, has a basic dependence on the electron density and temperature gradients play a basic role, has an intrinsic frequency of oscillation related to these gradients, and is characterized by both a slower growth rate and significantly different eigenfunctions when compared to the purely resistive tearing mode [2,3]. The mode primary driving factor is the longitudinal current density gradient and can lead to the formation of relatively large magnetic islands. The excitation threshold is the same as that of the resistive tearing mode, as long as the relevant electron equation of state is adiabatic [1]. However, the mode was found to be practically impossible to excite in collisionless regimes [4] as a result of the combined effects of electron Landau damping and temperature gradient [4] or of the equivalent effects of parallel longitudinal electron conductivity and temperature gradient [5] in weakly collisional regimes. At the same time, as experiments with lower degrees of collisionality have been undertaken [6], modes of this kind, that produce reconnection, and their effects have been observed to persist. In order to resolve this paradox [7] we consider [8] that "mesoscopic reconnecting" modes develop from a background of "micro-reconnecting" modes with short scale distances ($\lesssim d_e \equiv c/\omega_{pe}$) that generate a series of strings of small magnetic islands and are driven by the electron temperature gradient. The envisioned effects of the latter modes are to produce a significant increase of the ratio of the transverse to the longitudinal thermal conductivity. This is shown to restore the excitation of drift-tearing modes involving the combined effects of finite resistivity, electron thermal conductivities, and temperature gradients.

2. Micro-reconnecting Modes and Their Role

In view of extending our results to more complex configurations we refer to a plane geometry where the magnetic field around a surface $x = x_0$ is represented as $\mathbf{B} \simeq B_z(x)\mathbf{e}_z + (x - x_0)B_y'\mathbf{e}_y$. A background of excited microscopic modes, localized over around successive surfaces $x = x_j$ contained in a finite interval centered on $x = x_0$, is considered to form. These "micro-reconnecting" modes are collisionless and driven by the electron temperature gradient. The relevant longitudinal field $\hat{E}_{\parallel} = -\nabla_{\parallel}\hat{\Phi} - (1/c)\partial\hat{A}_{\parallel}/\partial t$, while $\hat{\mathbf{E}}_{\perp} \simeq -\nabla_{\perp}\hat{\Phi}$, where $\hat{A}_{\parallel} = \tilde{A}_{\parallel}(\Delta x_j)\exp(-i\omega t + ik_y y)$, $\Delta x_j \equiv x - x_j$ and $\hat{A}_{\parallel} = \tilde{A}_{\parallel}(\Delta x_j)$ is an even function of Δx_j. The frozen-in condition is broken by the effects of finite electron inertia and as we consider $k_{\perp}^2\rho_e^2 < k_{\perp}^2 d_e^2 \lesssim 1$. Moreover $\omega^2 \gtrsim k_{\parallel}^2 \mathrm{v}_{the}^2$ for $\mathrm{v}_{the}^2 = 2T_e/m_e$ and $\omega^2 < k_{\perp}^2 \mathrm{v}_{thi}^2$ where $\mathrm{v}_{thi}^2 = 2T_i/m_i$ and $k_{\perp}^2\rho_i^2 \gg 1$. Here ρ_e and ρ_i indicate the electron and the ion gyroradius, respectively. We note that $\omega = \omega_R + i\omega_I$ and ω_R depends on the values of $\eta_e = (\mathrm{d}\ln T_{e\parallel}/\mathrm{d}x)/(\mathrm{d}\ln n/\mathrm{d}x)$. The dispersion equation for $\tilde{A}_{\parallel}$, for $\omega \lesssim \omega_{*Te} \equiv -k_y c/(eB)\, dT_e/dx$ and $\eta \gg 1_e$, is derived by the "drift" approximation considering that $\hat{n}_i \simeq \hat{n}_e \simeq -ne\hat{\Phi}/T_i \simeq (k_{\parallel}\hat{u}_{e\parallel}/\omega)n$, $\hat{u}_{e\parallel} = -\hat{J}_{\parallel}/(ne)$ is the longitudinal electron flow velocity, $\hat{J}_{\parallel} = -c/(4\pi)\left[\partial^2/\partial x^2 - k^2\right]\hat{A}_{\parallel}$, $k_{\parallel} \simeq k_y B_y'(\Delta x_j)/B$ where $B_y'/B \equiv 1/L_s$. The quadratic form that can be obtained from the dispersion equation in order to estimate the mode frequency and growth rate is

$$d_e^2\left\langle\left|\frac{d\tilde{A}_{\parallel}}{dx}\right|^2 + k^2\left|\tilde{A}_{\parallel}\right|^2\right\rangle \simeq \left\langle\left|\tilde{A}_{\parallel}\right|^2 \frac{\omega^2\omega_{*Te}\Im_0^0}{\omega^3 + k_{\parallel}^2 c_{se}^2\omega_{*Te}\Im_0^0}\right\rangle. \tag{1}$$

Here

$$\langle\ \rangle \equiv \int dx\,, \Im_0^0 \equiv \frac{1}{n}\int_{-\infty}^{+\infty} d\mathrm{v}_{\parallel} F_{Me}(\mathrm{v}_{\parallel}^2)\left(\frac{m_e \mathrm{v}_{\parallel}^2}{T_e}\right)\frac{\mathcal{L}(\mathrm{v}_{\parallel}^2)\bar{\bar{\omega}}^2 - \bar{\bar{\omega}}^3}{\bar{\bar{\omega}}^2 - k_{\parallel}^2\mathrm{v}_{\parallel}^2/\omega_{*Te}^2}, F_{Me} = \left[1/\left(\sqrt{\pi}\mathrm{v}_{the}\right)\right]n\exp\left(-\mathrm{v}_{\parallel}^2/\mathrm{v}_{the}^2\right),$$

$\bar{\bar{\omega}} \equiv \omega/\omega_{*Te}$, $c_{se} \equiv (T_i/m_e)^{1/2}$, $\mathcal{L}(\mathrm{v}_{\parallel}^2) = \mathrm{v}_{\parallel}^2/\mathrm{v}_{the}^2 - 1/2$. The "fluid limit" corresponds to $\Im_0^0 \simeq 1$. The resulting effective transverse thermal diffusion coefficient on the basis of the relevant quasi-linear theory, can be expected to be the order of $D_{\perp}^e \sim \alpha_d\,(d_e/r_{Te})cT_e/(eB)$, where α_d is a numerical coefficient, and $1/r_{Te} \equiv -\mathrm{d}\ln T_e/dx$, and we consider the ratio $\Delta_{th} \equiv D_{\perp}^e/D_{\parallel}^e$ to be increased further relative to the classical value by the expected reduction of the longitudinal thermal diffusivity $D_{\parallel}^e$ resulting from the excitation of the same modes.

3. Mesoscopic Mode and Relevant Electron Thermal Energy Balance Equation

The perturbed magnetic field, for the mesoscopic mode, is represented by $\hat{\mathbf{B}} = \tilde{\mathbf{B}}(x)\exp(-i\omega t + ik_y y)$. The theory of this mode involves the analysis of three asymptotic regions of which the outer (macroscopic) region involves scale distances of the order of the radius a of the plasma column. In this region the "hyperconductivity" condition $\hat{\mathbf{E}} + \hat{\mathbf{v}} \times \mathbf{B}/c = 0$ is held to be valid and the perturbed magnetic field is described by the quasi-neutrality condition $\nabla \cdot \mathbf{J}\big)_{tot} \simeq \nabla_{\parallel} J_{\parallel}\big)_{tot} = 0$, as the effects of finite ion inertia can be neglected. In the transition region, the relevant longitudinal momentum conservation equation is

$$0 \simeq -\nu_{ei}^{\parallel} n m_e \hat{u}_{e\parallel} - en\hat{E}_{\parallel} - \mathbf{B} \cdot \left(\nabla p_e + \alpha_T n \nabla T_e\right) \big/ \mathrm{B} - \hat{\mathbf{B}} \cdot \left(\nabla \hat{p}_e + \alpha_T n \nabla \hat{T}_e\right) \big/ \mathrm{B}, \qquad (2)$$

whose components have standard definitions. The adopted electron thermal energy balance equation is $(3n/2)\left(\partial \hat{T}_e/\partial t + \hat{V}_{E*}\, dT_e/dx\right) \simeq nT_e \nabla_{\parallel} \hat{u}_{e\parallel} \simeq -\nabla_{\perp} \cdot \hat{\mathbf{q}}_{e\perp} - \nabla \cdot \left(\hat{q}_{e\parallel}\, \mathbf{B}/\mathrm{B} + \hat{\mathbf{B}} q_{e\parallel}/B\right)$, $\hat{V}_{Ex} \simeq -ick_y \hat{\Phi}/B$, $\nabla_{\perp} \cdot \hat{\mathbf{q}}_{e\perp} \simeq -D_{\perp}^{e}\left(\partial^2 \hat{T}_e/\partial x^2\right) 3n/2$, $\quad -\nabla \cdot \left(\hat{\mathbf{B}} q_{e\parallel}/B + \mathbf{B}\hat{q}_{e\parallel}/B\right) \simeq ik_{\parallel}\left\{-D_{\parallel}^{e}\left[ik_{\parallel}\hat{T}_e + \left(\hat{B}_x/B\right)\left(dT_e\right)/dx\right]\right\}$ and $k_{\parallel} \simeq k_y B_y' \left(x - x_0\right)/B \equiv k_y \left(x - x_0\right)/L_s$. We note that the terms associated with the thermal conductivities for the mesoscopic mode are prevalent. It is not contradictory to envision weakly collisional mesoscopic modes excited from a background of collisionless microscopic modes as the relevant frequencies and growth rates have very different values. In fact, the typical frequency of both modes is closely related to $\omega_{*Te} \equiv k_y c T_e / \left(eBr_{Te}\right)$. Therefore we require that the mean free path $\lambda_{ei} = \mathrm{v}_{\mathrm{the}}/\nu_{\mathrm{ei}}$ be in the following interval $1/\beta_e^{1/2} < \lambda_{ei}/\left(2r_{Te}\right) < \left(a/\rho_e\right)\left|\omega_*^T\right|/\gamma$ that corresponds to a realistic range of plasma parameters. We note also that while in the case of the microscopic modes $\gamma \equiv \mathrm{Im}\,\omega \gtrsim \mathrm{Re}\,\omega$, in the case of the mesoscopic modes $\gamma \ll \left|\mathrm{Re}\,\omega\right| \sim \omega_{*Te}$. Thus we identify an intermediate and an innermost region by the scale distances

$$\delta_I \equiv \Delta_{th}^{1/4}\left(\frac{L_s}{k}\right)^{1/2}, \qquad \delta_c \equiv \left(\frac{\varepsilon_*}{\Delta_{th}}\right)^{1/4} \delta_I \qquad (3)$$

where

$\Delta_{th} \equiv D_{\perp}^{e}/D_{\parallel}^{e}$, $\varepsilon_* \equiv D_m/D_A$, $D_m = \nu_{ei}^{\parallel}\, c^2/\omega_{pe}^2$, $D_A \equiv \mathrm{v}_A^2 \big/ \left|\omega_{e\parallel}^{T} - \omega_{di}\right|$, $\omega_{di} = c\left(dp_i/dx\right)k_y/\left(eBn\right)$, $\omega_{e\parallel}^{T} = \left(1 + \alpha_T\right)\omega_{*Te} + \omega_{*e}$ and $\omega_{*e} = -k_y c T_e \left(dn/dx\right)/\left(enB\right)$. Matching the solution of the outer region to that of the inner regions leads to find [8]

$$\omega \simeq \omega_{*\parallel}^{T} + \delta\omega_1 + \delta\omega_2 \text{ where } -i\delta\omega_1 \simeq D_m \frac{\Delta' k}{\varepsilon_m^{1/4}} \frac{1}{\Im_0} \sim D_m^{\;3/4} D_A^{\;3/4} \frac{k^{3/2}}{L_s^{1/2}}, \qquad (4)$$

for $\Delta' \sim k$, and

$$-i\delta\omega_2 \simeq \left|\omega_{*\parallel}^{T}\right| \frac{\varepsilon_*^{3/4}}{\Delta_{th}^{3/4}} G_0 \frac{1}{\Im_0} \sim \left|\omega_{*\parallel}^{T}\right| \left(\frac{D_m}{D_{\perp}^{e}}\right)^{3/4} \left(\frac{D_{\parallel}^{e}}{D_A}\right)^{3/4} \qquad (5)$$

where $\Im_0 \equiv \int d\bar{x}_c \left\{1+\alpha_*^T \bar{x}_c \bar{\mathcal{L}}_c\right\} \equiv \Im_{0R} + i\Im_{0I}$, $\Im_{0R}$ and G_0 are positive [8] dimensionless quantities $\left(\Im_{0R} \simeq 2/\left|\alpha_*^T\right|^{1/4} G_0 \simeq 2.2\right)_0$. Likewise, we can verify that $\mathrm{Re}\left(\delta\omega/\omega_{*\parallel}^T\right) < 0$. Thus the considered modes, are excited even when $\Delta' < 0$ in the considered regime where $\Delta_{th} > \varepsilon_*$, and the basic drift-tearing modes cannot be excited. We note that the width of the innermost region δ_c decreases quite slowly with temperature and it is clear that the linearized theory breaks down when the width of the macroscopic magnetic islands that it can produce, as indicated by $\delta_{Is} \equiv \left|\tilde{B}_{x0}/B'_y k\right|^{1/2}$, becomes the order of δ_c. This corresponds to $\left|\tilde{B}_{x0}\right|/B \sim \varepsilon_*^{1/2}$ that is equivalent to $\tilde{\Omega}_{ci}\tilde{\Omega}_{ce} \sim \nu_{ei}\omega_{*\parallel}^T$, where $\tilde{\Omega}_{ci} \equiv e\tilde{B}_{x0}/(m_i c)$ and $\tilde{\Omega}_{ce} \equiv \tilde{\Omega}_{ci}\, m_i/m_e$. Thus we may conjecture that when this limit is reached the effective value of δ_c will continue to grow until that of δ_I is reached and that this will be the island saturation size. It is evident that using the so-called Rutherford model, that is appropriate for the highly resistive tearing mode [2], is not justified for the weakly collisional regimes to which the present theory [8] applies.

It is a pleasure to thank C. Crabtree and V. Roytershteyn for their collaboration on different parts of the subject treated in this paper. In particular, V. Roytershteyn [9] described first numerically, the transition from the case where $D_\perp^e/D_\parallel^e$ is negligible to where it is not. *Sponsored in part by the U.S. Department of Energy.

[1] B. Coppi, *Phys. Fluids* **8,** 2273 (1965).

[2] H.P. Furth, J. Killeen, M.N. Rosenbluth, *Phys. Fluids* **6,** 459 (1963).

[3] B. Coppi, J.M. Greene, and J.L. Johnson, *Nucl. Fus.* **6**, 101 (1965).

[4] B. Coppi, J.W.-K. Mark, L. Sugiyama, G. Bertin, *Phys. Rev. Letters* **42,** 1058 (1978).

[5] J. Drake, T.M. Antonsen, A.B. Hassam, *et al*., *Phys. Fluids* **26,** 2509 (1983).

[6] J.A. Snipes, R.S. Granetz, R.J. Hastie, *et al*., *Plasma Phys. Cont. Fus.* **44,** 381(2002).

[7] B. Coppi, C. Crabtree, P.H. Diamond, O. Gurcan, C.J. McDevitt, V. Roytershteyn, Paper TH/R2-19, *2006 I.A.E.A. Conf. Fus. Energy, Chengdu, (October 2006).*

[8] B. Coppi, MIT (LNS) Report 06/11, in print for "*Collective Phenomena in Macroscopic Systems*" Eds. G. Bertin *et al., Publ. World Scientific, 2007.*

[9] V. Roytershteyn, Massachusetts Institute of Technology, Physics Department, Ph.D. Thesis (October 2006).