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Kovásznay modes in stability of self-similar ablation flows of ICF

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Exact self-similar solutions of gas dynamics equations with nonlinear heat conduction for semi-infinite slabs of perfect gases are used for studying the stability of ablative flows in inertial confinement fusion. Both the similarity solutions and their linear perturbations are numerically computed with a dynamical multidomain Chebyshev pseudo-spectral method. Laser-imprint results, showing that maximum amplification occurs for a laser-intensity modulation of zero transverse wavenumber have thus been obtained [1]. Here we pursue this approach by proceeding to an analysis of perturbations in terms of Kovásznay modes [2]. We have derived the exact propagation equation for the acoustic mode and the exact evolution equations for the entropy and vorticity modes.

Self-similar mean flow

The hydrodynamic equations for one-dimensional compressible flow with non linear heat conduction [3], written in terms of the Lagrangian variable m with $\mathrm{d}m = \overline{\rho}\,\mathrm{d}x$, come as

$$\begin{aligned} &\partial_t\left(1/\overline{\rho}\right) - \partial_m \overline{v_x} = 0, \qquad && \partial_t \overline{v_x} + \partial_m \overline{p} = 0, \\ &\partial_t\left(\overline{v_x^2}/2 + \overline{\mathscr{E}}\right) + \partial_m\left(\overline{p v_x} + \overline{\varphi_x}\right) = 0, \quad \text{with} \quad \overline{\varphi_x} = -\overline{\chi}\,\overline{\rho}^{1-\mu}\,\overline{T}^{\nu}\,\partial_m \overline{T}, \quad \mu \geq 0,\ \nu \geq 1, \end{aligned} \tag{1}$$

along with the perfect gas equation of state $p = R\rho T$, where the symbols have their usual meanings. Initial and boundary conditions are of the form

$$\overline{\rho}(m,0) = \overline{\rho}_{\mathrm{i}}, \quad \overline{v_x}(m,0) = 0, \quad \overline{T}(m,0) = 0, \quad \text{for} \quad m \geq 0, \tag{2}$$

$$\overline{p}(0,t) = \overline{p}_*\,(t/t_*)^{2(\alpha-1)}, \quad \overline{\varphi_x}(0,t) = \overline{\varphi}_*\,(t/t_*)^{3(\alpha-1)}, \quad \text{with} \quad \alpha = \frac{2\nu-1}{2\nu-2}, \tag{3}$$

where $\overline{p}_*$, $\overline{\varphi}_*$ and t_* are some characteristic pressure, heat flux and time. With these initial and boundary conditions, system (1) admits a self-similar formulation. Considering particular solutions of the form $\overline{f}(m,t) = \overline{F}(\xi)$ where $\xi = m t^{-\alpha}$ leads to recasting these equations under the form of a system of ODEs

$$\frac{d\,\mathbf{Y}}{d\xi} = \mathscr{F}(\mathbf{Y},\xi), \tag{4}$$

where $\mathbf{Y} = (\overline{G}\;\overline{V}\;\overline{\Theta}\;\overline{\Phi})^{\mathsf{T}}$ and $\overline{G}$, $\overline{V}$, $\overline{\Theta}$, $\overline{\Phi}$ denote the self-similar variables associated to $\overline{\rho}$, $\overline{v_x}$, $\overline{T}$, $\overline{\varphi_x}$, respectively. Initial and boundary conditions become

$$\overline{G} = 1, \quad \overline{V} = 0, \quad \overline{\Theta} = 0, \quad \text{for } \xi \to \infty, \tag{5}$$

$$\overline{P} = \overline{G}\,\overline{\Theta} = \mathscr{B}_p, \quad \overline{\Phi} = \mathscr{B}_\varphi, \quad \text{for } \xi = 0, \tag{6}$$

$\mathscr{B}_p$ and $\mathscr{B}_\varphi$ being dimensionless numbers based on $\overline{p}_*$ and $\overline{\varphi}_*$.

Perturbed flow

The equations for the linear perturbations are written using an Eulerian description, in the (m,y,z)-coordinate system, as [1]

$$\partial_t \rho + \bar{\rho}\,(\partial_m \bar{\rho}\, v_x + \bar{\rho}\partial_m v_x + \partial_m \bar{v_x}\,\rho + \nabla_\perp \cdot \vec{v}_\perp) = 0, \tag{7a}$$

$$\partial_t v_x + \bar{\rho}\partial_m \bar{v_x}\, v_x + \partial_m p - \partial_m \bar{p}\,\frac{\rho}{\bar{\rho}} = 0, \tag{7b}$$

$$\partial_t \vec{v}_\perp + \frac{1}{\bar{\rho}}\nabla_\perp p = \vec{0}, \tag{7c}$$

$$C_v\,(\partial_t T + \bar{\rho}\partial_m \bar{T} v_x) + \bar{\rho}\,\partial_m \bar{v_x}\, T + \bar{p}\,\partial_m v_x + \partial_m \varphi_x - \partial_m \bar{\varphi_x}\frac{\rho}{\bar{\rho}} + \frac{(\bar{p}\nabla_\perp \cdot \vec{v}_\perp + \nabla_\perp \cdot \varphi_\perp)}{\bar{\rho}} = 0, \tag{7d}$$

where $\nabla_\perp \cdot = (\partial_y ., \partial_z .)^{\mathsf{T}}$. The above system of partial differential equations in physical space is replaced by a 1D system in the *yz*-Fourier space, consisting of (7a), (7b), (7d) along with

$$\partial_t\,(\nabla_\perp \cdot \vec{v}_\perp) - k_\perp^2\, p/\bar{\rho} = 0, \quad \text{with} \quad k_\perp = \sqrt{k_y^2 + k_z^2}, \tag{8}$$

where the same notation has been used for a quantity and its Fourier transform. Boundary conditions are provided at $\xi = 0$ by imposing arbitrary time-dependent density and incident heat-flux perturbations, and at the shock front ($\xi = \xi_s$) by the non-isothermal Rankine-Hugoniot conditions for linear perturbations [1]. System (7a), (7b), (7d) and (8) is solved in the variables $(\xi = m t^{-\alpha}, k_\perp)$. Numerical approximation in the ξ variable is carried out with a dynamical multidomain Chebyshev spectral method [5].

Kovásznay modes

Considering laser irradiation asymmetries in the context of direct drive irradiation, we have obtained space-time evolutions of flow perturbations for a wide range of wavenumbers [1, 4]. Complex wave-like structures of acoustic, vorticity and entropy types have been observed, enticing us to study Kovásznay mode evolutions. The analysis of ablation-front distortion results [1] leads us to focus at first on acoustic mode. The exact propagation equation for the acoustic perturbation has been derived for unsteady and nonuniform flows with, on the left-hand side, d'Alembert's operator, and on the right-hand side, 51 source terms, which reads in compact form

$$\Box p \equiv \left[\frac{\partial^2}{\partial t^2} - \gamma \bar{p}\bar{\rho}\left(\frac{\partial^2}{\partial m^2} - \frac{k_\perp^2}{\bar{p^2}}\right)\right] p = f(\rho) + g(v_x) + k_\perp\, h(\nabla_\perp . \vec{v}_\perp) + q(p, T; k_\perp)\,. \tag{9}$$

Space-time evolutions of these source terms are plotted in the case $k_\perp = 5$ and $t \in [0.05, 10]$. Three regions in space—the shocked fluid region $(0.12 \lesssim \xi < 0.3)$, the ablation layer $(\xi \simeq 0.12)$ and the conduction zone $(0 \leq \xi \lesssim 0.12)$—and two regions in time—short and long times—stand out (Fig.1 a). In particular, it turns out that acoustic wave generation takes principally place in the conduction zone and at the ablation front. In effect, for the shocked fluid

region, the sum of source terms is almost equal to zero, testifying of a quasi-free propagation, namely

$$\Box p \simeq 0. \tag{10}$$

In the conduction zone and in the ablation layer, source terms exhibit complex patterns for short and long times. For these two regions, evaluating and comparing the contribution of the different source terms of the acoustic propagation equation should enable us to propose approximate acoustic wave propagation equations for each zone of the flow for $k_\perp \neq 0$. A previous analysis of such source terms for the particular case $k_\perp = 0$ leads us to the same conclusion for the shocked region.

To complete the Kovásznay mode analysis, the exact evolution equations have been derived for the entropy and vorticity modes. The equation for the entropy perturbation reads

$$\frac{\partial s}{\partial t} = -\overline{\rho} v_x \frac{\partial \overline{s}}{\partial m} + p \frac{\overline{\rho}}{\overline{p}^2} \frac{\partial \overline{\varphi_x}}{\partial m} - \frac{1}{\overline{T}} \frac{\partial \varphi_x}{\partial m} - \frac{1}{\overline{p}} \nabla_\perp \cdot \varphi_\perp, \tag{11}$$

and the equation for the vorticity transverse component $\omega_\perp$ comes as

$$\frac{\partial}{\partial t}\left(\frac{\omega_\perp}{\overline{\rho}}\right) = i k_\perp \frac{1}{\overline{\rho}^2} \left\{ \frac{3\rho}{\overline{\rho}} \left[\overline{\rho} \frac{d\overline{p}}{dm} - \overline{p} \frac{d\overline{\rho}}{dm} \right] + \overline{p} \frac{\partial \rho}{\partial m} - \overline{\rho} \frac{\partial p}{\partial m} + p \frac{d\overline{\rho}}{dm} - \rho \frac{d\overline{p}}{dm} \right\}. \tag{12}$$

Looking at the figures (Fig.1 b, c), we distinguish the two preponderant regions in time and the different regions of the flow. For short times, and especially in the conduction zone and the ablation layer, the entropy and vorticity source contributions are significantly amplified. The behaviour at the origin shows that sums of source terms increase with time for the three modes but with changes of sign for vorticity. Besides the conduction zone is a place where the sums of entropy and vorticity source terms have essentially an oscillatory regime with amplitudes decreasing with the distance from the origin. As for the acoustic mode, the sums of source terms are almost equal to zero in the shocked region. Moreover, the same plot for a wavenumber $k_\perp = 0$ does not show any oscillations of this type, and any damping mechanism in the shocked region. This suggests that these oscillatory phenomenons are probably due to transverse perturbations.

Summary

In this paper, we have presented the exact evolution equations of Kovásznay modes and a preliminary analysis for transverse wavenumber $k_\perp \neq 0$. Approximate equations for acoustic, entropy and vorticity modes may thus be proposed for each region of the flow.

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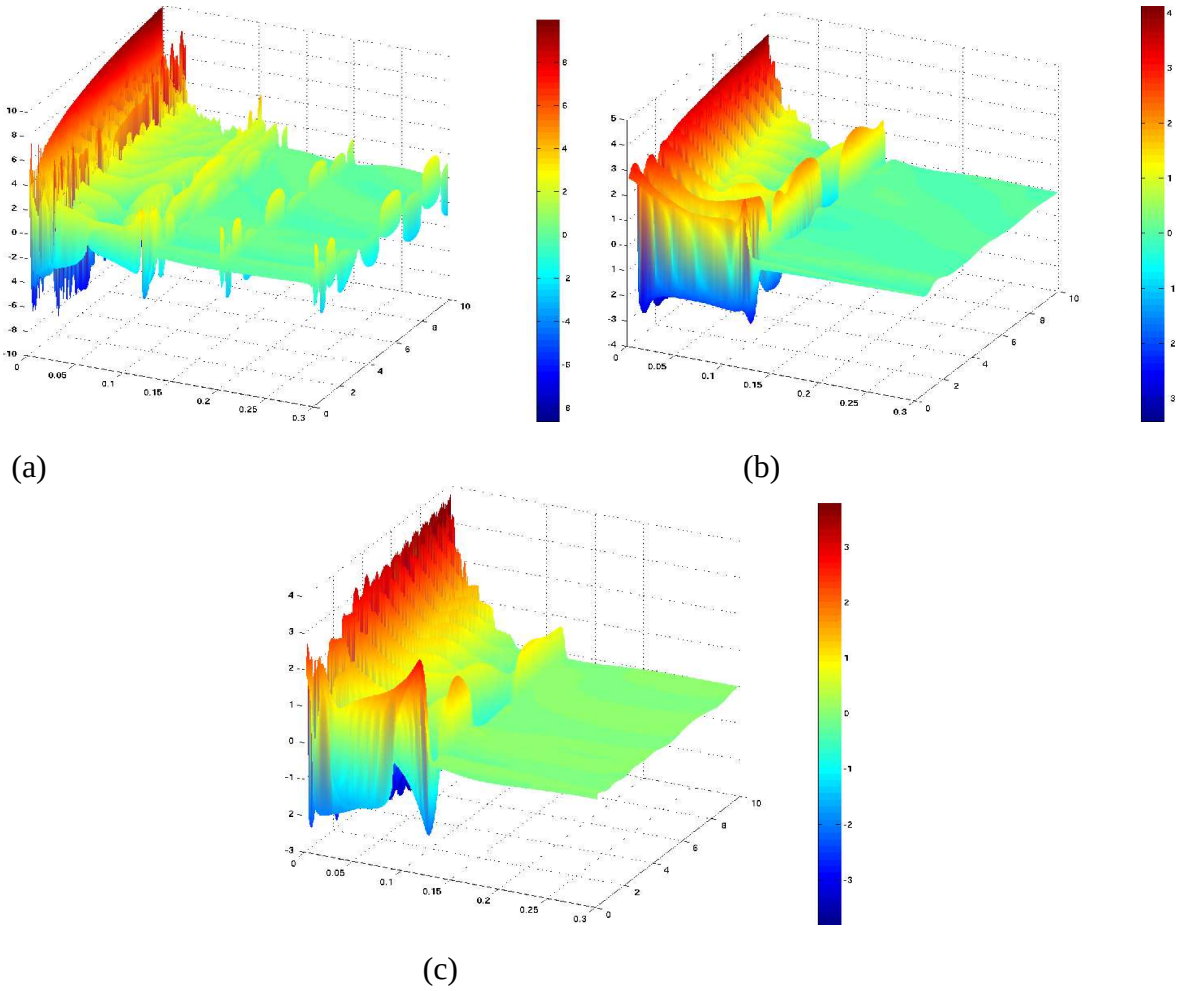

Figure 1: Space-time evolutions (ξ, t) of source terms of, respectively, (a) the acoustic propagation equation (9), (b) the entropy evolution equation (11) and (c) the vorticity evolution equation (12) for $k_\perp = 5$ and $t \in [0.05, 10]$.

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