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THEORY OF ION-MATRIX-SHEATH DYNAMICS

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Abstract

The problem of the time-dependent one-dimernsional unipolar ion sheath (ion-matrix-sheath) is solved analytically. The time dependence of the sheath parameters in the initial stage of the plasma response to the step potential applied to the wall is investigated. The characteristic time of relaxation towards the stationary state is found.

1. Introduction

Due to their much smaller inertia the electrons follow changes of the wall potential instantaneosly as compared to the ions. If in particular, a high negative voltage is applied to the wall, they are repelled from the wall and leave a positive ion sheath (’’matrix’’). The ions extracted from the matrix sheath give rise to a current towards the wall. The width of the ion matrix is assumed to be much smaller than the ion mean free path, therefore the ion motion within the matrix sheath can be considered as collisionless.

To our knowledge, there exist only few publications investigating the different aspects of the time-dependent plasma-wall transition layer [1-4]. We have succeeded to solve the problem of the one-dimensional time-dependent ion matrix sheath analytically in the general form, i.e., for an arbitrary time dependence of the wall potential. We analyse the case when high negative step potential is applied to the wall. The temporal evolution of the matrix parameters at the initial stage of the response is determined. The chartacteristic time for the relaxation to the stationary state, described by the Child-Langmuir law, is found.

2. Model and basic equations

We consider a semi-bounded plasma occupying the half space, $z < 0$. The absorbing plane wall is placed at $z = 0$. For times $t < t_0$ the electrode is floating and at $t = t_0$ a large negative voltage φ_w is applied to it, wich remains constant for $t > t_0$. We assume $|\varphi_w| >> kT_e / e$ (with k the Boltzmann constant, T_e the electron temperature, and e the positive elementary charge) and therefore neglect the floating potential. The Debye length λ_D is smaller than the sheath thickness required to shield the wall potential φ_w. Since the electrons are repelled by the sheath, their density n_e falls sharply (within a few Debye lengths) from $n_e = n_0$ at the plasma side to $n_e = 0$ at the electrode side. Hence the electron density can be approximated in the form $n_e = n_0 \Theta(z_s(t) - z)$, where n_0 is the undesturbed plasma density, $z_s(t)$ is the position of the matrix sheath edge, and Θ is the Heaviside step function. The ion gas is cold, $T_i << T_e$. The characteristic time scale for the sheath's relaxation is determined by the ion dynamics. Bearing in mind that the total current density is the sum of the convection and displacement currents, $J = en_i v_i + \varepsilon_0 \partial E/\partial t$, Ampere's law, $(1/\mu_0)\nabla\times B = J$, involves that in an one-dimensional case $\partial J_z/\partial z = 0$, i.e., the total current density depends on time only, $J_z = J_z(t)$ [5]. In our case the convection current is provided by the ions. Introducing the normalized quantities

$$\omega_{pi} t \to t, \quad z/\lambda_D \to z, \quad v_{iz}/c_s \to u, \quad c_s = \sqrt{kT_e/m_i}, \quad \lambda_D = \sqrt{\varepsilon_0 kT_e/e^2 n_0}, \qquad (1)$$

$$E_z/\sqrt{n_0 kT_e/\varepsilon_0} \to E, \; J_z/en_0 c_s \to J, \quad -e\varphi/kT_e \to \varphi \quad n_i/n_0 = n,$$

the above assumptions result in the basic equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} = 0, \qquad \text{and} \qquad \frac{\partial E}{\partial t} + u\frac{\partial E}{\partial z} = J(t), \qquad (2)$$

with $J_z(t) = 0$ for $t < t_0$ and $z \geq z_0(t)$. The second equation in (2) represents Ampere's law in the one-dimensional case and has been obtained using Posson's equation, $\partial E/\partial z = n$.

3. Initial and boundary condition

At $t = t_0$ the matrix sheath is characterized by a uniform distribution of ions (there are no electrons). For the electric field and the potential at $t = t_0$ we have $E = z - s_0$, and $\varphi = (z - s_0)^2/2$ for $-|s_0| \leq z \leq 0$. Hence, the initial sheath-edge position, with $E = 0$, is given by $z_s(0) = s_0 = -\sqrt{2\varphi_w}$. With increasing time the sheath will expand until the sheath edge reaches its final position, $s_m = -2^{5/4}\varphi_w^{3/4}/3$, defined by the Child-Langmuir law. The boundary conditions at the sheath edge, $z = z_s(t)$, follow from the constant values $\varphi = 0$, $E = 0$, and $n = 1$, in the unperturbed plasma region, $z \leq z_s(t)$. We also have to introduce the ion velocity at the sheath edge, $u = u_s(t)$. The boundary condition at $z = 0$, $\varphi(0.t) = \varphi_w = const$, is provided by the prescribed potential at the wall.

4.Solutions at the initial stage after application of the step voltage

Introducing the coordinate and the ion velocity as functions of the electric field at the inital moment, $z_0(E) = z(E, t = t_0)$ and $u_0(E) = u(E, t = t_0)$, we find

$$u = u(E,t) = u_0(\mathcal{E}_0) + \mathcal{E}_0 \cdot (t - t_0) + \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' J(t''), \quad z_0(\mathcal{E}_0) = s_0 + \mathcal{E}_0 , \qquad (3)$$

$$z = z(E,t) = z_0(\mathcal{E}_0) + \int_{t_0}^{t} dt' u(\mathcal{E}_0, t'), \qquad \mathcal{E}_0 = E - \int_{t_0}^{t} dt' J(t'). \qquad (4)$$

Inverting (4) we obtain the explicit dependence $E = E(z,t)$, and after substitution in (3) we also find $u = u(z,t)$.Using the solution (4) one can also find the potential distribution in the sheath. The constancy of the wall potential φ_w allows to find the time dependence of the current $J(t)$ at the initial stage, $t - t_0 << 1$, and then find all the sheath perameters. Assuming that for such times the deviation of the electric field on the wall, $E_w(t)$, from its initial value is small, $E_w(t) = |s_0| + \Delta(t)$, $\Delta << |s_0|$, we obtain

$$J(t) \approx u_s(t_0) + (t - t_0)|s_0|/2 , \qquad (5)$$

$$z \approx z_0(t) + \{1 + (t - t_0)^2/2\}E , \qquad z_0(t) = -\{1 + (t - t_0)^2/4\}|s_0| \qquad (6)$$

$$u \approx u_s(t_0) + (t - t_0)E , \qquad \varphi \approx \{1 + (t - t_0)^2/2\}E^2/2 . \qquad (7)$$

. From (7) we have for the electric field at the wall $E_w \approx \{1 - (t - t_0)^2/4\}|s_0|$. This expression coincides with the result of [4] (see Eq. (25) of [4]).

5. Approaching the time-independent state

For purpose of applying the boundary conditions we represent the solutions (2) in form:

$$u = u_s\{F^{-1}[F(t) - E]\} + \int_0^E \frac{E' dE'}{J\{F^{-1}[E' - E + F(t)]\}}, \qquad (8)$$

$$z = z_s\{F^{-1}\lfloor F(t) - E\rfloor\} + u_s\{F^{-1}[F(t) - E]\}\int_0^E \frac{dE'}{J\{F^{-1}[E' - E + F(t)]\}} + \qquad (9)$$

$$+ \int_0^E \frac{dE'}{J\{F^{-1}[E' - E + F(t)]\}} \int_0^{E'} \frac{E'' dE''}{J\{E'' - E + F(t)\}},$$

where $z_s\{F^{-1}[F(t) - E]\}$ and $u_s\{F^{-1}[F(t) - E]\}$ originate from the position $z_s(t)$ of the sheath edge and the related ion velocity $u_s(t)$. The auxiliary function $F(t)$ is defined by the relation $J(t) = \partial F / \partial t$ and $F^{-1}(x)$ is the inverse function to $F(x)$. Morover we assume that the velocity at the sheath edge does not change with time, $u_s = const$. It is also expected that close to the time-independent state the sheath parameters change with time slowly, so that in the partial integration in (9) we can neglect all terms with higher than the first time derivative of $J(t)$. Also the function $z_s\left\{F^{-1}\left[F(t)-E\right]\right\}$ can be expanded in powers of E, keeping terms up to $\partial^2 |z_s| / \partial t^2$. Finally we use the small parameter, $\alpha \equiv \left(\sqrt{2}J_0/\sqrt{\varphi_w}\right) << 1$, corresponding to a strong electric field in the sheath [6]. From (9) we construct the following equation for the current deviation $\delta J(t)$ from its stationary value value J_0:

$$\frac{\partial^2 \delta J}{\partial t^2} + \frac{5}{12}\sqrt{\alpha}\,\frac{\partial \delta J}{\partial t} + \frac{10}{16}\alpha^2 = 0, \qquad J(t) = J_0 + \delta J(t). \tag{10}$$

For the rate λ ($\delta J \propto \exp(\lambda t)$) of the relaxation to the stationary state we find $\lambda \approx -\frac{3}{2}\alpha^{3/2} = -12\left(\frac{J_0^{\,2}}{8\varphi_w}\right)^{3/4}$, which corresponds to the slow change with time, assumed above. In the time-independent state we have the relations $J_0 = u_s$, $\varphi_w \approx E_0^{\,4}/8J_0^{\,2}$, and the sheath width is given by $|z_0| \approx (2/3\sqrt{\alpha})\,|s_0|$, which obviously exceeds the initial sheath's width $|s_0|$.

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