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Theory of the magnetic presheath-sheath transition

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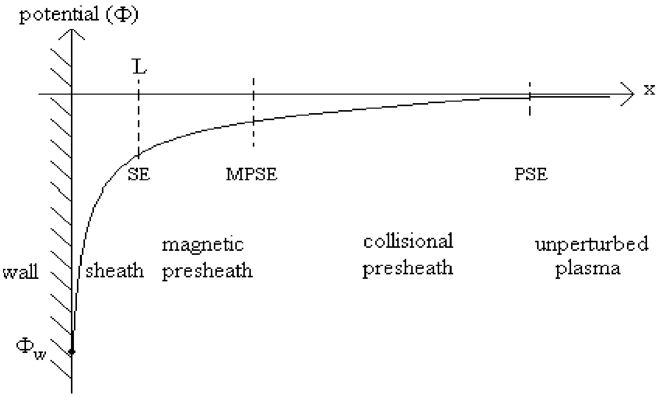
Abstract

The relation between the magnetic presheath (MPS) and Debye sheath (DS) regions of the plasma-wall transition (PWT) layer is investigated. The corresponding intermediate scale is found which allows to match the potential in these two regions avoiding the singularity arising from asymptotic analysis. It is expressed as an opinion that in the hydrodynamic approximation the intermediate scales between the transition layers of PWT have the similar structure in non-magnetized as well as magnetized case.

1. Introduction

In tokamaks, it is desirable to construct oblique magnetic field configurations so that the risk of melting and sublimation is reduced [1]. In such configurations, the plasma-wall transition (PWT) layer can be divided into three regions, namely: Debye sheath (DS), magnetic presheath (MPS) and collisional presheath (CPS).

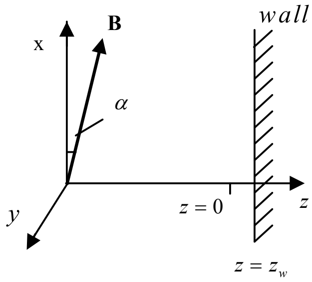


The classical PWT problem without magnetic field is well known, when the monotonic shape of the electric potential requires the fulfilment of the Bohm criterion at the interface of the CPS ($\sim\lambda$: chosen in the relevant form) and the DS ($\sim\lambda_D$). Chodura [2] was the first to show the existence of a magnetic presheath in the case of an oblique magnetic field and investigated its behaviour without any collisional effects. This region was found to be scaled with the ion gyro-radius ρ_i. Hence for the limiting values of scale variables, i.e., $\lambda_D >> \rho_i >> \lambda$, the problem contains three distinguished scales and in this "asymptotic three-scale limit", i.e. for $\varepsilon_{Dm} = \lambda_D / \rho_i \to 0$ and $\varepsilon_{mc} = \rho_i / \lambda \to 0$, the DS can be characterized as collisionless and non-neutral, the MPS as collisionless and quasi-neutral ($n_i \approx n_e$) [2] and the CPS as collisional and quasi-neutral.

The DS and the MPS regions are separated by the sheath edge, which is characterized by a field singularity from the MPS side, and by the marginal form of the Bohm criterion, $v_z = c_s = \left[k\left(T_e + \gamma T_i\right)/m_i \right]^{1/2}$ from the DS side; c_s is the ion-sound velocity, k is the Boltzmann constant, $\gamma = 1 + \left(n_+ / T_i\right)\left(dT_i / dn_+\right)$ is the local polytropic coefficient [3]. The MPS and CPS are separated by a relatively less distinguished boundary surface defined as the "MPS entrance". The condition imposed from the CPS side is similar to the Bohm criterion, but the ion velocity must be directed along the magnetic field line, $v_\parallel = c_s$. This is called as Bohm-Chodura criterion. Hence the dominant effect of the MPS is to deflect the ion orbits, so that the velocity component v_z, can fulfil the Bohm criterion at the DS entrance [1,4].

2. Model and basic equations

The surface of the conducting wall is placed at the position $z = z_w$, and the plasma occupies the region $z < z_w$. The problem is 1D with z, the relevant coordinate. The electric potential decreases towards the wall monotonically, where its value is $\Phi = \Phi_w$, and the electric field $E(z) = -\partial\Phi / \partial z$ is directed along the z-axis. The magnetic field is assumed to be uniform and lying in the $xz-$plane, making a small angle α with the wall. The electrons follow the Boltzmann distribution $n_e = n_s \exp\left(e\Phi / kT_e\right)$. Introducing the dimensionless variables



$$(\boldsymbol{v} / c_s) \Rightarrow \mathbf{v}, \qquad \left(-e\Phi / kT_e\right) \Rightarrow \varphi, \qquad \left(n_+ / n_s\right) \Rightarrow n_i, \tag{2.1}$$

n_s is the ion density at the DS edge, chosen here as an origin of the coordinate system, $z_s = 0$, and $\varphi(z_s) = 0$. The thermal motion of ions is neglected, $T_i \to 0$.

The three components of ion momentum equation can be represented in the form:

$$v_z \frac{dv_x}{dz} = \frac{1}{\rho_i} \sin\alpha \cdot v_y, \tag{2.2}$$

$$v_z \frac{dv_y}{dz} = \frac{1}{\rho_i}\left(\cos\alpha \cdot v_z - \sin\alpha \cdot v_x\right), \tag{2.3}$$

$$v_z \frac{dv_z}{dz} = \frac{d\varphi}{dz} - \frac{1}{\rho_i} \cos\alpha \cdot v_y. \tag{2.4}$$

where $\rho_i = c_s / \omega_c$ (ω_c is the ion cyclotron frequency). Sources and sinks are neglected in ion continuity equation, thanks to the smallness parameters ε_{Dm} and ε_{mc}, and we get $n_i \cdot v_z = 1$, which along with Boltzmann distribution gives shape to Poisson's equation as:

$$\frac{1}{v_z} - \exp(-\varphi) = \lambda_D \frac{d^2\varphi}{dz^2}, \tag{2.5}$$

Eq. (2.3) leads to the boundary conditions $v_y \to 0$, $v_x \to v_{\parallel}\cos\alpha$, and $v_z \to v_{\parallel}\sin\alpha$ for the velocity components at $z \to -\infty$, when $(\partial/\partial z) \to 0$. After straightforward calculations the system (2.2) - (2.5) can be reduced to the one equation in the form [4]:

$$\frac{d}{dz}\left[\left(\frac{v_z^2-1}{v_z}\right)^2\left(\frac{dv_z}{dz}\right)^2 - \frac{1}{\rho^2{}_i} f(v_z)\right] = 2\lambda_D{}^2 \frac{v_z^2-1}{v_z}\frac{dv_z}{dz}\left[\frac{d}{dz}v_z\frac{d}{dz}\Phi + \frac{1}{\rho_i^2}\frac{\sin\alpha}{v_z}\Phi\right], \tag{2.6}$$

where $$\Phi = \frac{1}{2}\left(\frac{d\varphi}{dz}\right)^2 + \frac{d^2\varphi}{dz^2},$$

and $$f(v_z) = \cos^2\alpha\left[1 + 2\ln(v_z/\sin\alpha) - v_z^2 - \left(\frac{2}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}\left(\frac{1}{v_z} + v_z\right)^2\right)\right] \tag{2.7}$$

3. Asymptotic analysis for the magnetic presheath and the sheath

(I) Using the ion gyro-radius $x = z/\rho_i$, as scale variable, Eq. (2.6) gives MPS solution:

$$\left(\frac{v_z^2-1}{v_z}\right)^2\left(\frac{dv_z}{dx}\right)^2 = f(v_z) + O(\varepsilon_{Dm}{}^2), \tag{3.1}$$

where $\varepsilon_{Dm} = (\lambda_D/\rho_i) << 1$, which provides with the fulfilment of the quasi-neutrality condition. At $v_z \to 1$ there might be an electric field singularity. Now, close to the sheath edge $v_z = 1 + u$, $(u << 1)$ and then from the Poisson's equation we have $\varphi \cong u$, $(u < 0)$ and (3.1) gives $\frac{d\varphi^2}{dx} \cong -\sqrt{f(1)}$, or $\varphi \cong -[f(1)]^{1/4}\sqrt{-x}$. According to (2.7), $f(1) > 0$.

(II) For the DS we use the variable $\xi = z/\lambda_D$ and from (2.7) it follows

$$\frac{1-v_z^2}{v_z}\frac{dv_z}{d\xi} = -v_z\frac{d}{d\xi}\left[\frac{1}{2}\left(\frac{d\varphi}{d\xi}\right)^2 + \frac{d^2\varphi}{d\xi^2}\right] + \mathrm{O}(\varepsilon_{Dm}{}^2) \tag{3.2}$$

Close to the sheath boundary, $v_z = 1 + u$ $(u > 0)$ and $\varphi \cong u << 1$, from (3.2) we find

$$\frac{d^2\varphi}{d\xi^2} \cong \varphi^2. \tag{3.3}$$

Hence the difference of characteristic scales, $\rho_i >> \lambda_D$, leads to the fracture on the shape of the electric potential at the MPS-DS interface. We have found the way to bridge these distinguished region in the smooth form.

4. Intermediate scale analysis

As mentioned above in case (I) the *rhs* of Eq. (2.6) can be neglected. Therefore we can apply any transformation, on *rhs*, keeping the results for the case (II) unchanged. So we use the (3.2), valid only for the case (II), and substitute into (2.6). After integration we get

$$\left(\frac{1-v_z^{\,2}}{v_z}\right)^2\left(\frac{dv_z}{dz}\right)^2 = \frac{1}{\rho_i^{\,2}} f(v_z) + \lambda_D^{\,4}\left[\left(v_z\frac{d\Phi}{dz}\right)^2 + \frac{1}{\rho_i^{\,2}}\sin^2\alpha\cdot\Phi^2\right], \qquad (4.1)$$

which gives the same results for the both (I) and (II) cases. Introducing the new variable $\zeta = z/l,$ by choosing the characteristic scale l, we can make the contribution of the magnetic field having the same order as that of the charge separation. Close to the sheath edge, $v_z = 1+u$, $\varphi \cong u$ $(u << 1)$ for the re-normalized potential $w = s\cdot\varphi$ from (4.1) we obtain

$$\left(\frac{dw^2}{d\zeta}\right)^2 = f(1) + \left(\frac{d^3w}{d\zeta^3}\right)^2 . \qquad (4.2)$$

The intermediate scale is found as $l = \left(\rho_i\cdot\lambda_D^{\,4}\right)^{1/5}$ and $s = \left(\rho_i/\lambda_D\right)^{2/5}$. For intermediate scale in the classical transition with no magnetic field, it was found as $l_m = \left(\lambda\cdot\lambda_D^{\,4}\right)^{1/5}$ [5]. It looks like that in the hydrodynamic approach the power (1/5) of the characteristic length of the intermediate scale remains same. Eq. (4.2) describes the smooth transition between the MPS and the DS. The electric field in the transition region equals $E \approx \left(kT_e/e\right)\left(\lambda_D^{\,2}\cdot\rho_i^{\,3}\right)^{-1/5}$.

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