*34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, P-4.032 (2007)*

Generation of meso-scale convective structures in tokamak edge plasma

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1. Introduction

Both blobs and ELM filaments have routinely been observed experimentally in edge and SOL plasmas and the comparison of the structure of blobs [1] and ELM filaments [2] reveal their striking similarities. This suggests that the mechanism of the convection of meso-scale structures at the outer side of the torus due to plasma polarization (caused by magnetic field curvature) and subsequent $\mathbf{E}\times\mathbf{B}$ radial drift studied in [3] is rather universal and can be applied to both blobs and ELM filaments. In particular, the estimates for the radial velocity and characteristic size of meso-scale structures based on this convection mechanism seem to be consistent with experimental observations.

The generation of ELM is typically attributed to the peeling-ballooning instabilities (see Ref. 4 and the references therein), however the mechanism (-s) of blob formation is not clear. It is plausible that the blobs are the manifestation of avalanches (see Ref. 5 and the references therein) developed somewhere deeper in the core and ultimately coming to the edge. Here we present a mechanism of the blob formation specifically operating in the edge region.

We should note that in L-mode regime the edge plasma is considered to be stable with respect to the interchange drive due to stabilization by good curvature region and the magnetic field line bending (Alfven waves) as well as ion finite Larmor radius effects. Therefore, the interchange drive is not readily available as a mechanism of blob generation (contrary to what is considered in some 2D simulation models, which neglect stabilization effects).

Nevertheless, an interchange drive can play a crucial role in blob generation. In this paper we show that the interplay of the interchange drive and nonlinear effects associated with drift wave turbulence (which is rather strong at the edge in L-mode) can lead to the blob formation. In particular, we discuss blob generation due to synergetic effects of a) the interchange drive, b) turbulent Reynolds stress (e.g. see Ref. 6 and the references therein), and c) relatively modest local plasma perturbations associated with inverse cascade of the drift-wave turbulence.

It is important to note a vital distinction between the ballooning-type instability considered in our paper and the structure of the modes in conventional ballooning mode theory. In standard ballooning mode stability analysis background plasma parameters are considered to be the functions of the magnetic flux surfaces and the structure of the mode is determined by the ballooning equation. In our case the background plasma parameters are fluctuating due to drift turbulence. These fluctuations are driven by the instabilities that occur at short cross-field length scale (of the order of the ion gyro-radius) and extend to longer length scales due to inverse cascade. As a result, plasma parameters may be inhomogeneous in both parallel and cross-field directions. Therefore, we consider the ballooning-type stability of the plasma situated within a given magnetic flux tube of a meso-scale size, Δ, and having parallel gradient scale lengths $\sim qR$ (here q and R are the safety factor and major tokamak radius). Such approach is legitimate when the time-scale of the instability is shorter than the characteristic life-time of these plasma structures (limited by e.g. density/pressure equilibration along the magnetic field lines).

Our model relies on the interaction of the electromagnetic interchange modes with the bath of small-scale drift-wave fluctuations. The nonlinear equations that describe both the electron drift waves (primarily electrostatic) and electromagnetic Alfven type perturbations in inhomogeneous magnetic field have the form

$$\partial A/\partial t + c\nabla_{\|}\left(\phi - (T_e/e)\ln n\right) = 0, \tag{1}$$

$$\partial n/\partial t + V_E\cdot\nabla n + V_D\partial\left(n - n_0(e\phi/T_e)\right)/\partial y - \nabla_{\|}J/e = 0, \tag{2}$$

$$-(n_0 c/B_0\omega_{ci})d_0\nabla^2\phi/dt + \nabla_{\|}J/e - V_D n/\partial y = 0, \tag{3}$$

where, n, ϕ, and A are plasma density, the electrostatic and magnetic potentials, respectively; B_0 is the magnetic field stress, ω_{ci} is the ion cyclotron frequency; the total gradient along the magnetic field is given by $\nabla_{\|}(...) = \partial(...)/\partial z - \left(\mathbf{b}\times\nabla A/B_0\right)\cdot\nabla(...)$; the fluid (substantive) derivative is $d_0/dt = \partial/\partial t + c\mathbf{b}\times\nabla\phi/B_0$; the parallel electric current is defined as $J = -(c/4\pi)\nabla_{\perp}^2 A$; and $V_D = 2cT_e/eB_0R$ is the magnetic drift velocity in the vertical direction. Ions are assumed to be cold in these equations, $T_i = 0$, however we

consider the finite ion temperature (Finite Larmor Radius stabilization) when analyzing the conditions for the blobs formation in Sec. 2.

The equations (1-3) describe the standard electron drift-wave dispersion relation, $\omega = \omega_{*e}/(1+k_\perp^2\rho_s^2)$ and the ballooning mode $\omega(\omega-\omega_{*i}) - k_\parallel^2 V_A^2 + \omega_D\omega_{*e}/k_\perp^2\rho_s^2 = 0$ (modified to account for the FLR effects). Here V_A is the Alfven speed, $\omega_{*e} = k_y cT_e/(eB_0L_n)$, $L_n^{-1} = -d\ell n(n_0)/dx$, $\rho_s^2 = T_e/(M\omega_{ci}{}^2)$, M is the ion mass and $\omega_D = k_y V_D$.

2. Scale separation and turbulence driven ballooning modes

We consider dynamics of large-scale perturbations in presence of small-scale drift wave turbulence using the standard assumption of time and spatial scale separation and exploiting the wave kinetic equation (e.g. see Ref. 6 and the references therein). The plasma variables are assumed in the form $n = n_0 + n_K + n_k$, $\phi = \phi_K + \phi_k$, $A = A_K + A_k$, where X_K and X_K are respectively large and small-scale components, $K << k$. The X_K is taken in the form $X_K \propto \exp(iK_y y - i\Omega t)$. The large-scale modes are essentially electromagnetic. However the electromagnetic effects are not important for small-scale fluctuations, so that one can neglect A_k. The time scale separation also requires the condition $\Omega_K << \omega_k$. The condition $K_\parallel V_A \approx V_A/qR << \omega_{*e}$ is assumed which is well satisfied at the plasma edge.

Large-scale perturbations are electromagnetic (of Alfvenic nature). The density perturbation for large-scale perturbations is found from the ion continuity equation $n_K = (\Omega_{*e}/\Omega)(e\phi_K/T_e)$, where $\Omega_{*e} = \omega_{*e}(K_y/k_y)$ and small dispersive corrections of the order of $K^2\rho_s^2 << 1$ have been neglected. The Ohms' law provides the following relation between the electrostatic and vector potentials

$$-(\Omega - \Omega_{*e})A_K + cK_\parallel(\phi_K - (T_e/e)(n_K/n_0)) = 0. \qquad (4)$$

Then neglecting of the disperson $K^2\rho_s^2 << 1$, the density gradient does not affect the ideal MHD relation between A_κ and ϕ_κ, $-\Omega A_K + cK_\parallel\phi_k = 0$. Quasi-neutrality equation gives the final equation for large-scale modes. This equation includes the contribution of the Reynolds stress tensor from small-scale fluctuations

$$-i\Omega\rho_s^2K^2\frac{e\phi_K}{T_e} - \frac{c}{B_0\omega_{ci}}\left\langle(\tilde{\mathbf{V}}\cdot\nabla)\nabla_\perp^2\frac{e\tilde{\phi}}{T_e}\right\rangle - i\Omega_D\frac{n_K}{n_0} + i\frac{K_\parallel^2V_A^2K_y^2\rho_s^2}{\Omega}\frac{e\phi_K}{T_e} = 0. \qquad (5)$$

Contribution of the Reynolds stress can be simply evaluated as

$$\delta\left\langle(\tilde{\mathbf{V}}\cdot\nabla)\nabla_\perp^2\frac{e\tilde{\phi}}{T_e}\right\rangle = -\frac{cT_e^2}{e^2B_0}K_y^2\int d^2kk_x\delta N_k\ , \qquad (6)$$

where δN_k is the perturbation of the wave action invariant, $N_k = |e\varphi_k/T_e|^2$, where we have switched to the integral over all spatial harmonics, $\sum k_xk_y|\phi_k|^2 = \int d^2kk_xk_y|\varphi_k|^2$ and neglect Larmor radius corrections of the order of $k_\perp^2\rho_s^2$. The wave action is modulated by large scale perturbations thus providing a feedback loop for the turbulent drive. The perturbation of N_k is most simply evaluated from the wave kinetic equation [6], giving

$$\delta N_k = \frac{ic}{\Omega B_0}K^2k_x\phi_q\frac{\partial\overline{N}_k}{\partial k_y}. \qquad (7)$$

It is important to note that the perturbation of the wave action is not resonant. Using the relation (7) in (6) and taking into account FLR stabilizing term one arrives to the instability criterion for large-scale modes

$$\Omega_{*e}\Omega_D/K_y^2\rho_s^2 > K_\parallel{}^2V_A{}^2 + \Omega_{*i}^2/4 - K_y^2c_s^2\rho_s^2\int d^2kk_x^2\overline{N}_k\,. \qquad (8)$$

From Eq. (10) one finds that the impact of turbulent Reynolds stress exceeds the stabilizations due to FLR for $(e\langle\phi\rangle/T_e)\langle k_x\rangle\rho_s > \rho_s/L_n$, and magnetic field bending for $(e\langle\phi\rangle/T_e)\langle k_x\rangle\rho_s > (L_n/qR)/(K_yL_n\sqrt{\beta})$, where $\langle\phi\rangle$ and $\langle k_x\rangle$ are the ensemble averaged magnitudes of the electrostatic potential fluctuation and the wave number of drift turbulence, and $\beta = (c_s/V_A)^2$ is the plasma's beta. For tokamak edge plasma in L-mode regime, where turbulence is strong both inequalities may be satisfied. In particular the FLR stabilization can be satisfied quite easily leading to the destabilization of the modes at the smaller scale than that typically considered in the conventional ELM's. As a result, these "mini" ELM events triggered by turbulence assisted interchange drive produce blobs, which then propagate into the SOL.

3. Inverse cascade and ballooning instability of meso-scale structures

In previous section we consider the model describing the destabilization of ballooning mode by small-scale drift turbulence assuming that the perturbations of plasma parameters at large scales are small. However, as we discussed in the Introduction, the inverse cascade of the drift turbulence may cause significant plasma perturbations at the meso-scale Δ between L_n and ρ_s. Let us consider what happens if a density/pressure perturbation δn at the intermediate scale $\Delta_x \approx \Delta_y \approx \Delta$ is introduced into the system. Such a meso-scale perturbation modifies the equilibrium profile of plasma density so the radial gradient of plasma density will affect the values of drift frequencies in (8), affecting both FLR and interchange terms. A simple, order of magnitude estimate can be obtained by taking $K_y\Delta \sim 1$ and writing the instability criterion (8) in the form

$$c_s^2/R\Delta - c_s^2\rho_s^2/\Delta^4 > V_A^2/q^2R^2 , \qquad (9)$$

For simplicity we consider the case $T_e = T_i$ and assume that plasma fluctuations are rather large, $\delta n \approx n_0$. We also neglect turbulent Reynolds stress effects. As it is obvious from (9) the FLR stabilization reduces faster for larger scale modes (compared to the reduction of the destabilizing interchange drive). The competition of the first and second terms in (9) defines the characteristic size of meso-scale structure for which the left hand side of Eq. (9) is maximal:

$$\Delta_m \approx \left(\rho_s^2 R\right)^{1/3} . \qquad (10)$$

(we assume that $\Delta_m < L_n$). Then the instability within this flux tube occurs for relatively high beta at the edge of plasma

$$\beta > q^{-2}\left(\rho_s/R\right)^{2/3} . \qquad (11)$$

The turbulent Reynolds stress, as we see from Eq. (8), assists the meso-scale structures instability and reduces the critical value of beta.

4. Results of numerical simulation

In order to verify our analytic estimates we solve 2D version of Eq. (1-3) numerically assuming that the parallel scale length of the perturbations is $\sim qR$. We chose sub-critical gradient of plasma density and seed a drift wave. Preliminary results show qualitative agreement with physical picture of blob generation formation described above and demonstrate blob formation and detachment for relatively large amplitude of drift waves (see Fig. 1).

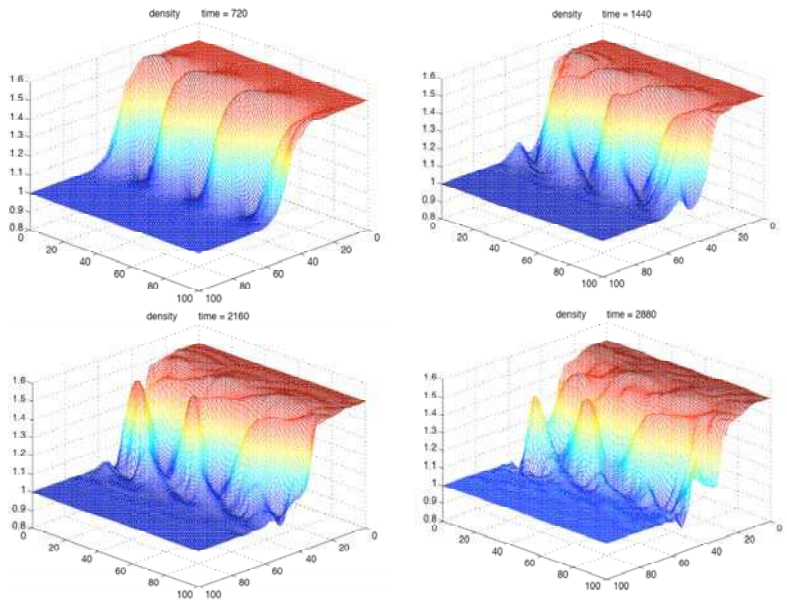

Fig. 1 Evolution of plasma density

5. Summary

We have presented a simple model of blob generation in edge plasmas of tokamaks based on the synergy of the interchange drive and nonlinear effects associated with drift wave turbulence. We have shown that the Reynolds stress reduces the threshold of the ballooning mode while inverse cascade of the drift turbulence may cause significant meso-scale perturbations that are subject to the ballooning-type instability. We suggest, that the blob formation is a result of these instabilities. Equations (10) and (11) define an approximate cross-field scale of the blob and beta threshold for the blob generation due to the ballooning-type instability facilitated by the inverse cascade.

We should note that the inverse cascade plays two-fold role in our model. On one hand, the growth of the large scale Alfvenic mode driven by turbulent Reynolds stress (as described by Eq. (16)) is a manifestation of the inverse cascade in the coupled system of electrostatic and electromagnetic fluctuations. The large-scale electromagnetic mode grows at the expense of the energy stored in small-scale electrostatic fluctuations. Essentially, this process is similar to those considered previously in the context of zonal flows

and streamer generation [5]. On the other hand, we invoke the inverse cascade processes as a mechanism generating meso-scale perturbations, which locally modify the plasma gradient length scale and alter ballooning-type stability. Also, we note that the blob formation and transport into SOL provide the energy sink at large scales that may affect inverse cascade. Preliminary results of numerical simulations show qualitative agreement with this physical picture of blob generation, although more work needs to be done.

It is interesting to note that recent experimental observations qualitatively agree with a physical picture of blob generation outlined in this paper. For example, study of blob generation on TORPEX device [7] clearly shows a strong increase of local plasma pressure gradient just before a blob formation. Another example comes from the papers [8, 9], where experimental data from C-Mod on effective "density limit" were compared with the results of 3D numerical modeling of plasma turbulence. According to [8], edge plasma transport in C-Mod was increasing drastically (effective "density limit") when parameter α_{MHD} exceeds of $\sim 3\times\alpha_d$ (see Fig. 13), where $\alpha_{MHD} \sim \beta q^2 (R/L_{pe})$, $\alpha_d \sim q^{-1}(\lambda_{ei}/R)^{1/2}(R/L_{pe})^{1/4}$, λ_{ei} is the electron mean free path, and L_{pe} is the cross-field pressure scale length. Meanwhile from Fig. 12 of the same paper we find that $R/L_{pe} \sim \alpha_d$. As a result, the inequality for effective "density limit" $\alpha_{MHD} >\sim 3\times\alpha_d$ can be simply written as $\alpha_{MHD}(L_{pe}/R) > \text{const.}$ or $\beta > \text{const.}/q^2$, which looks rather similar to (11), (we should note that q and magnetic field in experiment varied within factor 2 and 1.5 respectively, so that q dependence is the strongest one and the corrections due to term $(\rho_s/R)^{2/3}$ are within the experimental error bar).

Acknowledgements

The work was supported by the USDOE grant No. DE-FG02-04ER54739 at UCSD and NSERC Canada.

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