

## **Interaction of particles with systems of magnetic islands and edge turbulence in tokamaks in fully Hamiltonian approach**

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### **Introduction**

It is generally accepted that generation of transport barriers is connected with existence of a radial electric field and with the localization of the barrier in the region of rational magnetic surfaces. Moreover, stochastization of magnetic field lines around rational surfaces plays the key role behind the concepts of ergodic divertor and ELM mitigation by magnetic field perturbation [1]. Magnetic islands are generated just at the rational surfaces, thus it is interesting to investigate all features accompanying their existence.

Our contribution is based on numerical simulations from the first principles. Our goal is to compare the chaotic behavior of particles (ions) in a system of magnetic islands and to examine the differences between chaotization of magnetic field lines and particle trajectories.

### **Methods and results**

We have taken the perturbation of equilibrium magnetic field which creates either one magnetic island chain at  $q = 4$  or two island chains at  $q = 4$  and  $q = 5$ . For the former, the field lines are regular everywhere, but particles can behave chaotically [2]. The latter case can lead to chaotic behavior of field lines because of island overlap.

The canonical equations of particle motion resulting from the full Hamiltonian with this perturbed field were integrated numerically for 20000 revolutions around the major axis and the intersections with the poloidal plane were plotted. (See [2] for details.) Other parameters are those of the CASTOR tokamak, and charge and mass of a deuterium ion, to compare with the case of carbon ions from our previous work.

We have confirmed that our former result of a single island leading to stochastization of particle trajectories (originally shown for carbon ions) is also valid for deuterium ions. To determine how this effect depends on particle parameters, we did numerous calculations with varying perpendicular and parallel energy. We then classified the Poincaré plots as strongly stochastic (where the large chaotic sea appears) or weakly stochastic (when it does not). The results are shown in Fig. 1. We see that strong stochasticity sometimes disappears apparently randomly.

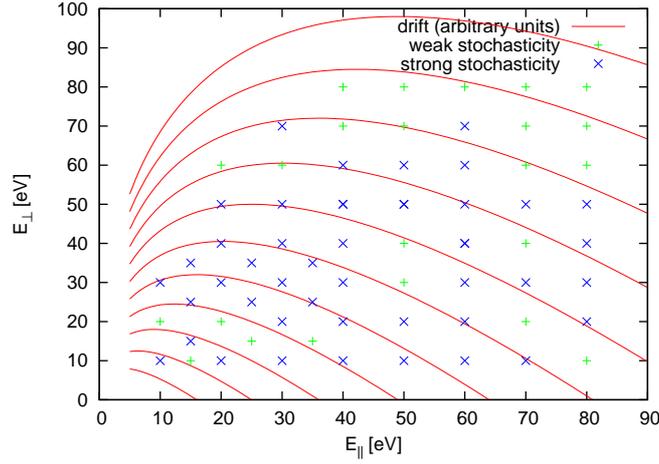


Figure 1: Initial conditions (perpendicular and parallel energy) which cause the motion of deuteron to be strongly stochastic (blue) or weakly stochastic (green), and lines of constant amplitude of drift (red).

We believe that this is caused by sensitivity to initial conditions and roundoff errors. However, in a certain region of parameters stochasticity disappears consistently. This happens for large values of drift, whose amplitude, proportional to

$$\sqrt{E_{\parallel}} + \frac{E_{\perp}}{2\sqrt{E_{\parallel}}}$$

is also drawn in Fig. 1. Thus the drift not only drives the stochastization, as shown in [2], but for larger values suppresses it, as well.

When a second mode of perturbation was added, we discovered a surprising effect. With only one mode of perturbation, we chose the parameters so that particle trajectory fills a large stochastic sea. With the second mode of perturbation and all other parameters identical, this stochastic sea disappeared. On the contrary, field lines are stochastic in the case of two modes and regular in the case of one mode, as expected. (Fig. 2) This disappearance of the particle chaos after the addition of a second magnetic island can probably be explained by the phase of this second ( $m = 5$ ) island, which has an O-point at the right (low field) side, see the plot of field lines (red). The phase of the  $m = 5$  island structure seen in the plot of particles (green) is opposite - the X-point is at the right side. When the  $m = 5$  magnetic island is added, it probably cancels the effect of chaos because of its opposite phase (blue).

We also calculated the diffusion coefficient of the particle motion in the stochastic sea created by one magnetic island. We have computed the variance, i.e. the mean square distance from the initial point for multiple trajectories:

$$S(t) = \frac{1}{N} \sum_{i=1}^N (x_i(t) - x_i(0))^2$$

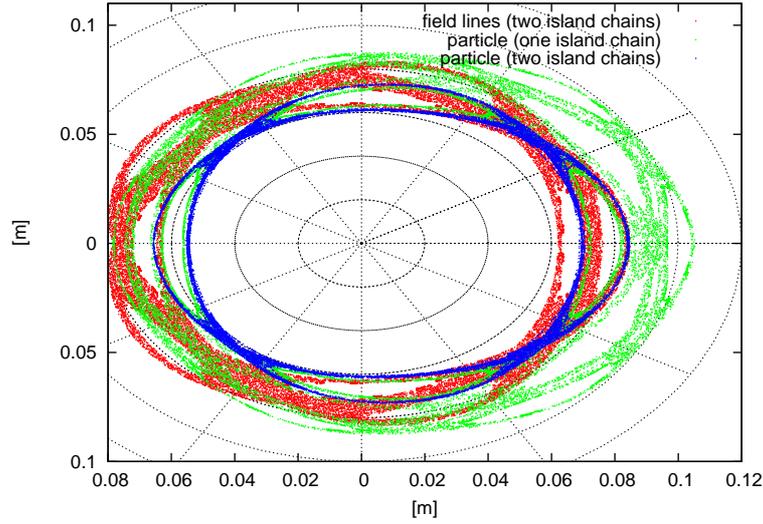


Figure 2: Intersections of field lines with the poloidal plane in the case of two island chains (red), intersections of particle gyration center for the same case (blue) and for only one island chain (green).

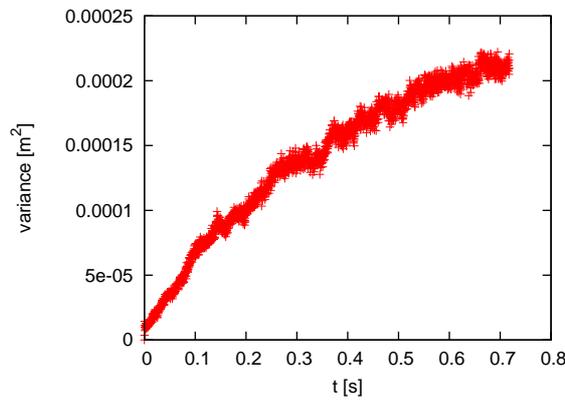


Figure 3: Time evolution of the mean square displacements from the initial position for a group of particle trajectories, for the case of chaotic motion in one magnetic island.

If the variance depends linearly on time (regular diffusion), its time derivative is the diffusion coefficient:  $D = dS(t)/dt$ . The time dependence of this variance is shown in Fig. 3. For short times, it is linear, indicating a random-walk diffusion with a diffusion coefficient with an order of magnitude of  $5 \cdot 10^{-4} \text{ m}^2 \cdot \text{s}^{-1}$ .

Further effects influence the dynamics besides magnetic islands; the most important ones being collisions and edge plasma turbulence. To estimate the latter effect, we model the action of the turbulence by means of a time-independent electrostatic potential, periodical in both the poloidal angle and the radial coordinate. We used a cylindrical configuration for simplicity, disregarding the curvature of the tokamak. Our focus was set on the diffusion of once-ionized carbon in the radial direction.

We therefore consider the potential in the form

$$V = U_0 \cos(kr) \cos \left[ m \left( \theta - \frac{z}{R_0 q(r)} \right) \right].$$

The choice of  $k$  and  $m$  stems from the following physical considerations. Given that a lot of phenomena in plasma occur on the scale of millimeters, the parameter  $k$  is set to  $1000 \text{ m}^{-1}$ . Then  $m$  is chosen to give a similar scale in the poloidal direction. The introduction of the safety factor  $q$  ensures that the potential remains constant on the unperturbed field lines. The amplitude of the potential  $U_0$  is in range  $0 - 100 \text{ V}$ .

Numerical simulations show that the addition of the potential has a dramatic impact on the value of radial diffusion coefficient. Firstly, the coefficient rises significantly with the magnitude of the potential. Secondly, the presence of the potential significantly increases the value of the diffusion coefficient from the unperturbed case.

$U[\text{V}]$	$D[\text{m}^2 \cdot \text{s}^{-1}]$
10	2.2
15	5.5
25	13.9
40	17.1

## Conclusion

We have investigated cases where field lines are chaotic, while the particle motion is not and vice versa, and found nontrivial dependencies on particle and perturbation field parameters. We note that a similar investigation of ion motion in the perturbed field of the TEXTOR DED has been described in [3]. In the case where we computed diffusion coefficient, we have found that its value is rather low. It is an open question if in a different regime a diffusion important enough to have a tangible impact on the plasma would appear. We are currently investigating the impact of an additional electrostatic potential similar to [4]. Preliminary results show that such potential has a strong impact on diffusion of carbon ions, with diffusion coefficients exceeding  $10 \text{ m}^2 \cdot \text{s}^{-1}$ .

Table 1: Dependence of radial diffusion coefficient on the potential amplitude.

## Acknowledgments

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## References

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