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# ITG-driven momentum transport in a plasma slab with a sheared longitudinal flow

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## Abstract

The one-dimensional theoretical model, previously developed for stability studies of ITG modes in a plasma slab, is used for the calculation of the momentum and energy quasi-linear (QL) fluxes. In the first part of this work, the relevant transport equations are solved numerically in order to investigate the QL relaxation of initial velocity and ion pressure profiles. In the second part the global solution of the 2nd order eigenvalue differential equation for the complex electrostatic ITG fluctuation amplitude $\tilde{\phi}_{\mathbf{k}}$ is calculated, and a comparative discussion for different toroidal velocity profiles is presented.

## Introduction

The interrelation between energy and angular momentum transport observed in different tokamak experiments [1,2], and the appearance of a toroidal net plasma rotation during auxiliary heating without angular momentum input [3,4], suggest a common origin of the two processes. According to a theoretical model on spontaneous generation of plasma toroidal rotation [5], the mechanism of momentum generation is associated with the turbulence of ion temperature gradient (ITG) modes made unstable by the ion pressure gradients. In the quasi-linear evolution of the ITG modes, both momentum and energy fluxes turn out to depend not only on the mode energy density, but also on the background rotation and ion pressure radial profiles. In the present paper, we extend the slab model developed for ITG mode stability studies [6] *(a)* by solving numerically the relevant system of two transport equations for the parallel momentum and for the ion pressure, taking into account the saturated quasi-linear fluxes, and *(b)* by developing the corresponding "global" analysis, that is by integrating the 2nd order ordinary differential equation for the complex fluctuation amplitude $\tilde{\phi}_{\mathbf{k}}$, as an eigenvalue problem.

## Transport equations

The model, previously developed to study the linear ITG stability in a plasma slab (-a < x +a), is based on a two-fluid guiding-center approximation, and includes the spatial variation of the magnetic field and its curvature, by means of a gravitational-like drift velocity [6]. In this frame, we have calculated the QL momentum and ion pressure fluxes, transverse to **B**, as spatial averages of the products of the advection electric drift velocity, $\tilde{v}_{Ex} = -\frac{c}{B}\frac{\partial\tilde{\phi}}{\partial y}$, times the advected fluctuating physical quantities, $\tilde{v}_z$ and $\tilde{p}_i$, respectively, that is

$$\Gamma^{\perp}_{v_z}(x,t) = \langle \tilde{v}_{Ex}\tilde{v}_z \rangle_{y,z} = \lim_{L_y,L_z\to\infty}\frac{1}{L_yL_z}\int_{-L_y/2}^{+L_y/2}dy\int_{-L_z/2}^{+L_z/2}dz\,\tilde{v}_{Ex}\tilde{v}_z = \lim_{L_y,L_z\to\infty}\frac{1}{L_yL_z}\int_{-\infty}^{+\infty}dk_y\int_{-\infty}^{+\infty}dk_z\,\tilde{v}_{E,\mathbf{k}}\tilde{v}^*_{z,\mathbf{k}}$$

$$\Gamma^{\perp}_{p_i}(x,t) = \langle \tilde{v}_{Ex}\tilde{p}_i \rangle_{y,z} = \lim_{L_y,L_z\to\infty}\frac{1}{L_yL_z}\int_{-L_y/2}^{+L_y/2}dy\int_{-L_z/2}^{+L_z/2}dz\,\tilde{v}_{Ex}\tilde{p}_i = \lim_{L_y,L_z\to\infty}\frac{1}{L_yL_z}\int_{-\infty}^{+\infty}dk_y\int_{-\infty}^{+\infty}dk_z\,\tilde{v}_{E,\mathbf{k}}\tilde{p}^*_{i,\mathbf{k}}.$$

These fluxes enter the evolution equations for the regular parts of the macroscopic velocity $V_z(\mathbf{r},t) = \overline{V}_z(x,t) + \tilde{v}_z(\mathbf{r},t)$, and of the ion pressure $P_i(\mathbf{r},t) = \overline{P}_i(x,t) + \tilde{p}_i(\mathbf{r},t)$, that is

$$\frac{\partial\overline{V}_z}{\partial t} + \frac{\partial\Gamma^{\perp}_{v_z}}{\partial x} = -\left\langle \tilde{v}_x\tilde{v}_{y'} \right\rangle\frac{d\vartheta}{dx} + \frac{1}{M\bar{n}_i^2}\left\langle \tilde{n}_i\frac{\partial\tilde{p}_i}{\partial z'} \right\rangle_{y,z} + \frac{1}{M\bar{n}_i^2}\left\langle \tilde{n}_i\hat{\mathbf{b}}\cdot\left(\nabla\cdot\underline{\underline{\tilde{\Pi}}}\right) \right\rangle_{y,z} + S_{v_z}, \tag{1}$$

$$\frac{\partial\overline{P}_i}{\partial t} + \frac{\partial\Gamma^{\perp}_{p_i}}{\partial x} = (\gamma-1)\left\langle \tilde{v}_{\parallel}\frac{\partial\tilde{p}_i}{\partial z'} \right\rangle_{y,z} + S_{p_i}, \tag{2}$$

where $\langle A(\mathbf{r},t) \rangle_{y,z} = \overline{A}(x,t)$ and $\langle \tilde{a}(\mathbf{r},t) \rangle_{y,z} = 0$. In the frame of our collisionless model, the QL density flux is zero and $\bar{n}_i(x) = \text{const}$. The RHS of Eqs.(1,2) contain as well average terms, quadratic in the fluctuating potential, which cannot be written as a divergence of a flux, and source terms. In the present study, of all these contributions, which are assumed to be small, a parallel velocity source term, $S_{v_z}$, localized at the boundaries $x = \pm a$, has been included. Notice that the first term in the RHS of Eq.(1) contains the *x*-derivative of the pitch angle of the magnetic line of force $\vartheta(x)$, representing magnetic shear. The linearized equations for the Fourier coefficients $\tilde{n}_{i,\mathbf{k}}$, $\tilde{v}_{z,\mathbf{k}}$, and $\tilde{p}_{i,\mathbf{k}}$ allow one to express the quadratic forms to be integrated in ***k*** in the QL fluxes as proportional to $|\tilde{\phi}_{\mathbf{k}}|^2$ [6]. Finally, in order to solve Eqs.(1,2), the saturation level of the modes is determined by a "mixing-length" criterion, that is by assuming that the potential amplitude $|\tilde{\phi}_{\mathbf{k}}|$, after its exponential growth, stabilizes at a level such that the non-linear term $\tilde{v}_{Ex}\frac{\partial\tilde{n}_i}{\partial x}$ becomes of the same order as $\frac{\partial\tilde{n}_i}{\partial t}$. This gives approximately $\left(\frac{e|\tilde{\phi}_{\mathbf{k}}|}{T_e}\right)_{sat} \cong \frac{1}{D_B}\frac{\gamma_{\mathbf{k}}}{k_xk_y}\frac{1}{\Delta k_z\Delta k_y}$, where $\Delta k_{y(z)}$ is the effective spectral width in $k_y$ ($k_z$), $\gamma_{\mathbf{k}}$ is linear growth rate of the modes, and $D_B = \frac{cT_e}{eB}$ is the Bohm diffusion coefficient. In the course of the numerical integration of Eqs.(1,2), $|\tilde{\phi}_{\mathbf{k}}|^2$ appears in a time $\tau \approx \gamma_{\mathbf{k}}^{-1} \cong 10^{-5}\,\mathrm{s}$, linearly up to $\left(|\tilde{\phi}_{k}|^2\right)_{\mathrm{sat}}$ and then taken constant.

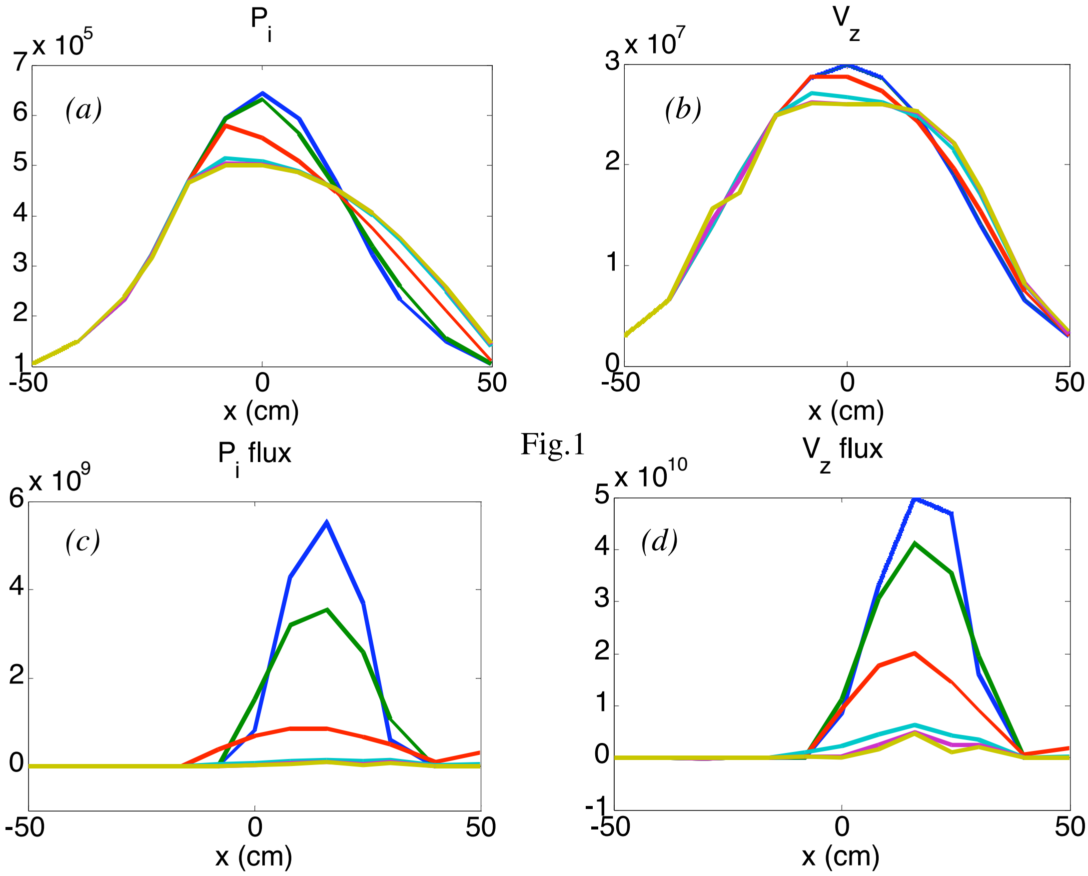


Fig.1

In Fig.1 the spatial profiles of the regular parts $\overline{\mathrm{P}}_{\mathrm{i}}\left[\mathrm{dyne/cm^2}\right]$ (*a*) and $\overline{\mathrm{V}}_{\mathrm{z}}[\mathrm{cm/s}]$ (*b*), and the corresponding QL fluxes (*c*,*d*) are shown at several times, t = $10^{-5}$s, $10^{-4}$s, $10^{-3}$s, $10^{-2}$s, 0.1s 1s.

## The "global" analysis and the eigenvalue problem

The linearized equations in Ref.6 are combined into a homogeneous 2nd order ordinary differential equation for the complex Fourier coefficient of the fluctuating potential $\tilde{\phi}_{\mathbf{k}}(\mathrm{x})$:

$$\frac{d^2\tilde{\phi}_{\mathbf{k}}}{dx^2} + A(x)\frac{d\tilde{\phi}_{\mathbf{k}}}{dx} + B(x)\tilde{\phi}_{\mathbf{k}}(x) = 0. \tag{3}$$

At any *x* a local Cartesian system is defined where *z* is along the equilibrium magnetic field and *y* is orthogonal to both *z* and *x* axes. In Eq.(3) the coefficients A(x) and B(x) depend on the complex frequency $\omega_{\mathbf{k}} = \omega_{\mathbf{k}}^{R} + i\gamma_{\mathbf{k}}$ (the eigenvalue), on $k_{\parallel} = k_z$ and $k_{\partial} = k_y$, on the equilibrium profiles, and their *x*-derivatives, these dependencies being in general strongly non-linear. We choose a set of equilibrium profiles and then solve Eq.(3) between –a and +a by a shooting method, reducing the problem from a boundary to an initial-value problem.

Localized solutions around the singular surface, where $k_{\parallel} = 0$, and such that $k_x \rho_s < 1$ are found for $\partial^{R}_{\mathbf{k}}$ and $\gamma_{\mathbf{k}}$ of the same order. With the objective of finding the response of the ITG turbulence level to the background ion momentum gradient, a comparative analysis is made by integrating Eq.(3) with fixed boundary conditions and different velocity profiles of the type $V_{\parallel}(x) = (V_{\parallel 0} - V_{\parallel a})(1-\xi^{\alpha})^{\beta} + V_{\parallel a}$, depending on two parameters $\alpha$ and $\beta$ and where $\xi = \dfrac{x}{a}$, $V_{\parallel 0}$ and $V_{\parallel a}$ are the on-axis and boundary velocities, respectively.

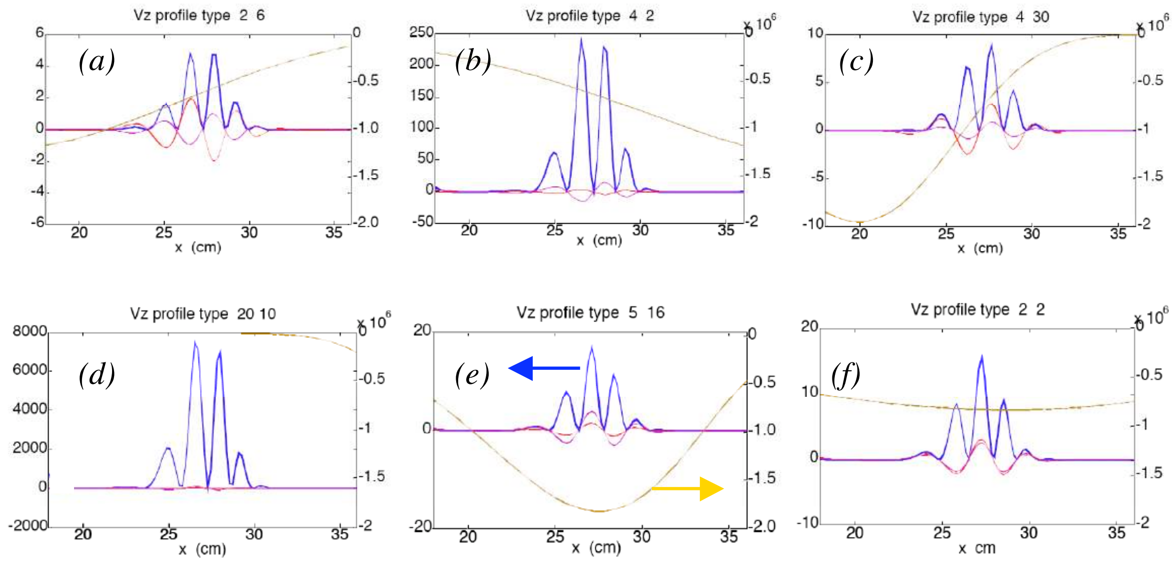


Fig.2

In Fig.2 $\mathrm{Re}\,\tilde{\phi}_k$, $\mathrm{Im}\,\tilde{\phi}_k$, and $|\tilde{\phi}_k|^2$(blue line) are shown *vs* x, for different $V_{\parallel}$ profiles, (*a*) $\alpha = 2, \beta = 6$ (2-2), (*b*) 4-2, (*c*) 4-30, (*d*) 20-10, (*e*) 5-16, and (*f*) 2-2; moreover $V_{\parallel 0} = 3\times 10^7$ cm/s and $V_{\parallel a} = 3\times 10^6$ cm/s, a = 50 cm, and R = 160 cm; $dV_{\parallel}/dx$ is plotted in yellow. We notice that in the interval where the solution is localized $k_y < 0$, $dV_{\parallel}/dx = V_{\parallel}' < 0$, and $k_{\parallel}$ goes through the null value, from small positive to small negative values. The trend seems to be that the excited turbulence level $|\tilde{\phi}_k|^2$ is higher when values of $k_y V_{\parallel}'/k_z$, assumed over the range where the solution is localized, are dominantly negative.


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