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Self consistent MHD equilibrium in turbulence simulations

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Introduction: In the edge region of the tokamak, the plasma parameters are such that the equilibrium and the turbulence interact strongly, and evolve together. In particular, the magnetic vector potential ($A_{\|}$) is a dynamical quantity evolved by an electromagnetic turbulence model like GEM Ref. [1]. This quantity takes part in the MHD equilibrium, defined by the balances (forces and divergences) describing the Pfirsch-Schlüter currents, via the Àmpere's law. Its axisymmetric component yields changes to the q-profile (field pitch), as well as the Shafranov shift. The challenge is to avoid double counting these effects, which are also set by an MHD equilibrium solver. For the case of the simplified geometry of an $S-\alpha$ model (which does not require an MHD solver), the treatment for the self-consistent evolution of the MHD equibilibrium with the turbulence has been given in Ref. [2]. Here, a the treatment to do so on simplified zeroth order in the inverse aspect ratio ($\varepsilon = a/R$) Grad-Safranov solutions (nested circular flux surfaces) is reported. Furthermore, since turbulence is affected by the deformation of its spatial domain, an X-point is also included in the model to deform the circular flux surfaces. This serves as a starting basis to tackle the full problem, which will employ real tokamak geometry.

Model: The tokamak axisymmetric magnetic field can be expressed as, $\mathbf{B} = I\nabla\phi + \nabla\Psi \times \nabla\phi$, where ϕ represents the toroidal angle, $I = I(\Psi)$ the poloidal current and Ψ the poloidal flux, with the sign convention that $\nabla\Psi$ is negative. The expression for the local helicity of the magnetic field (field pitch) is found from the ratio of the contravariant components of the magnetic field $q = B^{\phi}/B^{\eta}$, where η is the geometrical poloidal angle. With the assumption $\varepsilon \ll 1$, one obtains the cylinder approximation, for which $I = RB \approx B_0 R_0$ and $\Psi'(r) \approx -(rB_0)/q$. Assuming a constant field pitch and integrating over the radial coordinate r yields

$$\Psi_C(R,Z) = -\frac{B_C}{2}\left[r^2 - a^2\right] \tag{1}$$

with the additive constant $B_C a/2$ chosen to fulfil $\Psi_C(a) = 0$, where a stands for the tokamak minor radius and $B_C = B_0/q$ has units of a magnetic field. One can invert the Eq. (1) to have the contours $R = R(\Psi_C)$ and $Z = Z(\Psi_C)$ in the poloidal plane (R,Z), since $r^2 = (R-R_0)^2 + (Z-$

$Z_0)^2$. Doing so yields the equation for circular flux surfaces

$$\begin{cases} R(\Psi_C,\eta) = r\cos\eta + R_0 \\ Z(\Psi_C,\eta) = r\sin\eta + Z_0 \end{cases} \tag{2}$$

The Shafranov shift constitutes a $\mathscr{O}(\varepsilon)$ correction to such a circular equilibrium. In practice, it corresponds to a shift of the flux surfaces in the R-direction, such that they become centered on $R_1 = (R_0 + \Delta, Z_0)$, with $\Delta(r)$ representing the shift, as ilustrated in the Fig. 1. The equation for the new flux surfaces can be obtained directly from Eq. (2) by substituting R_0 with R_1. Introducing a new radial coordinate to describe the shifted surfaces, namely, $\rho^2 = (R - R_0 - \Delta)^2 + (Z - Z_0)^2$, allows expressing the poloidal magnetic flux, including the Shafranov shift, as

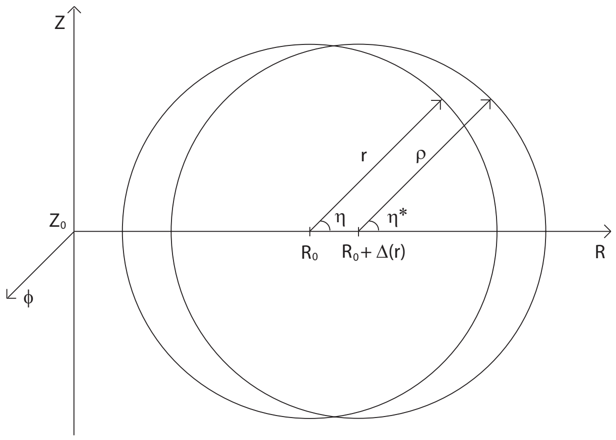

Figure 1: Graphical representation of the relation between the cylindrical coordinate system (r,η,ϕ) and the shited coordinate system (ρ,η^*,ϕ).

$$\begin{aligned} \Psi_C(\rho) = -\frac{B_C}{2}(\rho^2 - a^2) &= -\frac{B_C}{2}(r^2 - a^2) + B_C\Delta r\cos\eta - \frac{B_C}{2}\Delta^2 \\ &= \Psi_C(r) + \Psi_1(r)\cos\eta + \mathscr{O}(\Delta^2) \end{aligned} \tag{3}$$

where the relation between ρ and the original (r,η) coordinates, $\rho^2 = r^2 - 2r\Delta\cos\eta + \Delta^2$, has been used. The first term in the second line corresponds to the zeroth order model given by Eq. (1) and the last term is a second a order correction that can be dropped. The middle term introduces the definition

$$\Psi_1(r) = B_C r\Delta(r) \tag{4}$$

and corresponds to the first order correction due to $\Delta(r)$.

As mentioned before, the GEM model provides a contibution to the MHD part of the equilibrium through the magnetic vector potential. Its axisymmetric part yields the Shafranov shift, through the Pfirsch-Schlüter current, whereas its zonal component (flux surface average) provides a contribution to magnetic the field pitch. These can be expressed in terms of poloidal magnetic flux as

$$\Psi^{\mathrm{GEM}} = \Psi_0^{\mathrm{GEM}} + \Psi_1^{\mathrm{GEM}} + \mathscr{O}(\varepsilon^2) \approx R_0\langle A_\parallel\rangle + R_0\langle A_\parallel \cos s\rangle + \mathscr{O}(\Delta^2) \tag{5}$$

where the angle brackets denote the flux surface average, and s is the field aligned parallel (poloidal) coordinate used in GEM. Comparing the second term on the right-hand side to Eq. (4) leads directly to expression for the Shafranov shift coming from GEM

$$\Delta^{\mathrm{GEM}} = \frac{R_0 \langle A_{\parallel} \cos s \rangle}{r B_C} \tag{6}$$

that can be included in Eq. (2), with $R_0 + \Delta^{\mathrm{GEM}}$ instead of R_0. The complete expression for the poloidal magnetic potential providing the magnetic field geometry becomes time dependent since it now includes the changes to the MHD equilibrium due to the turbulence

$$\Psi(\rho, t) = \Psi_C(r) + \Psi_0^{\mathrm{GEM}}(r,t) + \Psi_1^{\mathrm{GEM}}(r,t) \cos \eta \tag{7}$$

To handle the problem consistently, and avoid double counting the shift, the axisymmetric part must be stripped out of the magnetic vector potential $A_{\parallel}$ within GEM, as it yields a perturbed magnetic field that acts on the variables (through the parallel gradient operator), and such information has already been accounted for by the magnetic field geometry through Eq. (7). Further discussion on this subject can be found in Ref. [2].

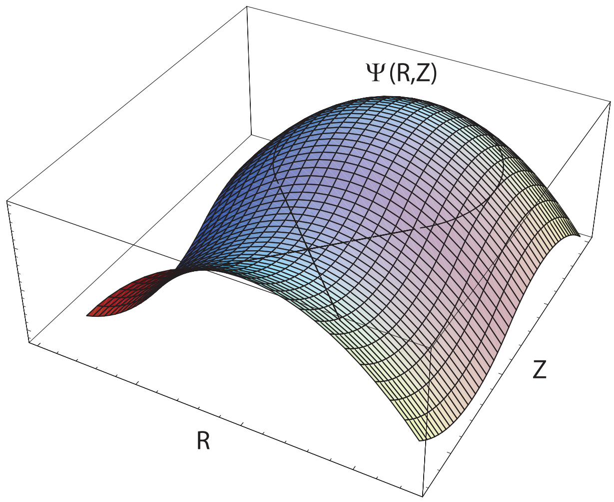

Figure 2: Matched poloidal magnetic flux Ψ from both the circles and hyperbola models. The separatrix is also represented.

To include an X-point that deforms the circular flux surfaces yield by Eq. (2), one needs to find the appropriate poloidal flux function which contains such a singularity, and then patch it together continuously with Eq. (1) across a given boundary curve in the poloidal plane. The following function fulfils such requirements

$$\Psi_H(R,Z) = -\frac{B_C}{2}\left[(R-R_0)^2 - (Z-Z_X)^2\right] \tag{8}$$

provided that the location of the X-point is set to be $(R_0, Z_X = Z_0 - \sqrt{2}a)$, and that the boundary between both models Ψ_C and Ψ_H is placed at $Z_B = Z_0 - \frac{a}{\sqrt{2}}$. Such a model, with $\Psi = \Psi_C$ for $Z < Z_B$ and $\Psi = \Psi_H$ otherwise (Ref. [3]), is $\mathscr{C}^1$ (continuous up to the first derivative), as can be seen from Fig. 2. This property holds even when a Shafranov shift is considered since it is flux function $\Delta = \Delta(\Psi)$. Hence, the previous considerations on this matter apply to Eq. (8) as well.

Preliminary results: Here, the first results from the first GEM simulations in the edge region including the self-consistent MHD equilibrium evolution outlined in the previous paragraphs are reported. These were made as follows: the initial flux surfaces were constructed from Eqs. (1) and (8) and yield the geometrical information needed for GEM that calculated using the METRICS code [5]. Then, as the turbulence was evolved in time, its contribution to the field pitch and Shafranov shift was calculated from Eqs. (5) and (6) and included back into the flux surfaces, which provide the geometry for the next time step, while the axisymmetric part stripped out of $A_{\|}$ within the current time step. The METRICS code was then run on the new flux surfaces, and the next GEM time step was calculated. The loop was repeat afterwards until the end of the simulation. Another simulation using the $S-\alpha$ geometry of Ref. [2] was also performed for the same parameters ($T = 80$eV, $n = 2.2 \times 10^{13}$cm^{-3}, $M_i = 3670 m_e$, $R =$ 165cm, $a = 50$cm, $L_\perp = 4.62$cm, $B = 1.2$T and $q = 3.9$), to allow a comparison between the two geometry models. Although the results obtained require further analysis, it is still noteworthy that the reduction observed in the electron heat flux compared to the $S-\alpha$ geometry case is consistent with the ability of the local shear, which is particularly strong near the X-point in the model of Eq. (7) and absent in the $S-\alpha$ model, in facilitating processes of nonlinear decorrelation of vortical turbulent structures [4].

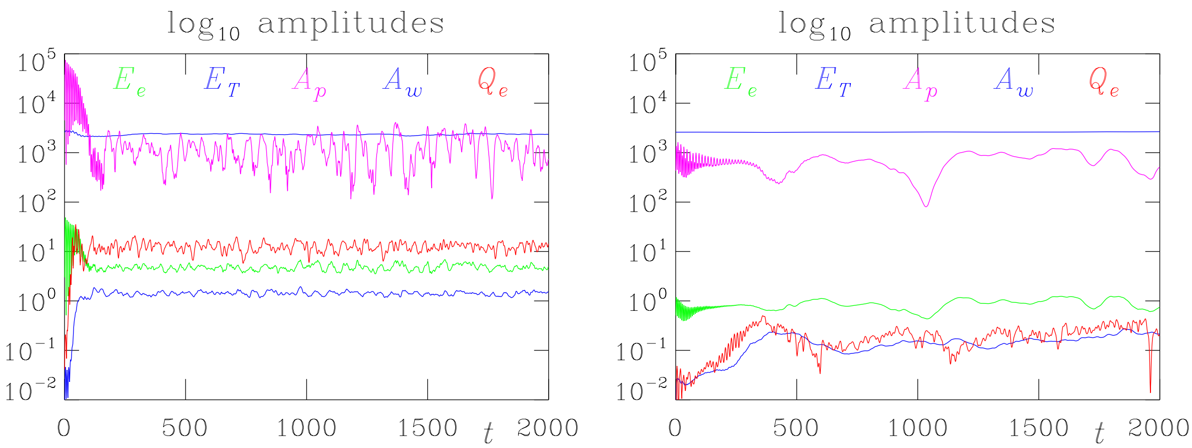

Figure 3: Time traces of free energy amplitudes and transport for (left) $S-\alpha$ geometry and (right) the geometry given by Eqs. (1) and (8), in units of $L_\perp/c_s$. The domain averages of the $E \times B$ energy, total energy, $\tilde{\phi}^2/2$ (electrostatic potential), $\tilde{\Omega}^2/2$ (vorticity) and electron heat flux are denoted by E_e, E_T, A_p, A_ω and Q_e, respectively.

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