

Study of Resistive Edge Modes in an ITER-like Geometry

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Abstract

The linear stability of the collisional η_i mode and the resistive ballooning mode in the electrostatic limit is studied in an ITER-like geometry. The reduced Braghinskii equations are used as a model for the electrons together with an advanced fluid model for the ions. Using the ballooning mode representation, the drift wave problem is set as an eigenvalue equation along a field line. The eigenvalue equation is solved numerically using a standard shooting technique and applying WKB type boundary conditions. The effects of various quantities, such as collisions, wave vector, density gradients and temperature gradients on mode stability are examined.

The drift wave eigenvalue problem

The reactive ion-temperature-gradient driven drift mode (or η_i mode) is a promising candidate for explaining the anomalous transport in the core of tokamak plasmas. The situation at the edge is different where strong influence of electron-ion collisions gives a resistive nature to the drift modes. The objective of the present paper is to study the linear stability of resistive edge modes in an ITER-like geometry. A full magnetic field equilibrium that corresponds to ITER Scenario 4 [1] is computed using the variational moments equilibrium code VMEC and is then mapped to the Boozer coordinate system (s, θ, ζ) [1, 2], where $s = 2\pi\psi/\psi_p$ is the normalized flux (radial) coordinate and θ, ζ are the generalized poloidal and toroidal angles, respectively. Here, $2\pi\psi$ is the poloidal magnetic flux bounded by the magnetic axis and $\psi = \text{constant}$ surface, and $\psi_p = \pi B_0 \bar{a}^2/q$ is the total poloidal magnetic flux, where B_0 is the magnetic field at the magnetic axis, q is the safety factor and \bar{a} is the average minor radius. Equilibrium quantities like field line curvature $\kappa (\equiv \mathbf{e}_{\parallel} \cdot \nabla \mathbf{e}_{\parallel})$ and the local magnetic shear (LMS) S [1] can be expressed as

$$\kappa = \frac{\kappa_n}{\sqrt{g^{ss}}} \nabla s + \frac{\kappa_g}{\sqrt{g^{ss}}} \left(\frac{\dot{\psi} g^{ss}}{B} \right) (\nabla \alpha - \wedge \nabla s), \quad (1)$$

$$S = (\hat{s} \times \mathbf{e}_{\parallel}) \cdot \nabla \times (\hat{s} \times \mathbf{e}_{\parallel}) = (\mathbf{e}_{\parallel} \cdot \nabla) \wedge, \quad (2)$$

where, κ_n is the normal component and κ_g is the geodesic component of the field line curvature and are defined as, $\kappa_n = \boldsymbol{\kappa} \cdot \hat{s}$, $\kappa_g = \boldsymbol{\kappa} \cdot \hat{s} \times \mathbf{e}_{\parallel}$. Here, $\mathbf{e}_{\parallel} = \mathbf{B}/B$ is a unit vector along the magnetic field, $\hat{s} \equiv \nabla s/|\nabla s|$, $\wedge = g^{s\alpha}/g^{ss}$ is the local magnetic shear integrated along the field line and $g^{ij} = \nabla i \cdot \nabla j$ is the dot product of metric coefficients of the flux coordinates.

Investigating the resistive edge modes instability an advanced fluid model is used for the ions and the reduced Braginskii equations are used as a model for the electrons. After deriving the drift wave equation in a standard way [3], it is then reduced to an ordinary differential equation along a field line by employing WKB assumptions in the coordinates (ψ, α, ζ) and by using the standard ballooning mode formalism [1, 2]. The resulting eigenvalue equation for the potential, Ψ , in the extended ballooning mode coordinate, ζ , is written as

$$\frac{d^2 \Psi}{d\zeta^2} + U(\zeta, \Omega) \Psi = 0. \quad (3)$$

with the effective potential, $U(\zeta, \Omega)$, given by

$$U(\zeta, \Omega) = \nu_{ei}^* \left(\frac{JB}{q\bar{R}\psi} \right)^2 \frac{D_2(\Omega)}{D_1(\Omega)} \quad (4)$$

where

$$\begin{aligned} D_1(\Omega) &= \Omega^2(1+b) - \Omega \left[(1-b\alpha)\Omega_{*e} + \left(1 + \frac{10}{3}\tau\right)\Omega_d \right] \\ &\quad + \epsilon_n \Omega_{*e} \tau \left[(\eta_i - \frac{7}{3})\Omega_{*e} - \frac{5}{3}(1+\tau)\Omega_d \right] \\ D_2(\Omega) &= \Omega^2 [\Omega + \alpha\Omega_{*e}] b - \Omega \left[(1+\alpha)\Omega_{*e} + \left(1 + \frac{5}{3} + \epsilon_n\right)\Omega_d \right] \Omega_d \\ &\quad + \Omega_d^2 \tau \left[\left(\frac{7}{3} + \frac{5}{3}\tau - \eta_i\right)\Omega_{*e} + \frac{5}{3}(1+\tau)\Omega_d \right] \end{aligned}$$

and

$$\begin{aligned} \Psi(\zeta) &= \frac{e\phi}{T_{e0}} - \frac{\delta n}{n_0}, & \Omega_d &= \Omega_d(s, \alpha, \zeta) = \chi B_0 \bar{R} \left(\frac{\mathbf{B} \times (\boldsymbol{\kappa} + \nabla \ln B)}{B^2} \right) \cdot \hat{\mathbf{k}}_{\perp}, \\ \Omega &= \frac{\bar{R}\omega}{c_s}, & \Omega_{*e} &= \frac{\bar{R}\omega_{*e}}{c_s}, & \omega_{*e} &= T_{e0} \left(\frac{\mathbf{B} \times \nabla \ln n_0}{eB^2} \right) \cdot (i\nabla_{\perp}), & \alpha &= (1 + \eta_i)\tau, \\ \chi &= \epsilon^{-1} \frac{q\rho_{s0}}{\bar{a}} k_{\alpha}, & \epsilon_n &= 2L_n/\bar{R}, & b &= (k_{\perp}\rho_s)^2 = \left(\frac{\chi B_0}{B} \right)^2 \hat{k}_{\perp}^2, & c_s &= \left(\frac{T_{e0}}{m_i} \right)^{1/2}, \end{aligned}$$

$$\rho_{s0} = \frac{c_s}{eB_0/m_i}, \quad \nu_{ei}^* = 0.51 \frac{\bar{R}}{\lambda_e} \left(\frac{m_e}{m_i}\right)^{1/2}, \quad \lambda_e = \frac{v_{th}}{\nu_{ei}}, \quad \tau = \frac{T_{i0}}{T_{e0}}, \quad v_{th} = \left(\frac{T_{e0}}{m_e}\right)^{1/2},$$

$$L_n^{-1} = -\frac{d \ln n_0}{ds} \hat{s} \cdot \nabla s|_{\zeta=0}, \quad \hat{\mathbf{k}}_{\perp}(s, \alpha, \zeta; \theta_k) = \frac{\bar{a}}{q} \left[\nabla \zeta - q \nabla \theta - \left(\frac{\zeta - \zeta_0}{q} - \theta_k \right) \hat{q} \nabla s \right].$$

In the weak collisionality limit, equation (3) gives the toroidal η_i mode dispersion relation [4] while in the opposite limit it gives the resistive ballooning mode dispersion relation.

Results

The eigenvalue problem, equation (3), is solved numerically in an ITER-like magnetic field configuration [1] for a magnetic flux surface, $s = 0.73$, with global magnetic shear, $\hat{q}(s)/q(s) = 2.026$. Details of the boundary conditions and the numerical method used are given in Ref. [2]. From the $k_{\perp}\rho_s|_{\zeta=0}$ spectrum shown in figure (a), we find that the maximum growth rate occurs for $k_{\perp}\rho_s|_{\zeta=0} = 0.12$, which is close to the typical value (i.e., $k_{\theta}\rho_s \approx 0.15$) found for these resistive modes in an analytical tokamak equilibrium [3]. However, we also find an initial stabilization in the low k regime of the spectrum, even though we have not included the additional stabilizing effects such as shear damping and parallel ion motion in the model.

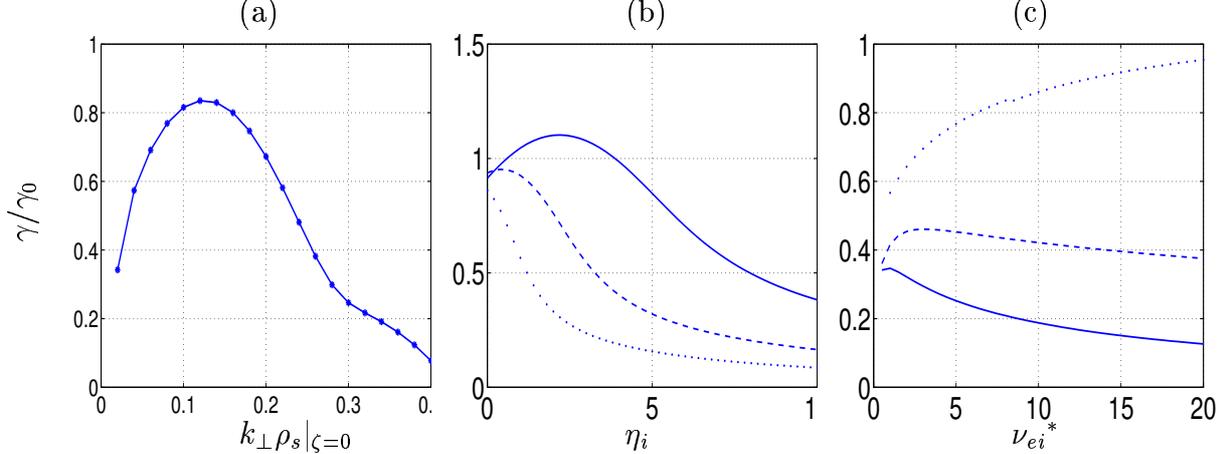


Figure: (a) The growth rate (normalized to $\gamma_0 = [2c_s^2/L_n\bar{R}]^{1/2}$) as a function of $k_{\perp}\rho_s|_{\zeta=0}$ for $s = 0.73$ magnetic surface with $\epsilon_n = 0.04$, $\tau = 1$, $\theta_k = 0$, $\nu_{ei}^* = 3$ and $\eta_i = 2.0$. (b) The normalized growth rate as a function of η_i with $\epsilon_n = 0.01$ (dotted), $\epsilon_n = 0.02$ (dashed) and $\epsilon_n = 0.04$ (solid) and for $\nu_{ei}^* = 10$, $k_{\perp}\rho_s|_{\zeta=0} = 0.15$, $\tau = 1$, and $\theta_k = 0$. (c) The normalized growth rate as a function of ν_{ei}^* for $\eta_i = 0$ (dotted), $\eta_i = 1.5$ (dashed) and $\eta_i = 4.0$ (solid). The other parameters are $\epsilon_n = 0.01$, $\tau = 1$, $\theta_k = 0$ and $k_{\perp}\rho_s|_{\zeta=0} = 0.15$

The effects of η_i on mode growth rate is depicted in figure (b) where ϵ_n is taken as a parameter. For steeper density gradients (for example, $\epsilon_n = 0.01$) the growth rate decreases monotonically whereas for larger ϵ_n values a small initial stabilization is evident in the figure. A strong stabilization for large ion temperature gradients for all cases is also observed. As mentioned earlier, the eigenvalue equation (3) contains two branches of the resistive modes. In order to study the transition, the normalised growth rate as a function of the collision frequency with η_i as a parameter is displayed in figure (c). In the large collisionality limit (for large ν_{ei}^* values), the resistive ballooning mode characteristics (decreasing growth rate for large η_i values) are evident in the figure. In the collisionless limit, the coupled mode reduces analytically to the toroidal η_i mode.

The main concern of the present work is to make a first attempt to study resistive η_i mode and the resistive ballooning mode in the electrostatic limit using a numerical VMEC equilibrium relevant to ITER Scenario 4. The trends of our results are somewhat similar to what has been found earlier for an analytical tokamak equilibrium [3]. It may be due the reason that in electrostatic limit studied here the modes are strongly localised.

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