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Two-fluid Extended-MHD Calculations of Collisionless Reconnection in Magnetized Plasmas and Toroidal Equilibrium

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I. Introduction

We have developed a high-order, implicit method for solving the time-dependent extended-magneto-hydrodynamic (X-MHD) equations in two dimensions [1-3] and have applied this to the problems of collisionless magnetic reconnection in the presence of a strong guide field and two-fluid stationary toroidal equilibrium. These calculations are performed with a new simulation code, M3D-C^1, which uses a fully unstructured triangular mesh that can be packed into regions with high gradients, but the time-step is limited only by resolution requirements. Each finite element contains a complete 4th degree polynomial with additional terms higher order terms to provide C^1 continuity across element boundaries as required to efficiently solve the stream function/ potential form of the equations including the high-order derivatives that appear in the electron and ion viscosities.

II. Magnetic reconnection in the presence of a strong guide field:

What has become a "standard problem" in 2-fluid magnetic reconnection was proposed in [4]. We define an initial equilibrium in 2D slab geometry as follows;

$$\begin{aligned}
&\text{Poloidal Magnetic Flux: } \psi^0(x,y) = \frac{1}{2}\ln\left(\cosh 2y\right) \\
&\text{Toroidal Field: } I^0(x,y) = B^0 \\
&\text{Total Pressure: } P^0(x,y) = \frac{1}{2}\left[\operatorname{sech}^2(2y) + 0.2\right] \qquad (2.1) \\
&\text{Electron Density: } n^0(x,y) = \left[\operatorname{sech}^2(2y) + 0.2\right] \\
&\text{Electron Pressure: } P_e^0(x,y) = 0.2P^0(x,y)
\end{aligned}$$

All other quantities are initialized to zero. A perturbation in the poloidal flux is applied at time $t=0$ as follows:

$$\psi(x,y) = \varepsilon \cos k_x x \cos k_y y\,. \qquad (2.2)$$

The initial equilibrium and perturbed current densities are just the Laplacian of the fluxes,

$J^0 = \nabla^2\psi^0, J = \nabla^2\psi$. The computation is carried out in a rectangular domain: $-L_x/2 \le x \le L_x/2$ and $-L_y/2 \le y \le L_y/2$. The system is taken to be periodic in the *x*-direction with ideal conducting boundaries at $y = \pm L_y/2$. The parameters are chosen such that $k_x = 2\pi/L_x$, $k_y = \pi/L_y$, with $L_x = 25.6, L_y = 12.8,$ and $\varepsilon = 0.1$. The nominal values of resistivity and viscosity are $\eta=0.005$ and $\mu=0.05$.

The reconnection calculations presented here extend the GEM [4-5] reconnection problem to include a strong background (guide) magnetic field B^0 as is present in a fusion plasma. We find that the background field significantly delays the onset of the fast reconnection phase and reduces the maximum reconnection rate and the amplitude of the velocities that develop in the reconnection region. This is illustrated in Figure 1.

The four curves in Figure 1a show the magnetic reconnection rate as a function of time for the GEM problem defined above, but with different values of the background toroidal field: 0.2, 1.0, 2.0, and 5.0. It can be seen that the case with $B^0 = 0.2$ exhibits a fast reconnection phase which is very similar in structure and in magnitude to the $B^0 = 0$ case presented in [4]. However, as B^0 is increased to 1.0, 2.0 and then to 5.0 and beyond, the onset time of the fast reconnection phase is delayed substantially and the maximum rate is significantly reduced. In Figure 1b we plot the density at the reconnection point as a function of time for the four cases of Figure 1a. It is seen that as the toroidal field increases and the solution becomes more like the incompressible solution, the density at the center (reconnection region) decreases much slower to the far-field value.

The explanation for the marked difference between the low (or zero) and high guide-field cases has to do in part with the evolution of the plasma density. In the zero or low guide field cases, the compressibility of the flow causes the density to quickly deplete in the reconnection region, increasing the effect of the Hall term and thereby accelerating the reconnection. The faster the fluid reconnects, the lower the density at the reconnection point becomes, and thus the sudden shock-like transition. As the guide field is increased, the fluid is forced to become more incompressible, removing this effect and thus greatly decreasing the maximum reconnection rate.

III. Stationary Toroidal Equilibrium.

We have computed toroidal equilibrium that are stationary on all timescales by solving for the steady state of the 2D (X-MHD) equations including the effects of gyroviscosity [2], Hall-terms, anisotropic thermal conductivity and self-consistent flow. A loop voltage is applied at the computational boundary to balance the resistive flux consumption. The calculation is extended across the separatrix into a vacuum region that is represented as a high-resistivity plasma.

Figure 2 shows the sequence of meshes used in an equilibrium calculations. An initial triangular mesh with uniform resolution is used to compute the first approximate equilibrium. From this approximate equilibrium solution the mesh is adapted according to gradients in the resistivity profile and the entire simulation is rerun. This process is repeated until a satisfactory result is obtained. Note that only an approximate equilibrium sufficient enough to drive the mesh adaptation procedure is needed for the simulations leading up to constructing the final mesh. The criteria for determining the final mesh is its appropriateness for resolving edge pedestal effects, which will be used for the future calculation of edge phenomena such as Edge Localized Modes.

We show in Figure 3 contours of the plasma resistivity, the stream function associated with the in-plane velocity, and the toroidal velocity on the final mesh. These velocity fields are driven by the plasma resistivity and other non-ideal terms in the equations. We find that exact up-down symmetry is not maintained in the flow fields, but that small asymmetries will grow to a finite level and then stabilize in magnitude and develop stable oscillations consistent with previous results. [5,6] A linear extended MHD code is being developed to study the stability of the 2D toroidal stationary equilibrium as described here. This will be reported in a future publication. This work was supported by U.S. DoE contract DE-AC02-76CH03073

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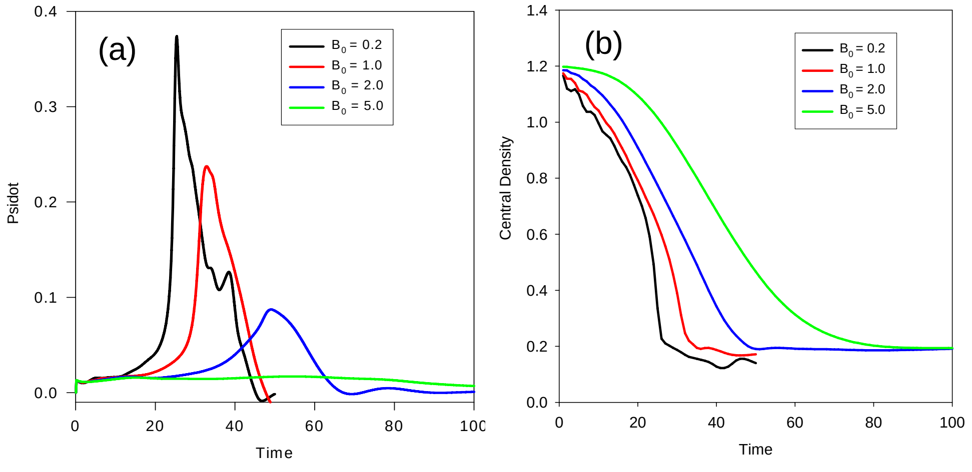

Figure 1: (a) Reconnection rate vs. time for GEM reconnection case with different guide fields. (b) Central density vs. time for different guide fields.

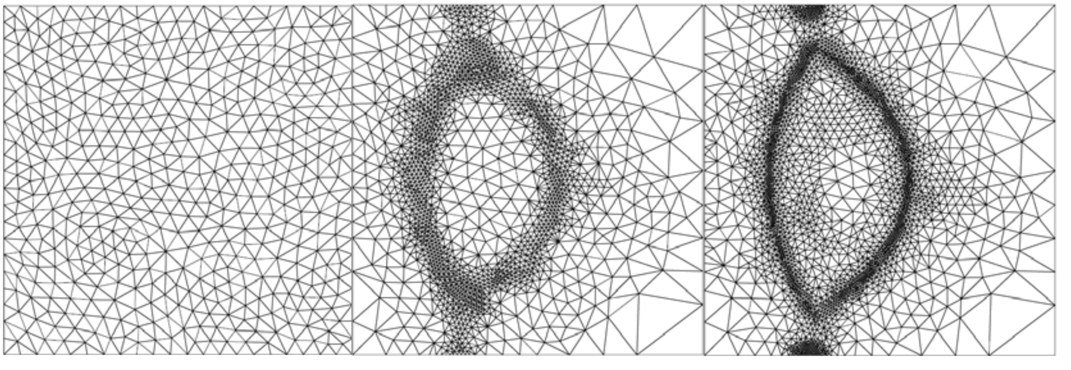

Figure 2: Sequence of meshes used in toroidal equilibrium calculation. Meshes are adapted to resistivity profile with 956(l), 3554(c), and 5512(r) elements.

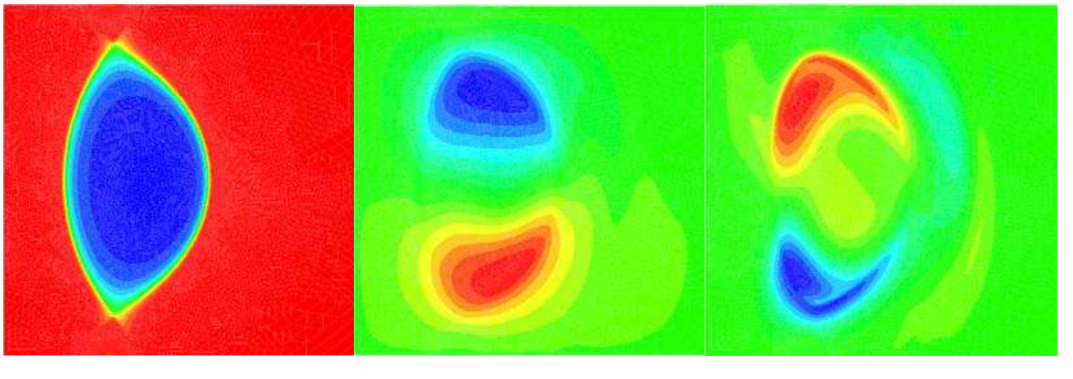

Figure 3: Contours of resistivity (l), stream function for in-plane incompressible velocity (c), and toroidal velocity (r) for the final mesh