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Tokamaks with Reversed Current Density: Current Holes, AC Operation and Axisymmetric Stability

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The "current hole" tokamak regimes with nearly zero or negative toroidal current in the central region feature good plasma confinement properties and may be naturally attained in the advanced tokamak (AT) regime.

The family of equilibrium configurations corresponding to eigenfunctions of Grad-Shafranov operator seems wide enough to model various topologies of axisymmetric magnetic islands and finite pressure effects. In particular, dipole-type equilibria with circular or shaped cross-sections were considered as a simplest model of AC tokamak operation . Such equilibria with shaped cross-sections and with the internal separatrix orientated along the minor semi-axis were found to be stable against fixed-boundary axisymmetric modes.

The effects of finite pressure on the topology of magnetic islands and on axisymmetric stability are demonstrated.

1. Ideal MHD stability: the problem formulation and approximation

For stability modelling without use of special coordinates and magnetic surface projections the original form of plasma potential and kinetic energy functionals can be used:

$$W_p = \frac{1}{2}\int \left\{ |\vec{Q}|^2 + \vec{j}\cdot\vec{\xi}\times\vec{Q} + (\nabla\cdot\vec{\xi})(\vec{\xi}\cdot\nabla p) + \gamma p(\nabla\cdot\vec{\xi})^2 \right\} d^3r, \quad \vec{Q} = \nabla\times(\vec{\xi}\times\vec{B}), \qquad (1)$$

$$K_p = \frac{1}{2}\int \rho|\vec{\xi}|^2 d^3r. \qquad (2)$$

In the case of $\gamma = 0$ the functionals can be rewritten in terms of electric field perturbation amplitude $\vec{E} = i\omega\vec{e}$, $\vec{e} = -\vec{\xi}\times\vec{B}$ (time dependence $e^{i\omega t}$ is assumed for eigenvalue problem):

$$W_p = \frac{1}{2}\int \left\{ |\nabla\times\vec{e}|^2 - \frac{\vec{j}\cdot\vec{B}}{B^2}\vec{e}\cdot\nabla\times\vec{e} - \frac{\vec{j}\cdot\vec{e}}{B^2}\left[2\vec{B}\cdot\nabla\times\vec{e} - \vec{t}\cdot\vec{e}\right] \right\} d^3r, \qquad (3)$$

$$K_p = \frac{1}{2}\int \rho|\vec{e}|^2/B^2 d^3r, \quad \vec{t} = \vec{j} + B^2\nabla(\frac{1}{B^2})\times\vec{B}, \qquad (4)$$

combined with the requirement $(\vec{e}\cdot\vec{B}) = 0$. The surrounding vacuum region (free boundary) can also be taken into account (see [1]).

For tokamak modelling we use the standard equilibrium magnetic field representation, $\vec{B} = \nabla\psi\times\nabla\phi + f\nabla\phi$, with the poloidal flux function ψ being one of the eigenfunctions of Grad-Shafranov operator:

$$-R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = ((R/R_0)^2\beta_p + (1-\beta_p))\lambda_{eq}\psi, \qquad (5)$$

which can be considered as equilibrium configuration with $p' = (\beta_p/R_0^2)\lambda_{eq}\psi$, $ff' = (1-\beta_p)\lambda_{eq}\psi$. The family is an extension of force-free configurations considered in [2] to finite pressure cases determined by the values of the parameter β_p, i.e. poloidal β on axis, $R = R_0$.

The approach to approximate and to solve the stability problem on triangular grids includes:

- longitudinal and poloidal projections of the unknown vector $\vec{e}$: $\vec{e} = e_\phi\nabla\phi + \vec{e}_{pol}$ and different finite elements for them: standard node-based "hat"-functions W_i for e_ϕ, and edge-based Whitney elements W_{mn}, $W_{mn} = W_m\nabla W_n - W_n\nabla W_m$ for $\vec{e}_{pol}$;
- Lagrange multipliers introduced to approximate the constraint $(\vec{e}\cdot\vec{B}) = 0$ at each grid node of the plasma region;

- the postprocessing step: reconstruction and plotting of the displacement vector, $\vec{\xi}_\perp = \vec{e} \times \vec{B}/B^2$.

The equilibrium and stability calculations use the same unstructured grids which are optionally adapted to the solution features (e.g. jump in the current density) [1]. For the tests presented here we mostly used triangulated structured grids taken from the CAXE/KINX calculations [3].

2. Axisymmetric stability for reversed current configurations of dipole type

Previous computations performed for the configurations with reversed current density [1] showed that most of them are unstable against $n = 0$ modes even with the conducting wall at the plasma boundary. But some dipole type configurations are stable at least in the fixed-boundary case. Typically, they correspond to the second eigenvalue of Grad-Shafranov operator (the first one gives the conventional equilibrium with nested magnetic surfaces) for elliptic (elongation $E \neq 1$) shapes. The separatrix lies along the minor axis of the ellipse in this case (Fig.1). The toroidicity effects are quite weak: very similar results are obtained for shaped cylindrical plasma. Finite pressure also weakly influences the stability: the configurations remain stable for high values of the poloidal β at least up to $\beta_p = 4$.

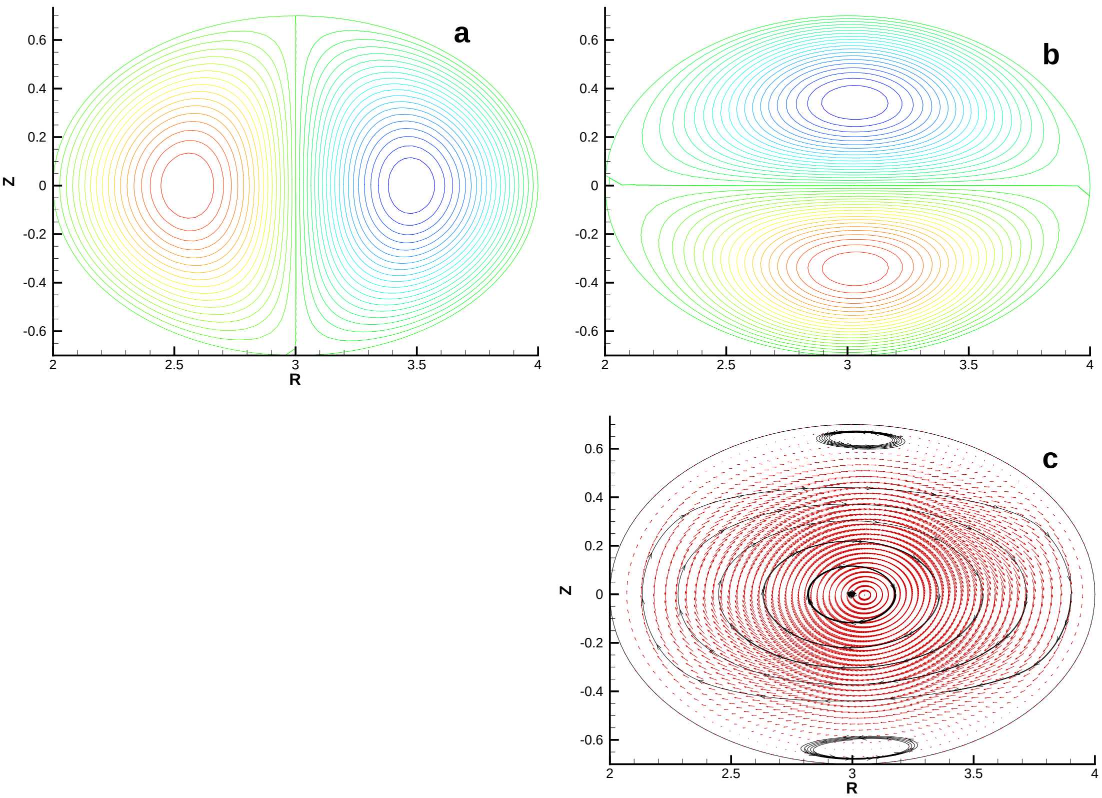

Fig.1. Reversed current density force-free equilibria (oblate cross-section, $E = 0.7$): a) stable configuration, $\lambda_{eq} = 18.5$; b) unstable configuration, $\lambda_{eq} = 26.2$; c) displacement vector $\vec{\xi}_{pol}$ with streamlines, ω^2=-1.1773.

For a purely circular cross-section toroidicity plays its role giving different stability properties to two related configurations (Fig.2). The stable force-free dipole equilibrium with horizontal separatrix (Fig.2,a) corresponds to slightly smaller eigenvalue of Grad-Shafranov operator. The force-free configuration with vertical separatrix is unstable and the increment increases significantly with growing β_p (Fig.2,b-e).

The eigenvalues ω^2 scale with squared poloidal Alfvén frequency which is close to unity since the poloidal flux function is normalized by its maximal absolute value.

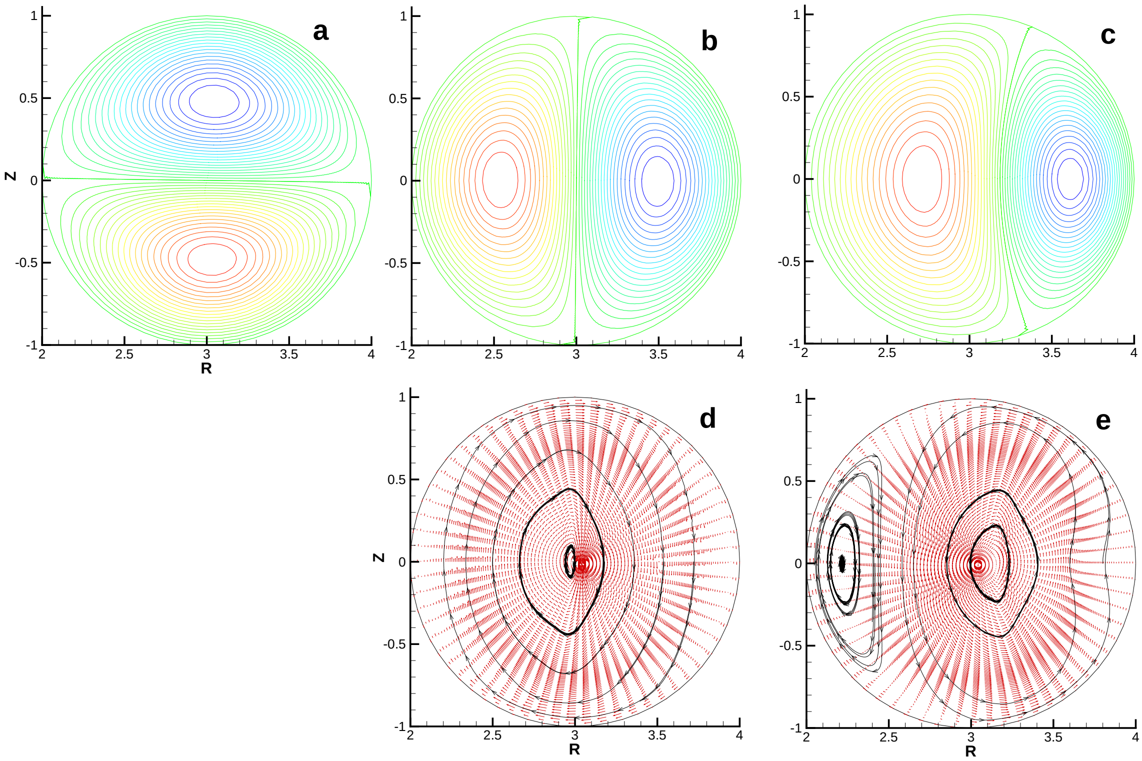

Fig.2. Reversed current density equilibria (circular cross-section, $E = 1$) of dipole type:
a) stable configuration, $\beta_p = 0, \lambda_{eq} = 14.833$;
b,c) unstable configurations: b) $\beta_p = 0,\ \lambda_{eq} = 14.835$; c) $\beta_p = 1.2,\ \lambda_{eq} = 14.503$;
d,e) corresponding displacement vectors $\vec{\xi}_{pol}$ with streamlines:
d) $\beta_p = 0,\ \omega^2 = -0.0009$; e) $\beta_p = 1.2,\ \omega^2 = -0.0072$.

3. Reversed current density configurations of "central" type. Finite pressure effects

The considered family of Grad-Shafranov eigenfunctions includes configurations modeling "current hole" equilibria: a central region with negative current density and non-nested flux surfaces embedded into a region with nested magnetic flux surfaces. The topology of axisymmetric magnetic islands and stability of $n = 0$ modes are rather sensitive to finite pressure. Fig.3 demonstrates these effects for a circular cross-section case. The force-free configuration (Fig.3,a,d) having one x-point is weakly unstable. Increasing pressure provokes an outward shift of magnetic axes and splits the x-point onto two x-points at about $\beta_p = 0.6$ (Fig.3,b,e). Further pressure increase gives rise to a second unstable eigenvalue/eigenfunction (Fig.3,c,f,g).

To conclude, the use of unstructured adaptive grids and a rather general approach to the approximation of the linear MHD equations open new capabilities for the stability analysis of unusual plasma equilibria. Further applications and developments of the method will include:

- the $n = 0$ stability analysis of more general reversed current equilibria, including cases with realistic current and pressure profiles and analytic configurations [4],[5];
- an extension of the approach to kink mode stability analysis.

Further research is needed also on the proposed numerical methods concerning, in particular, the role of grid alignment/anisotropy and a more efficient implementation of the constraint $(\vec{e} \cdot \vec{B}) = 0$.

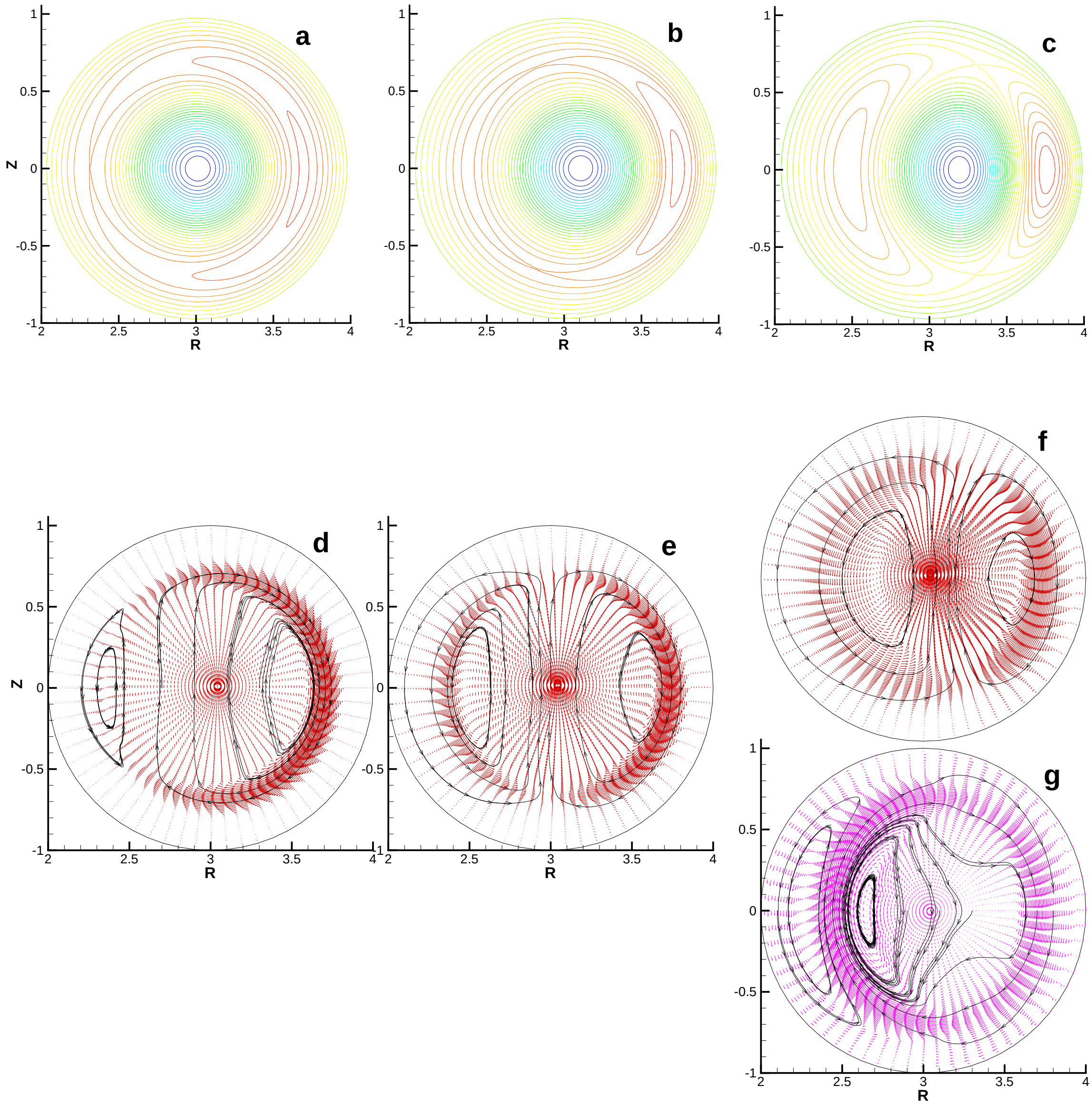

Fig.3. Reversed current density equilibria (circular cross-section, $E = 1$) of central type: equilibrium configurations: a) $\beta_p = 0$; b) $\beta_p = 0.6$; c) $\beta_p = 1.2$; displacement vectors: d) $\beta_p = 0,\ \omega^2 = -0.0063$; e) $\beta_p = 0.6,\ \omega^2 = -0.1697$; f) $\beta_p = 1.2,\ \omega^2 = -1.2389$; g) $\beta_p = 1.2,\ \omega^2 = -0.0623$.

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