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Radiative-Collisional Kinetics of Rydberg Atomic States in Astrophysical Plasmas

M.B. Kadomtsev, M.G. Levashova, V.S. Lisitsa

NFI RRC "Kurchatov Institute", Kurchatov sq. 1, Moscow, 123182, Russia

Computation of populations of highly excited (Rydberg) atomic states is actual both for astrophysical objects and for laboratory plasmas. Transition from one-dimensional model of atomic states population to two-dimensional kinetics (on n and l) makes the problem much more complicated due to distinctive increasing of number of atomic transitions which take part in radiative-collisional cascade. At the same time for highly excited states the classical description can be employed. This work is devoted to computing of two-dimensional distribution function based on classical kinetic equation modified with account of quantum nature of states population.

In classical description collisional processes results in diffusion of atomic electrons over atomic states. The effect of diffusion is taken into account by Landau collisional integral for collisions of atomic electron with those of plasma and considering plasma electrons having Maxwellian distribution over velocities [1]. As for radiative cascade, in classical approach it is described by continuous flow of distribution function along the characteristic [2]. The atomic electron's energy and angular momentum loss due to radiation emission are computed in the approximation of classical electrodynamics. In this approximation the electron's moving over atomic states is reduced to elementary diffusion equation with constant diffusion coefficient D:

$$\left[\hat{D}+\hat{R}\right]f(n,l)+q(n,l)=0 \qquad (1)$$

where the differential operator of the second order $\widehat{D}$ describes collisional diffusion and the differential operator of the first order $\widehat{R}$ describes radiative cascade [2], q is a source of atomic states population. This approach is valid only for those sources of populating of atomic states whose distribution in quantum numbers are broad enough so that statistical weight of states with small angular momenta is negligible: $l_{eff}^2 \propto n^{4/3} << n^2$.

Here we employ the method developed by Kukushkin and Lisitsa [2] that accounts jumps in quantum numbers for two-dimensional case (in nl-space). It also can be applied to radiative cascade accompanied by collisions.

Quantum kinetic equation for radiative-collisional cascade results from balance of atomic states population and takes the form:

$$\left|\hat{D}+\hat{Q}\right| f(n,l)+q(n,l)=0 \tag{2}$$

Acting of operator $\hat{Q}$ on distribution function is given by:

$$\hat{Q}f(n,l)=-A(n,l)\,f(n,l)+\Sigma A(n,l;n',l')\,f(n',l') \tag{3}$$

where $A(n,l;n',l')$ is radiative transition rate for atomic states with different quantum numbers, $A(n,l)=\sum\limits_{n',l'} A(n,l;n',l')$ is the total radiative decay rate from the given state to all lower ones, summing in (3) should be done with account of selection rules.

The iterative algorithm developed in [2] for radiative cascade could be extended for cascade accompanied by collisions and involves serching the solution f of eq. (2) in the form of successive iterations $f = f_0 + f_1 + f_2 + \ldots + f_n + \ldots$and find f_n on each step from the equation:

$$\hat{D}f_n(n,l)-A(n,l)\,f_n(n,l)+\sum_{n'=n+1}\sum_{l'=l\pm 1} f_{n-1}(n',l')A(n',l'\to n,l)=0 \tag{4}$$

where f_{n-1} is to be find on the preceding step, and f_0 is to be find from

$$\hat{D}f_0(n,l)-A(n,l)\,f_0(n,l)+q(n,l)=0 \tag{5}$$

The effective source $\langle q_n\rangle=\sum\limits_{n'=n+1}\sum\limits_{l'=l\pm 1} f_{n-1}(n',l')A(n',l'\to n,l)$ describes populating of state n,l by every possible n-quantum transitions. As long as every consequent averaging of the source leads to its smoothing, we can change quantum operator $\hat{Q}$ to the classical one $\hat{R}$ for the certain step of iterative procedure and solve the pure classical equation (1) with replacing of q by $<q_n>$.

Here we employ this method for photorecombination source. It takes the form [2] :

$$q(n,l)=\frac{4}{\pi^2 c^3 l^2 n^3}\exp\left(\frac{1}{2n^2 T}\right)\int_{x_0}^{\infty}\exp(-3x/Tl^3)G_0(x)dx \tag{6}$$

where $x_0 = l^3/6n^2, G_0(x)=x[K^2_{1/3}(x)+K^2_{2/3}(x)]$ - is a combination of McDonald functions. The source (6) is selective tangibly on l and to a lesser extent on n: it populate mainly the states with small n and l.

Solving of eqs. (1,2) was performed numerically by the grid method. Following [1] we used variables $\xi = n_0/n$ and $\eta = l/n$, where n_0 is a specific value of the principle quantum number where the effects of collision and radiative operators are of the same magnitude. First we solved these equations for three body recombination which corresponds to thermodynamic equilibrium population of atomic states near the continuum spectrum. This was used as a boundary conditions: $f_0(\xi \to 0) = 1$. Here and elsewhere f is given in units of thermodynamic equilibrium distribution. The results are shown in fig.1.

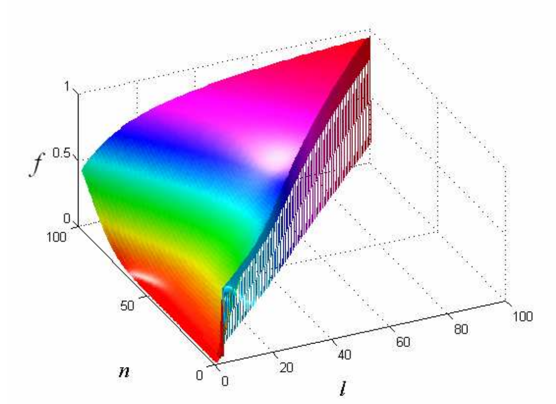

Fig. 1. Electron distribution function with account of collisions with plasma electrons at thermodynamic equilibrium population of atomic states near the continuum spectrum for typical conditions of astrophysical plasmas: N_e = 2500 cm^{-3}, T_e = 1 eV, n_0 ~ 50.

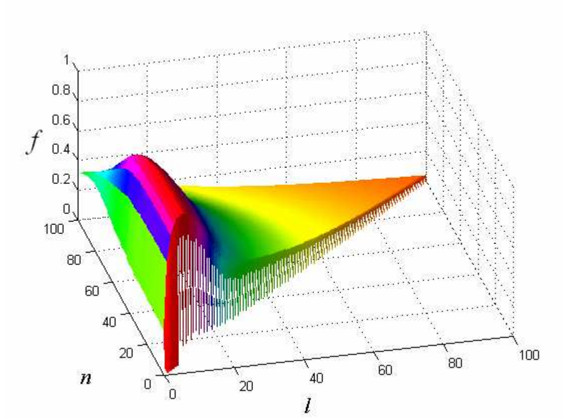

Fig. 2. Electron distribution function with account of collisions at zero boundary conditions with photorecombination source of population, N_e=2500 cm^{-3}, T_e=1 eV, n_0 ~ 50.

For estimation of the photorecombination source contribution it's interesting to obtain a solution for $f_{tot} = f_0 + f_1$ for zero boundary conditions with photorecombination source only. This result is given in fig.2. Comparing fig.1 and 2 shows, that these solutions are significant in completely different regions of variables ξ and η.

The solution for simultaneous acting of three body and photo recombination are shown in fig. 3.

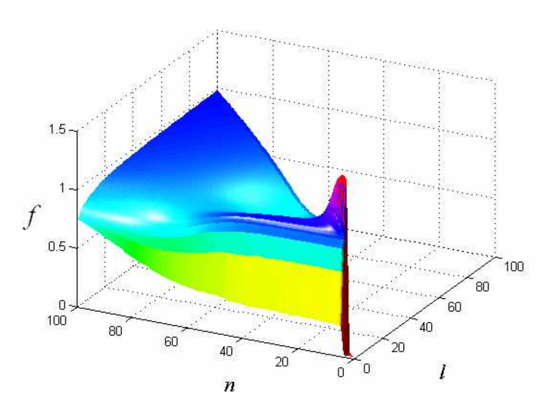

Fig. 3. Total distribution function with account of collisions with plasma electrons with photorecombination source of population and thermodynamic equilibrium population of atomic states at the approach to continuum spectrum, N_e=2500 cm^{-3}, T_e = 1 eV, n_0 ~ 50.

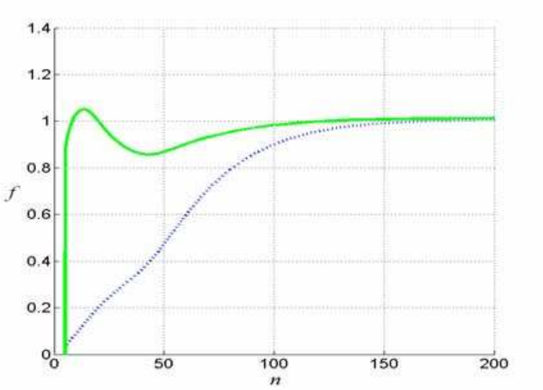

Fig. 4. One-dimensional distribution function with account of collisions with plasma electrons with photorecombination source of population and thermodynamic equilibrium population of atomic states near the continuum spectrum (continuous green), without photorecombination (dash blue), N_e= 2500 cm^{-3}, T_e = 1 eV, n_0 ~ 50.

It is interesting to notice the fall down of populations compared to thermodynamic equilibrium limit $f = 1$ in the intermediate region of variables ξ and η.

To compare results obtain here with those obtain previously in the frame of one dimensional models of atomic states population the total two-dimensional function was integrated over angular momentum quantum number. The graphs for one dimensional function $\overline{f}$ for three body recombination only and for the total (three body and photorecombination acting together) sources of population are shown in fig. 4. The comparison with numerical calculations performed in [3], shows a caving in population distribution for both methods of calculation, it caused by contention of radiative and collisional canals of population. Some discrepancy for small quantum numbers ($\xi >> 1$) caused by strong non-equilibrium of two dimensional distribution function in angular momentum, concerned with circular orbits predominating: *l* ~ *n*.

In conclusion, in this work the universal kinetics for two dimensional populations of Rydberg atomic states by three body and photorecombination sources was developed. The two-dimensional electron distribution function were calculated on the base of modified classical model of collisions and radiative cascade . It is shown that the account of direct population by photorecombination source is nessesary. The essentially non-equilibrium character of atomic states population both in principle and in angular momentum quantum numbers. This plays a key role in diagnostics of astrophysical and laboratory plasma; in particular, this is important for interpretation of radiation radiolines spectra observed in astrophysical plasmas [4].

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References

[1] S.T. Belyaev, G.I. Budker, in *Plasma Physics and Controlled Nuclear Reactions,* vol. 3, edited by M.A. Leontovich, Moscow: USSR Academy of Sciences, 1958, p.46.
[2] A.B. Kukushkin, V.S Lisitsa, *Sov. Phys. JETP,* 61, 937 (1985).
[3] V.S.Strelnitski, V.O.Ponomarev, H.A.Smith, Astroph.J. 470,1118 (1996).
[4] Sorochenko R L and Gordon M A 2004 *Radiorecombination Lines* (Springer)