

## Ion acceleration by a strong laser-driven quasi-static electric field

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### Abstract

A theoretical model of the quasi-static electric field that is formed at the sharp boundary between a solid target and vacuum, due to the appearance of a cloud of laser-produced hot electrons, has been developed. It allows one to describe the maximum energies and the energy spectra of the ions accelerated in the field, it and makes possible the comparison with experimental data and the previsions of regimes achievable in the future.

### Introduction

Laser-induced ion acceleration seems one of the most promising applications of the interaction of ultra-intense ultra-short laser pulses with solid matter [1]. Several processes can be responsible for laser ion acceleration, depending on the laser intensity. However, most of the experiments carried out in the last years rely on the so-called Target Normal Sheath Acceleration mechanism. The optimization of the interaction conditions (width of the target, energy and time duration of the laser pulse, effects of the laser pre-pulse) requires the formulation of a first-principle physical model of the acceleration, in order to properly choose the laser-target parameters, that maximize the laser-to-ion energy conversion efficiency. To this aim, we have developed a consistent theoretical description of the spatial distribution of the electron cloud at the rear surface, at  $x = 0$ , of a planar target irradiated by a powerful laser on the front surface. It is assumed that only electrons with total negative energy  $\varepsilon$  (kinetic plus potential), *i.e.*  $-\varepsilon\phi(x) < \varepsilon < 0$ , remain close to the surface forming the negative cloud (for  $x > 0$ ). Instead, the most energetic ones, with  $\varepsilon > 0$ , are lost by the system and do not participate to the ion acceleration process [2]. Here, the analytical solution of the relevant Poisson equation is calculated both in the non-relativistic and in the ultra-relativistic limits. The maximum energy acquired by a test-ion in the bound electron distribution, and the relevant spectrum are calculated and compared with experimental measurements.

### The non-relativistic “bound” electron distribution

Assume a uniform positive charge distribution for  $x < 0$ , representing the ionized target lattice. The self-consistent electrostatic potential  $\phi(x)$  produced by the hot electron distribution (with temperature  $T \ll mc^2$ ) satisfies the dimensionless Poisson equation

$$\frac{d^2\varphi}{d\xi^2} = \Phi(\sqrt{\varphi})e^\varphi \quad (1)$$

where  $\xi = x/\lambda_D$ ,  $\varphi = e\phi/T$ ,  $\lambda_D = \sqrt{T/4\pi\bar{n}e^2}$ ,  $\bar{n} = n_0/[\Phi(\sqrt{\varphi_0})e^{\varphi_0}]$ ,  $\varphi_0 = \varphi(0)$ , and  $\Phi(z) = 2/\sqrt{\pi} \int_0^z dt e^{-t^2}$ . The implicit solution of Eq.(1) writes

$$\int_{\varphi_0}^{\varphi(\xi)} d\varphi' \left[ \Phi(\sqrt{\varphi'})e^{\varphi'} - \frac{2\sqrt{\varphi'}}{\sqrt{\pi}} \right]^{-\frac{1}{2}} = -\sqrt{2}\xi, \quad (2)$$

which in the small amplitude limit  $|\varphi| \ll 1$ , that is far from  $x = 0$ , becomes

$$\varphi(\xi) \approx \left[ \varphi_0^{1/4} - (6\sqrt{\pi})^{-1/2} \xi \right]^4, \quad (3)$$

and in the large amplitude limit  $|\varphi| \gg 1$ , that is approaching  $x = 0$ , is approximately

$$\varphi(\xi) \approx \varphi_0 - 2 \ln \left[ 1 + \frac{\sqrt{\pi}}{\sqrt{2}} \exp\left(\frac{\varphi_0}{2}\right) \right]. \quad (4)$$

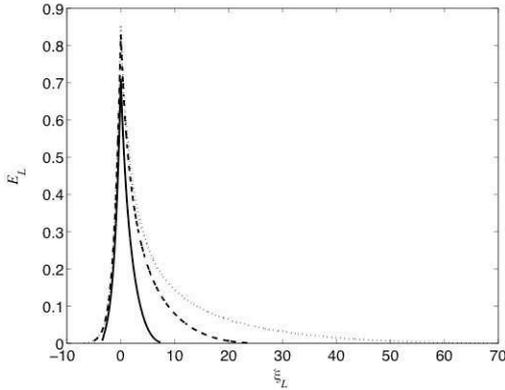


Fig.1 – The dimensionless electric field  $E_L = eE\lambda_L/T$  versus  $\xi_L = x/\lambda_L$ , for  $\varphi^* = 2$  (solid line), 4 (dashed line), 6 (dotted line). Here,  $\lambda_L = \sqrt{T/4\pi n_L e^2}$ , and  $n_L = N_{\text{tot}}/wA$  is the density of the laser-produced electrons,  $N$  is the total number of hot electrons,  $w$  is the width of the thin solid target (the distance between the front and rear surfaces),  $A$  the laser spot size (from ref.1).

Eq.(3) shows that the electrostatic potential decreases as the fourth power of the distance and becomes zero, with both the first (the opposite of the electric field) and the second (the electron density) derivatives, at a finite position,  $\xi_f = \sqrt{6\pi} \varphi_0^{1/4}$ , weakly dependent on  $\varphi_0$ . As a consequence of the finite spatial extent of the electron cloud, a test ion with charge  $Ze$ , placed initially at  $x = 0$ , is accelerated up to a finite maximum energy  $K_{\text{max}} = Z\varphi_0 T$ . In order to determine  $K_{\text{max}}$ ,  $\varphi_0$  should be found. To do that, the Poisson equation should be solved inside the target, for  $x < 0$ , and the two solutions matched at  $x = 0$ . So doing, the relationship  $\varphi_0 = -\Phi(\sqrt{\varphi^*}) + \frac{2}{\sqrt{\pi}} \sqrt{\varphi^*} e^{-\varphi^*} + \varphi^*$  is found, where  $\varphi^* = \varepsilon_{k,\text{max}}$  is the maximum kinetic energy of the hot electrons in the laser field [2].  $\varepsilon_{k,\text{max}}$  turns out to be a critical parameter for the determination of the complete solution of the problem. It depends on the physics of the laser-electron coupling mechanisms, a problem the solution of which is

out of the scope of the present analysis. Alternatively, it can be directly related to the experimental parameters, or determined on physical ground, for example as the maximum kinetic energy acquired by an electron in the laser field. Fig.1 displays examples of the spatial pattern of the self-consistent electric field around  $x = 0$  for different values of  $\epsilon_{k,\max}$ .

### The ultra-relativistic “bound” electron distribution

In order to make predictions on and comparisons with experimental data for laser intensities above  $10^{18}$  W/cm<sup>2</sup>, relativistic electron energies should be dealt with. Therefore, the electron

distribution in the limit  $p/mc \gg 1$  is considered, *i.e.*  $f_e^{\text{UR}} = \frac{\bar{n}c}{2T} \exp\left[-\frac{c|\mathbf{p}| - e\phi(x)}{T}\right]$

with  $T \gg mc^2$ , to calculate the bound electron density. The Poisson equation becomes

$$\frac{d^2\varphi}{d\xi^2} = e^\varphi - 1, \quad (5)$$

with the implicit solution

$$\int_{\varphi_0}^{\varphi(\xi)} \frac{d\varphi'}{\sqrt{e^{\varphi'} - \varphi' - 1}} = -\sqrt{2}\xi. \quad (6)$$

For  $|\varphi| \ll 1$ ,  $\varphi(\xi) \approx \varphi_0 e^{-\xi}$ . This behavior should go continuously into that of Eq.(3), however it can be shown that in the calculation of the energy acquired by a test-ion, the error introduced by the different dependences on  $\xi$  at small  $|\varphi|$ -values is negligible. For  $|\varphi| \gg 1$ , the solution coincides with Eq.(4). From now on, the ultra-relativistic limit of the electrostatic potential in Eq.(6) will be used to deal with the characteristics of the ion acceleration process, discarding the transition to the non-relativistic solution at low energies. Following the same procedure as in the previous Section,  $\varphi_0$  is now given by

$$\varphi_0 = \frac{(2\varphi^* - 1)e^{\varphi^*} - \varphi^* + 1}{2(e^{\varphi^*} - 1)}, \text{ with } \varphi^* = \epsilon_{k,\max}.$$

Consider at  $t = 0$  a small number of light test-ions distributed uniformly in a layer of width  $\Delta x \ll x_f$  on the rear surface ( $x = 0$ ) of the target made of high-Z material; their density is  $n_{\text{test}}(\xi) = n_0 [H(\xi) - H(\xi - \Delta\xi)]$ . Due to the conservation of the volume element in the phase space, the ion energy spectrum at  $\xi \geq \xi_f$  takes the form

$$n_{\text{test}}(\epsilon) = n_0 \frac{H(\epsilon - \varphi_0) - H(\epsilon - \varphi(\Delta\xi))}{2Z \left[ \exp\left(\frac{\epsilon}{Z}\right) - \frac{\epsilon}{Z} - 1 \right]^{1/2}}. \quad (7)$$

### The ion energy spectrum and the maximum ion energy

One of the critical issues to be faced in order to apply a theoretical model of ion acceleration is the determination of the maximum energy,  $\epsilon_{k,\max}$ , acquired by electrons under the action of the laser pulse. It is related to the laser energy,  $E_L$ , although it is difficult to single out a unique physical process of energy transfer from the laser to electrons. Therefore, a scaling law relating  $\epsilon_{k,\max}$  with  $E_L$  is proposed based on the analysis of published results on several experiments. In order to better fit the data, the scaling involves the hot electron temperature, as well, that is  $\epsilon_{k,\max} = E_{k,\max}/T \approx 3.8 + 0.8 \ln[E_L(\text{J})]$ . In Fig.2 the ion energy spectrum calculated according to our model (reddish line) compared to the experimental values relevant to the Nova experiment [1]. Besides this, several published ion energy spectra have been compared with the expectations of our model, in different laser intensity regimes, with satisfactory results. In Fig.3, the iso-lines at constant maximum ion energy are shown in the plane ( $I-E_L$ ) in the range of interest for proton acceleration in hadron-therapy, where  $K_{\max} = 250\text{MeV}$  is required. Two possibilities are envisaged, at higher (A) and lower (B)  $E_L$ .

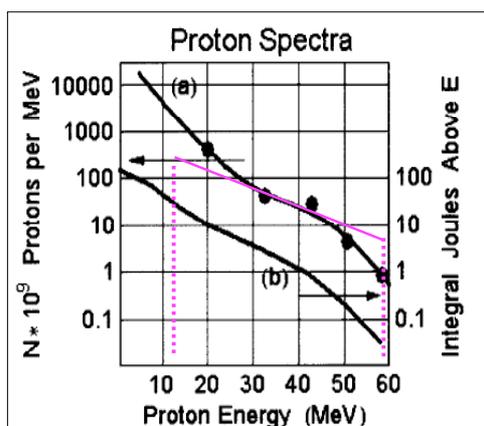


Fig.2 – Energy spectrum from Ref.1 compared with the result of Eq.(7).

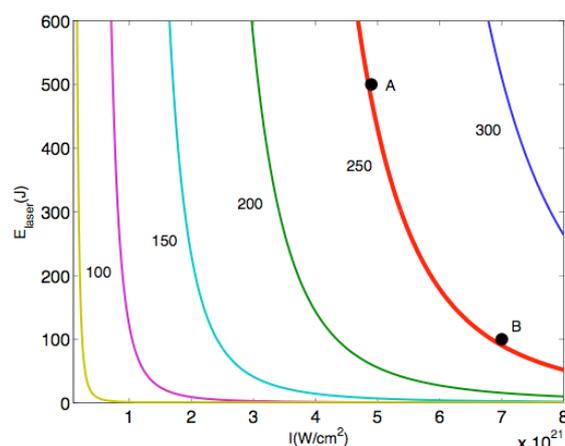


Fig.3 – Iso-lines at constant maximum ion energy in the plane intensity-energy of the laser pulse.

Notice that the present model predicts in a self-consistent way the existence of an upper cut-off in the spatial extension of the hot electron cloud, at  $x_r$ , a feature which has been observed in some experiments [3,4], and which has been introduced on physical ground in a previous quasi-static model [5].

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