Numerical Evaluation of Ripple-Induced Diffusion of Suprathermal Ion in Tokamaks

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Introduction

The confinement of fusion-produced alpha particles is important in tokamak reactors. In an axisymmetric field, alpha particles are well confined. In a practical system, however, a finite number of toroidal field (TF) coils cause TF ripples, which enhance the transport of alpha particles. Furthermore, particles can resonate with ripples, which also enhance diffusion coefficients. In this work, we evaluated the diffusion coefficients with the ripple rasonance condition.

Banana Drift

In a nonaxisymmetric field, the toroidal canonical momentum P_ϕ is not conserved, and it has large drifts at banana tips. This drift is approximately given by [1]

$$\Delta P_\phi = \Delta P_\phi^* \sin\left(N\phi_{\rm b} - \pi/4\right), \quad (1)$$

where N is the number of toroidal field coils, $\phi_{\rm b}$ is the toroidal angle of a banana tip and ΔP_ϕ^* is the drift amplitude at the banana tip due to the ripple and is given by an integral over a unperturbed trapped orbit, most of the contribution coming near the banana tip.

Because of the $\phi_{\rm b}$-depnedence of the toroidal canonical momentum drift, banana particles can resonate with toroidal ripples. This ripple resonance conditon is expressed as
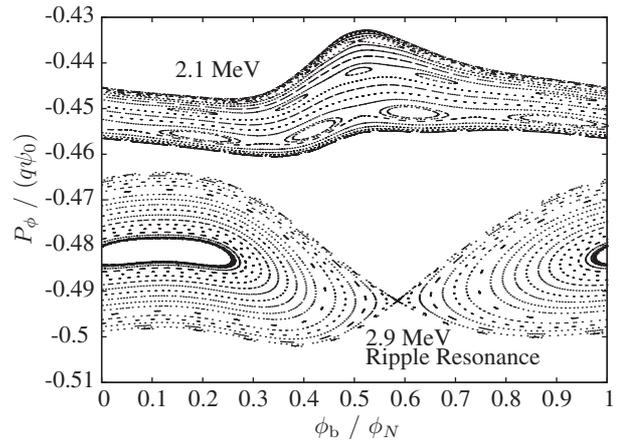

Figure 1: Poincaré map of particles with 2.1 MeV and 2.9 MeV (near resonance energy). The abscissa is the toroidal angle of banana tip where $\phi_N \equiv 2\pi/N$ and the ordinate is the normalized toroidal canonical momentum at banana tips.

$$\Delta\phi_{\rm b} \equiv \phi_{\rm b}^{i+1} - \phi_{\rm b}^i = \frac{2k\pi}{N} \ (k = \pm 1, \pm 2, \pm 3, \cdots), \quad (2)$$

where $\phi_{\rm b}^i$ denotes the toroidal angle of the ith banana tip. If a banana particle satisfies the ripple resonance condition, it bounces at the same toroidal angle normalized to $\phi_N = 2\pi/N$ at each

banana tip and it can make a large poincare surface in the (ϕ_b, P_ϕ) phase space (Fig. 1). Then such particles, which are near the ripple resonance condition, can enhance diffusion.

Ripple Diffusion

We calculated the diffusion coefficients in the magnetic field which has ripple resonance points. Calculation parameters are major radius $R_0 = 6.0$ m, minor radius $a = 2.0$ m with a circular cross section, safety factor $q_s = 4.2$, magnetic field at plasma center $B_0 = 3.0$ T, the number of TF coils $N = 18$, and toroidal ripple at plasma surface $\gamma \sim 0.5\%$. Coulomb collisions were simulated by a Monte-Carlo method [2]. Particles start from $R = R_0 + 0.9a$, $Z = 0$, and the initial toroidal angle is randomly given. Particles have the same magnetic moment μ_m whose pitch angle $\zeta = 0.7 \times 2/\pi$ radians if ripple is zero. The trajec-
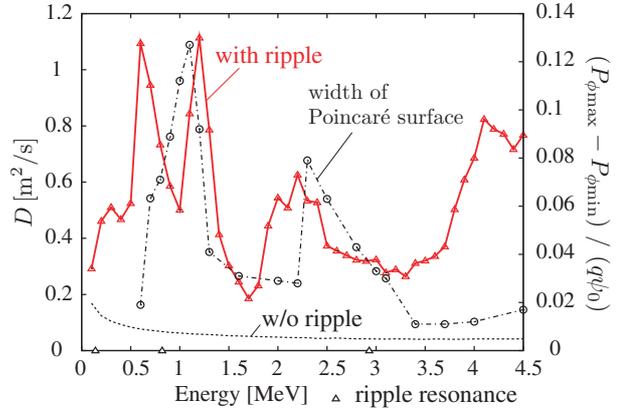

Figure 2: Energy dependence of the diffusion coefficients and the width of a Poincaré surfaces which are obtained with 25 initial toroidal angles. The resonance energies are 0.14, 0.80 and 2.93 MeV in an axisymmetric field.

tory of particles were calculated with the guiding center equations and diffusion coefficients were evaluated from 700 particles.

Results are shown in Fig. 2. Open triangles on abscissa mean ripple resonance energy in axisymmetric field. The diffusion coefficients are small at resonance energies and have peaks on both sides (M-shaped structure) and this M-shaped structure at higher energy is broader than that at lower energy. The peaks of the duffusion coefficients do not correspond to the peaks of the width of Poincaré surfaces.

M-shaped structure

To investigate the M-shaped structures of diffusion coefficients, we made a Poincaré map around a resonance energy. Figure 3 shows the variations of the Poincaré maps around a resonance energy, where the magnetic moment is constant and particles start from 25 initial toroidal angles.

Basically the Poincaré map gives open surfaces around closed surfaces as illustrated in Fig. 4 near the resonance energy. Since the changes in energy and magnetic moment by collisions are small (within about 5% and 1%, respectively), we consider particles move to adjacent surfaces in such Poincaré maps by collisions for simplicity. Particles on a closed surface spread to its
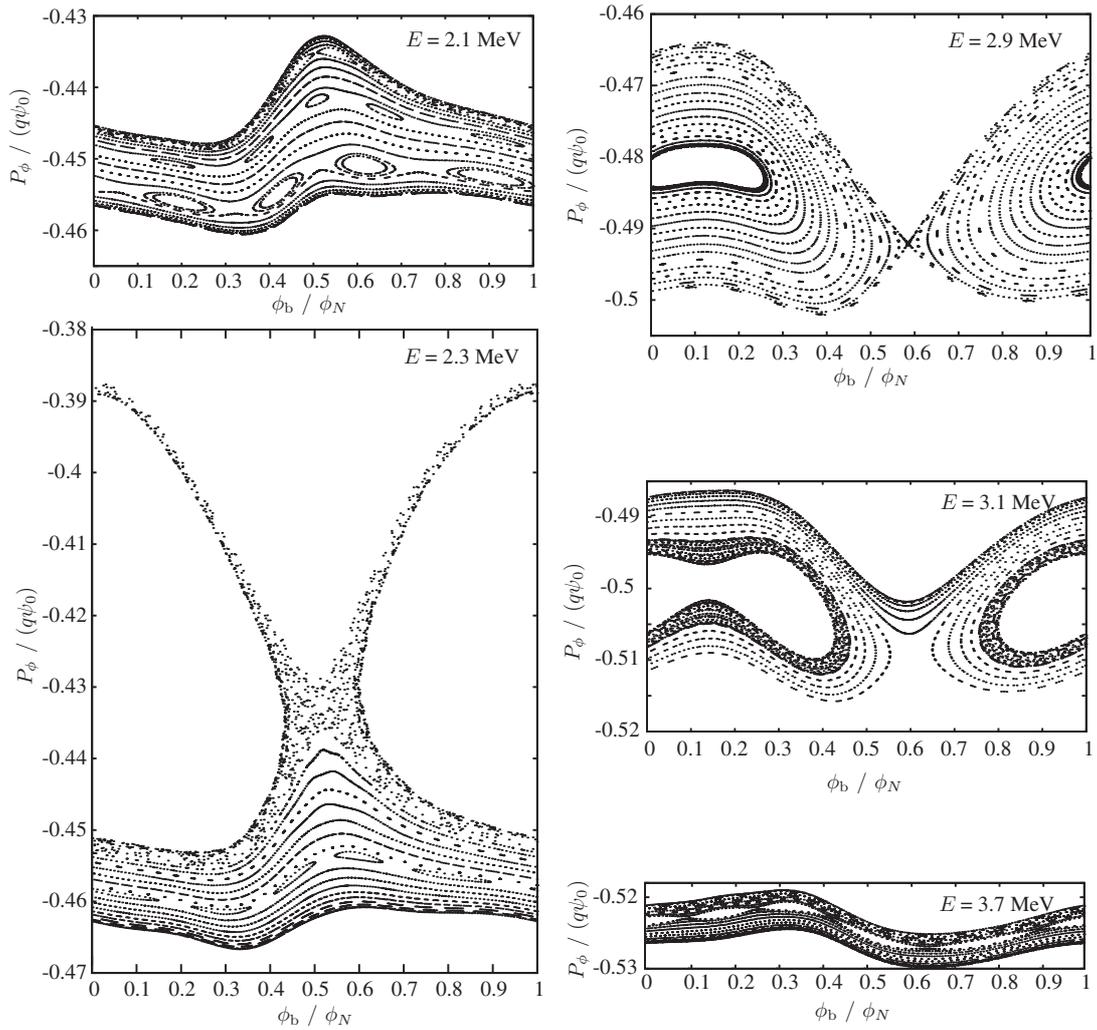
Figure 3: Variations of the Poincaré maps around a resonance energy of 2.9 MeV.

surface size at first, after that, however, diffusion by collisions is as small as that of particles on open surfaces. While the particles on open surfaces near the separatrix can jump over the large closed surface through the X-point by collisions and diffusion coefficients can be large. Hence diffusion coefficients exhibit M-shaped structures.

Diffusion coefficients were evaluated after particles spread to its surface size by ripple. In that time the standard deviation of the magnetic moment is about 0.5%. When particles jump from open surfaces to a large closed surfaces, large diffusion occurs. Therefore we investigated the ratio of open surfaces in Poincaré map when a separatrix exists including changes in the magnetic moment, and compared it with the diffusion coefficients. If the magnetic moment changes, the number of open surfaces is different especially at high energy (Fig. 5).
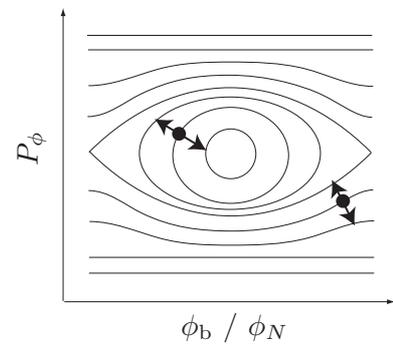

Figure 4: A simple picture of collisional diffusion in $(\phi_{\rm b}, P_\phi)$ phase space

Figure 6 shows the ratio of open surfaces with the three values of the magnetic moment shown in Fig. 5, which is also the ratio of particles on open surfaces, has a similar dependence on energy as the diffusion coefficients. At large energy, the M-shaped sturcture has wide peaks because separatrix exists in a wide range of energy around a resonance energy.

Conclusions

In nonaxisymmetric field, some particles can resonate with toroidal ripple and it has a large change of toroidal canonical momentum. Then diffusion coefficients are large near resonance energies. Furthermore, the diffusion coefficients exhibit an M-shaped structure around a resonance energy because closed Poincaré surfaces alone do not contribute to the diffusion so much as open surfaces, and open surfaces just near separatrix enhance the diffusion. On the other hand, diffusion coefficients drop when most of particles are on closed surfaces and exhibit an M-shaped structure.

References

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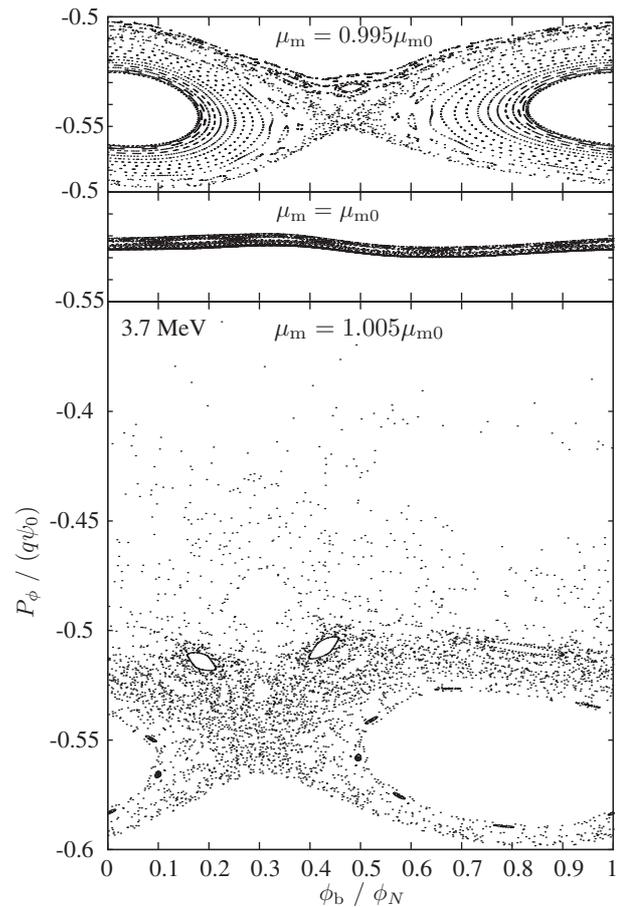[2] K. Tani, T. Takizuka, M. Azumi and H. Kishimoto, J. Phys. Soc. Jpn **50**, 1726 (1981)

Figure 5: Mangetic moment dependence of Poincaré map at 3.7 MeV.
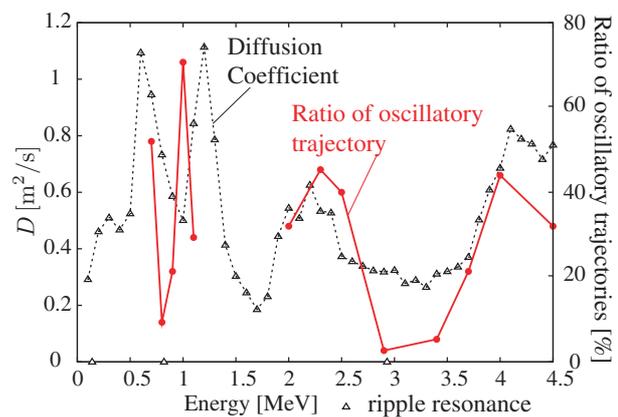

Figure 6: Comparison of diffusion coefficients and the ratio of open surfaces when a separatrix exists.