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FUGACITY AND LINEAR WAVES IN SELF-GRAVITATING NON-IDEAL DUSTY PLASMAS

K. Annou[1], S. Bahamida[1,2] , R. Annou[1]

[1]*Theoretical physics laboratory, faculty of physics,USTHB*

BP 32, El Alia, Bab-Ezzouar, Algiers, Algeria.

[2]*Faculty of sciences, Dep. Physics, UMB Boumerdes, Algeria*

Abstract

The existence of ultra low-frequency wave modes in dusty plasmas has been investigated over a wide range of dust fugacity [defined by $f = 4\pi n_{d0}\lambda_D^2 R$, where n_{d0} is the dust number density, λ_D is the plasma Debye length, and R is the grain size (radius)] and the grain charging frequency (ω) by numerically solving the dispersion relation obtained from the kinetic (Vlasov) theory. A detailed comparison between the numerical and the analytical results applicable for tenuous (low fugacity, $f << 1$), the dilute (medium fugacity, f ¡ö1), and the dense (high fugacity, $f >> 1$) [1,2]. In this work, the electrostatic modes in non-ideal dusty plasmas comprising of electrons, ions and dust grains has been studied including the grain charge variations. The general case of arbitrary fugacity corresponding to dust charge-density waves has been discussed.

Introduction

In recent years, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas. An important novel feature of dusty plasmas, when compared with the usual electron-ion plasmas with different ion species or with electrons having different temperatures, is the high charging of the grains which can fluctuate due to the collection of plasma_electron and ion.currents onto the grain surface. In the absence of charge fluctuations, dusty plasmas support ultra low-frequency waves which propagate as normal modes. On the other hand, when the grain charge fluctuations are self-consistently included, it is found that the dust modes are weakly damped. When self-gravitational interaction due to the heavier dust component is included, dusty plasmas are subject to macroscopic instabilities of the Jeans type. In the present paper, we consider Jeans instabilities in self-self-gravitating dusty plasma and specifically at the tenuous fugacity, to study the combined effects dust charge fluctuation and the non-ideal

Model equations

We consider e three-component dusty plasma having electrons, ions and dust grains, and include the self-gravitation due to heavier dust component. For the ultra low-frequency

regime, we use the standard dusty plasma model. Accordingly, the electron and ion number densities are given by Boltzmann distributions

$$n_e = n_{e0}\exp(e\phi/T_e) \quad (1)$$

$$n_i = n_{i0}\exp(-e\phi/T_i) \quad (2)$$

Here n_α and T_α are the respective number densities and temperatures of electrons $(\alpha = e)$, ions $(\alpha = i)$, $n_{\alpha 0}$ represents the respective equilibrium number density.

The dust grains, on the other hand, being the heaviest component, provide the inertia for the DAW and therefore the wave dynamics are governed by the fullest of dust fluid equations including the Poisson equation. Thus, we have the continuity, momentum balance and Poisson equation, respectively,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} n_d u_d = 0 \quad (3)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = -\frac{q_d}{m_d}\frac{\partial \phi}{\partial x} - \frac{T}{n_d m_d}\frac{\partial n_d}{\partial x} - \frac{\partial \psi}{\partial x} \quad (4)$$

where ψ is the gravitational potential, and the dust grains have charge q_d with equilibrium value Q_d. The description is closed by the electrostatic Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi(q_d n_d + e n_i - e n_e) \quad (5)$$

Together with the gravitational Poisson equation

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi G n_d m_d \quad (6)$$

The charge fluctuation obey the current balance equation

$$\frac{\partial q_d}{\partial t} + u_d \frac{\partial q_d}{\partial x} = I_e + I_i \quad (7)$$

Where the electron and the ion currents I_e and I_i respectively are given by

$$I_e = -\pi e a^2 n_e \sqrt{8T_e/\pi m_e}\exp(eq_d/aT_e) \quad (8)$$

$$I_i = \pi e a^2 n_i \sqrt{8T_i/\pi m_i}\,(1 - eq_d/aT_i) \quad (9)$$

The set equations (1)-(7) is closed with an appropriate choice of the equation of state for the dust fluid to describe the non-ideal behaviour of the finite sized grains. To keep the analysis tractable, we use the van der Waals equation of state **[1]** expressed in terms of the dust numberdensity

$$(p_d + An_d^2)(1 - Bn_d) = n_d k_d T_d \quad (10)$$

The constants A and B are given respectively $9k_d T_c/8n_c$ and $1/3n_c$, with the subscript 'c' indicating the respective values at the critical point.

Eqs. (1)-(9) constitute a complete set for dusty plasmas, by self-consistently including grain charge fluctuations as well as self-consistently effects. Assuming the perturbations to vary as $\exp i(kx-\omega t)$, we linearize Eqs. (1)-(10) and obtain the dispersion relation,

$$\frac{\omega^2}{k^2}=\frac{\omega_{pd}^2\lambda_D^2}{1+k^2\lambda_D^2+f\Delta}+v_{td}^2-\frac{\omega_{Jd}^2}{k^2}+\frac{1}{m_d}\left[k_dT_dBn_{d0}\frac{(2-Bn_{d0})}{(1-Bn_{d0})^2}-2An_{d0}\right] \qquad (11)$$

Here, $f=4\pi n_{d0}\lambda_D^2a$ is the dust fugacity parameter, $\Delta=\omega_2/(\omega_1-i\omega)$, ω_1 and ω_2 are the charging frequencies **[2],** $\omega_{pd}=(4\pi n_{d0}q_{d0}^2/m_d)^{\frac{1}{2}}$ is the dust plasma frequency, $\omega_{Jd}=(4\pi Gn_{d0}m_d)^{\frac{1}{2}}$ is the Jeans frequency, n_{d0} is the equilibrium dust density, $\vartheta_{td}=(k_dT_d/m_d)^{\frac{1}{2}}$ is the dust thermal speed and λ_D is a plasma Debye length defined hrough $1/\lambda_D^2=1/\lambda_{De}^2+1/\lambda_{Di}^2=4\pi e^2(n_{e0}/T_e+n_{i0}/T_i)$. The characteristic grain charging frequencies $\omega_1=\chi+\sqrt{(8\pi)}a\frac{e^2n_{e0}}{\sqrt{m_eT_e}}\exp\left(\frac{eq_{d0}}{eT_e}\right)$, $\omega_2=\chi\left(1-\frac{eq_{d0}}{eT_i}\right)+\sqrt{(8\pi)}a\frac{e^2n_{e0}}{\sqrt{m_eT_e}}\exp\left(\frac{eq_{d0}}{eT_e}\right)$

Substituting for the parameters *A* and *B*, the dispersion relation (11) may be expressed as,

$$\frac{\omega^2}{k^2}=\frac{C_{DA}^2}{1+k^2\lambda_D^2+f\Delta}+v_{td}^2+\varepsilon C_D^2-\frac{\omega_{Jd}^2}{k^2} \qquad (12)$$

with $\varepsilon=\varepsilon_{vr}+\varepsilon_{cf}$, $C_D^2=C_{DA}^2+\vartheta_{td}^2$ and $C_{DA}^2=\omega_{pd}^2\lambda_D^2$. Furthermore, we define, $\beta=\vartheta_{td}^2/C_D^2$, $\eta=n_{d0}/n_e$, $\alpha=T_e/T_d$, and write the quantities $\varepsilon_{vr}=\beta\eta(6-\eta)/(3-\eta)^2$ and $\varepsilon_{cf}=-9\beta\alpha\eta/4$. These denote, respectively, the contributions due to the volume reduction coefficient and the molecular cohesive force **[3].** Considering the terms on the right- hand side of Eq. (12), we note that the first term comes from the usual dust acoustic response, the second term are the thermal effects for ideal gas case, whilst the last term introduces the corrections due to the non-ideal gas.

Discussion

The dispersion law **(12)** governs the propagation of low-frequency dust modes in the presence of self-gravitational effects. To discuss the existence of Jeans type instability, first of all we consider the low-frequency case $\omega\ll\omega_1$, when the grains have sufficient time to achieve an equilibrium charge and hence the wave damping due to charge fluctuation and thermal effects. Accordingly, $\Delta\to\delta\equiv\omega_2/\omega_1$, a parameter which is of order unity over a wide range of dust fugacity. Thus the dispersion low (12) takes the form

$$\frac{\omega^2}{k^2}=\frac{C_{DA}^2}{1+k^2\lambda_D^2+f\delta}+v_{td}^2+\varepsilon C_D^2-\frac{\omega_{Jd}^2}{k^2} \qquad (13)$$

Neglecting thermal effects $(\varepsilon\to 0)$, we obtain the dispersion relation given by Rao and Verheest [4] in the cases of tenuous dusty plasmas, when the fugacity is low $(f\delta\ll 1)$.

As pointed out earlier, the Jeans instability is a purely growing instability which arise when the system is subjected to perturbations having wavenumbers $k < k_c$, when the critical wavenumber k_c is determined by equilibrium parameters. An analytical expression for $k_c$can be derived in the general case, without any approximation, by rewriting (12)

$$\Omega^3(1+k^2\lambda_D^2)-\Omega^2\omega_1(1+k^2\lambda_D^2+f\delta)+\Omega\left[k^2C_{DA}^2+(1+k^2\lambda_D^2)\left(k^2\vartheta_{td}^2+\varepsilon k^2C_D^2-\omega_{jd}^2\right)\right] - \omega_1\left[k^2C_{DA}^2+(1+k^2\lambda_D^2+f\delta)\left(k^2\vartheta_{td}^2+\varepsilon k^2C_D^2-\omega_{jd}^2\right)\right]=0 \quad (13)$$

Where we have denoted, for convenience, $\Omega = -i\omega$. Since the perturbations have assumed to vary as $\exp i(kx-\omega t)$, the Jeans instability occurs whenever (13) admits positive real root for Ω. Eq. (13) is a cubic in Ω. In particular, it follows that (13) always has one and only one positive real root whenever k satisfies the condition given,

$$\frac{k^2C_{DA}^2}{1+k^2\lambda_D^2+f\delta}+k^2v_{td}^2+\varepsilon k^2C_D^2<\omega_{jd}^2 \quad (14)$$

The critical wavenumber $k_c$is thus determined by the bi-quadratic equation

$$k_c^4[\vartheta_{td}^2(1+\varepsilon)+\varepsilon C_{DA}^2]\lambda_D^2+k_c^2\left[\vartheta_{td}^2(1+\varepsilon)(1+f\delta)+\left(1+\varepsilon(1+f\delta)\right)C_{DA}^2-\omega_{jd}^2\lambda_D^2\right] - \omega_{jd}^2(1+f\delta)=0 \quad (15)$$

Which obviously has only one positive root for k_c.

For the case when $k^2\lambda_D^2 \ll 1+f\delta$, the critical wavenumber is given by this expression

$$k_c=\omega_{pd}\left[\frac{C_{DA}^2}{1+f\delta}+\vartheta_{td}^2(1+\varepsilon)+\varepsilon C_{DA}^2\right]^{-\frac{1}{2}} \quad (16)$$

The critical wavenumber for tenuous regime, $(f\delta \ll 1)$ is given by

$$k_c=\omega_{pd}[C_{DA}^2+\vartheta_{td}^2(1+\varepsilon)+\varepsilon C_{DA}^2]^{-\frac{1}{2}} \quad (17)$$

In summary, we have analysed the behaviour of non-ideal dusty plasma, including grain charge Fluctuation effects over a range of dust fugacity. We point out that the wavenumber is proportional to the term of fugacity.

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