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Effect of a Gaussian and log-normal grain size-distributions on coherent structures in dusty plasmas

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Abstract

The investigation of dust-acoustic solitons when ions are adiabatically heated, whereas dust grains are size-distributed according to a Gaussian and log-normal distributions, is conducted. The solitary solutions are shown to undergo a transformation into snoidal waves. The Mach numbers allowing bounded oscillatory solutions to exist increase, whereas the allowed Mach numbers range decreases. The requirement for the existence of those structures is less stringent when the dust grains are following a Gaussian distribution in size, by opposition to a power-law distribution, [M.Ishak-boushaki, S.Bahamida, and R.Annou, Phys.Plasmas. **10**, 3418 (2003)].

The existence of ion-acoustic solitons has been investigated by many authors [1,2], in the framework of the Sagdeev quasi-potential method for large-amplitude solitons and the reductive perturbation method for small-amplitude ones. A balance is established between dispersion that is due to the self-consistent electric field when plasma approximation is dropped, and non-linearity that is due to the convective terms in the continuity and momentum equations. The conditions on corresponding Mach numbers have been derived, namely , the ion acoustic soliton can not exist for Mach numbers out of the range $1< M <1.6$. Moreover, it has been proved that the soliton characteristics may be modified by the inclusion of another ion component. that may become in some conditions an inhibitor factor for soliton formation [3]. the additional component may simply be a very massive dust grain, that acquires a very high charge, the presence of which is proved to influence, for example the Langmuir envelope soliton, where for a constant grain charge the soliton gets squeezed [4]. Let's recall as well, that it has been predicted by Rao *et al* [5] and experimentally proved later on, that charge dynamics introduces a new mode of a low phase velocity in the plasma, called dust-acoustic wave. The study of the non-linear features of this dust-acoustic wave has been conducted and the existence of compressive dust acoustic solitons has been reported [5,6]. When the ions are considered adiabatic the allowed Mach numbers range is sensitively modified, i.e., the lower limit is raised from unity to $\sqrt{\gamma}$, where γ is the ratio of specific heat capacity at constant pressure to that at constant volume, and the upper limit is accordingly modified [7]. The interval of the allowed Mach numbers is wider when ions are adiabatically heated. We showed in an earlier paper (c.f.Ref.[8]) that a power law grain size distribution beside the fact that it affect the modes supported by the plasma along with the growth rate of some parametric instabilities, it is found that the lower and upper limits of the allowed Mach numbers corresponding to bounded oscillatory solutions, get modified, and the interval of the allowed Mach numbers is strongly compressed. In this work we consider that the dust grains are following a Gaussian and log-normal distribution in size, by opposition to a power-law distribution [9].

Let's consider a many-component plasma with massless ions and size-distributed negatively charged dust grains. The time scale corresponding to the dust acoustic waves of low phase velocity is long with respect to the charging time, thus the charge reaches its equilibrium value quasi-instantly. By virtue of the same arguments, charge departure from equilibrium may be ignored in this note. Moreover, the plasma is assumed depleted from

electrons, thus the contribution of electrons on dust acoustic non-linear structures is ignored. The ions that are confined to a potential well, are adiabatically heated.
The governing equations are cast as follows,

$$\frac{\partial n_j}{\partial t}+\frac{\partial}{\partial x}n_j u_j=0 \quad (1) \qquad \frac{\partial u_j}{\partial t}+u_j\frac{\partial u_j}{\partial x}=\frac{Z_j}{m_j}\frac{\partial \phi}{\partial x} \quad (2) \qquad 0=\frac{\partial \phi}{\partial x}-\frac{1}{n_i}\frac{\partial p_i}{\partial x}, \quad (3\text{-a})$$

Eq.(3-a), may be put otherwise for $p_i = n_i^{\gamma}$, viz., $\frac{\partial \phi}{\partial x}+\gamma\, n_i^{\gamma-2}\frac{\partial n_i}{\partial x}=0$ (3-b

We close the system (1- 3.b) by Poisson's equation, $\frac{\partial^2 \phi}{\partial x^2}=\frac{Z_0}{1-\in}\sum_j Z_j n_j - n_i$, (4)

where the potential is normalized by T_{i0}/e, the dust fluid velocity by the dust acoustic speed $C_s=(Z_0T_{i0}/m_0)^{1/2}$, the time by $\tau=(m_0/e^2Z_0n_{i0})^{1/2}$, the space by $\lambda=(T_{i0}/e^2n_{i0})^{1/2}$ and the density by n_{i0}. Moreover, the dust charge Z_j and mass m_j are normalized by the charge and mass corresponding to the grain of the most probable radius r_0, viz., $Z_0=Z(r_0)$ and $m_0=m(r_0)$ and $\in=r_o^3n_{io}4\pi/3\sum_j (r_j^3/r_o^3)n_{jo}$, for impermeable grains, $\in=0$ being for permeable ones (c.f. Ref.[9]).

We look now for stationary solutions of Eqs. (1)-(4) , assuming the physical quantities to depend only on $\xi=x-Mt$ where M is the mach number.

We solve the system of equations (1-3b) taking into account the following boundary conditions, $\phi\rightarrow\phi_\infty$, $u_j\rightarrow 0$, and $n_j\rightarrow n_{j0}$ at $\xi\rightarrow\infty$, to obtain the ions density $n_i=[1-(\gamma-1)(\phi-\phi_\infty)/\gamma]^{1/(\gamma-1)}$, whereas dust density is given by, $n_j=Mn_{j0}/\sqrt{M^2+2Z_j(\phi-\phi_\infty)/m_j}$.
Consequently, the self-consistent potential should satisfy the condition,

$(\phi-\phi_\infty)\geq-(M^2/2)(m_j/Z_j)=-(M^2/2)(r_j/r_o)^2$ (5)

for dust density to remain real. A sufficient condition would be $(\phi-\phi_\infty)\geq(\phi_m-\phi_\infty)=-(M^2/2)(r_{\min}/r_o)^2$
The Poisson's equation is integrated with the boundary conditions $\phi\rightarrow\phi_\infty$, $\phi_\xi\rightarrow 0$ at $\xi\rightarrow\infty$, to yield, $(\partial\phi/\partial\xi)^2=-2V(\phi)$, (6)

where the Sagdeev potential is, $V(\phi)=1-[1-(\gamma-1)(\phi-\phi_\infty)/\gamma]^{\gamma/(\gamma-1)}-I(\phi)Z_0M/(1-\in)$ (7)

with, $I(\phi)=\sum_j m_j n_{j0}\left[\sqrt{M^2+2Z_j(\phi-\phi_\infty)/m_j}\;-M\right]$.

When the size distribution is continuous, discrete summation is replaced by an integration, to yield, $I(\phi)\approx\int_{r_{d1}}^{r_{d2}} r_d^3 n_0^0\left[\sqrt{M^2+2(\phi-\phi_\infty)/r_d^2}-M\right]f(r_d)dr_d$, (8)

where $r_d=r/r_0$ and $\delta n=n_0^0 f(r_d)dr_d$, is the number of grains having radii in the range r_d and r_d+dr_d .

For a Gaussian size distribution, i.e., $f(r_d)=C_g.\exp[\mu(r_d-1)^2]$
where $r_{d1}=(r_1/r_0)=0.1$ and $r_{d2}=(r_2/r_0)=10$ viz., Eq.(8) reduce, to,

$$I(\phi)=C_g n_0^0 M\left\{\int_{0.1}^{10} r_d^2\sqrt{r_d^2+2(\phi-\phi_\infty)/M^2}\exp[-\mu(r_d-1)^2]dr_d-\int_{0.1}^{10} r_d^3\exp[-\mu(r_d-1)^2]dr_d\right\}, \quad (9\text{-a})$$

Where, $C_g=0.55$ and $\mu=\ln(2)$.

Whereas, for a log-normal size-distribution, where $f(r_d)=C_{LN}\exp[-((\log r_d)/4\xi)^2]$.Eq.(8) reduces, to

$$I(\phi)=C_{LN}n_0^0M\left\{\int_{0.1}^{10}r_d^2\sqrt{r_d^2+2(\phi-\phi_\infty)/M^2}\exp[-((\log r_d)/4\xi)^2]dr_d-\int_{0.1}^{10}r_d^3\exp[-((\log r_d)/4\xi)^2]dr_d\right\}, \quad \text{(9-b)}$$

Where, $C_{LN}=0.88$ and $\xi^2=\left(\sqrt{\ln(2)}\right)/2$.

Due to physics consideration, the distribution has been truncated for the lower and higher limits of the grain size. In figure,1 we plot $V(\Phi)$ vs $\Phi=\phi-\phi_\infty$. A solution of Eq.(6) exists if $V(\phi)\langle 0$. For $\phi\rangle\,\phi_\infty$, a solution exists for every value of the Mach numbers, however it is an unbounded one and has to be discarded. On the other hand, viz., for $\phi\langle\phi_\infty$, Eq.(6) admits in only a selective Mach numbers range, periodic bounded solutions. For these Mach numbers the Sagdeev potential becomes negative between $\Phi=\Phi_1$ and $\Phi=\Phi_2$, where $\Phi=0$, Φ_1 and Φ_2 are the solutions of $V(\Phi)=0$. The lower Mach number M_l corresponds to $V(\Phi^*)=0$, where Φ^* is the solution of

$$\frac{\partial V}{\partial\Phi}=0 \quad \text{and} \quad \left[\frac{\partial^2V}{\partial V^2}\right]_{\Phi=\Phi^*}\rangle\, 0$$

Furthermore, at $\Phi_m=(\phi_m-\phi_\infty)=-M^2(r_{\min}/r_0)^2/2$ beyond which the dust density is no longer a positive definite quantity, the Sagdeev potential is required at least to vanish, i.e.,

$$V(\Phi_m)=1-\left(1+M^2(\gamma-1)(r_{\min}/r_o)^2/2\gamma\right)^{\gamma/(\gamma-1)}+\lambda M^2=0. \qquad (10)$$

Where, $\lambda=\dfrac{Z_on_o^o}{1-\varepsilon}\displaystyle\int_{r_{d1}}^{r_{d2}}r_d^2n_0^0\left[\sqrt{r_d^2-(r_{\min}/r_o)^2}-r_d\right]f(r_d)dr_d$, and M_U^2; solution of the equation (10), is the square of the upper Mach number, for $\gamma=3$.The solution would oscillate between Φ_1 and Φ_2 (c.f. fig.1), thus it is appropriate to Taylor expand $V(\Phi)$ around $\Phi=\Phi_1$, to get,

$$V(\Phi)=V^{'}(\Phi_1)(\Phi-\Phi_1)+\frac{1}{2}V^{''}(\Phi_1)(\Phi-\Phi_1)^2+\frac{1}{6}V^{'''}(\Phi_1)(\Phi-\Phi_1)^3, \qquad (11)$$

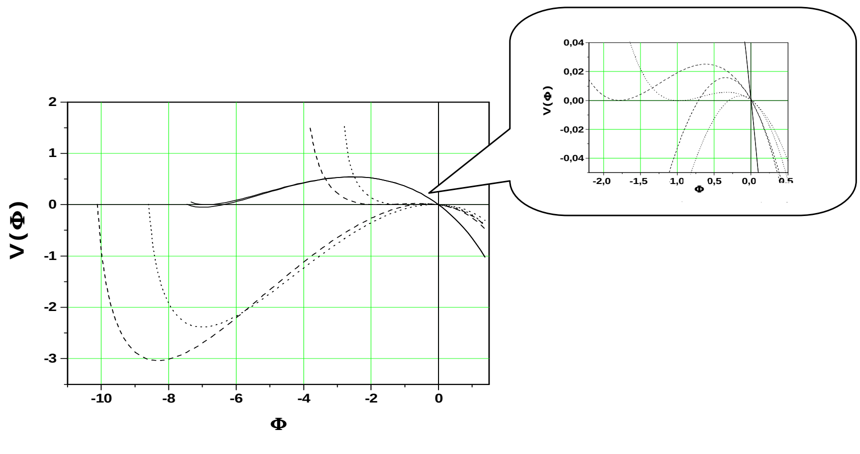

Fig.1. Sagdeev potential V(Φ) vs Φ, for different distributions,
with ε = 0 (permeable grains) and $\gamma=3$.
a) Log-normal distribution with $M_U = 2.46$ and $M_L = 1.53$ (dashed line)
b) Gaussian distribution with $M_U = 2.2$ and $M_L = 1.25$ (doted line)
c) Power-Law distribution[9] with $M_U = 3.878$ and $M_L = 3.846$ (solid line)

To solve Eq.(6), let's put it otherwise, $\quad d\psi/d\xi=\pm\left(f(\psi)/3k\right)^{1/2} \quad$ (12)

where $\psi=\Phi-\Phi_1$, $k=-1/V^{'''}(\Phi_1)$ and $f(\psi)=\psi^3-3C\psi^2-6A\psi=(\psi-\psi_0)(\psi-\psi_1)(\psi-\psi_2)$,

the coefficients being given by, $C = -V''(\Phi_1)/V'''(\Phi_1)$ and $A = -V'(\Phi_1)/V'''(\Phi_1)$. The zeros of the function f are the shifted zeros of the potential V, but $f(\psi)$ has to be positive rather, for the oscillatory bounded solutions to exist. The above mentioned zeros are given by, $\psi_1 = 0$ and $\psi_1 = 0$. Performing the necessary transformations [11], the oscillatory solution of Eq.(12) is given by, $\psi(\xi) = \psi_2 sn^2\left\{\sqrt{\psi_2(\psi_2 - \psi_0)/12k}\ \xi, S\right\}$, (13)
where $S^2 = \psi_2/(\psi_2 - \psi_0)$ and $\xi \longrightarrow sn$ being the Jacobian elliptic function, which is a periodic function with a period P given by, $P = 4\sqrt{3k/(\psi_2 - \psi_0)\psi_2}\, K(S^2)$, (14)

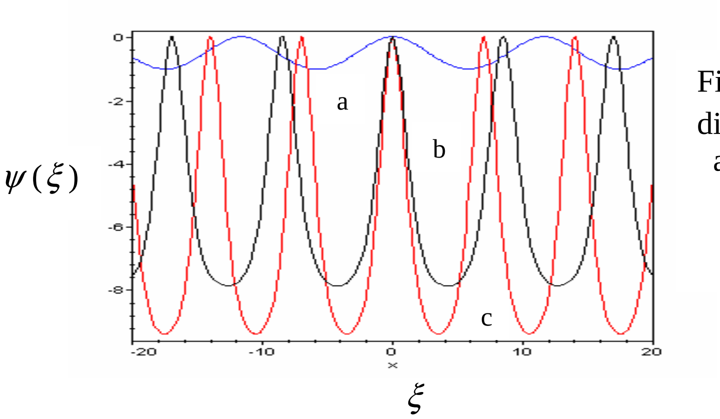

Fig.2. Shifted potential ψ vs ξ for different distributions, i.e.,
a/ Power-law(blue),
b/ Gaussian (black) and
c/ Log- Normal(red).

$K(x)$ is the complete elliptic integral of the first kind. The solution is called a snoidal wave. When $\psi_1 = \psi_0 = 0$ that is, $V'(\Phi_1) = 0$, the snoidal solution reduces to a solitary one $\psi(\xi) \rightarrow \sec h^2\left(\left(|\psi_2|/\sqrt{12k}\right)\xi\right)$. This case corresponds to a uniform grain size-distribution.

In summary, under certain conditions, viz., M_l =1< M< M_u =1.6, dust-acoustic solitary waves are proved to exist. The conditions on Mach numbers may be modified if ions are adiabatically heated, namely, M_l =1.73 and M_u=4.4 , for $\gamma = 3$ and $\in = 0$ (permeable grains). In this note we take into account the grain size-distribution that has been shown to be a cause of damping. It is found that the condition in terms of M, of the existence of oscillatory bounded solutions is strengthened; e.g., for the power law-distribution[9], M_l =3.846 and M_u=3.878. Whereas for the Gaussian as well as the Log-normal distribution, we have (M_l =1.25, M_u=2.2) and (M_l =1.53, M_u=2.46), respectively, and the range of allowed Mach numbers is given by ΔM_{PL}= 0.032, ΔM_G = 0.95, ΔM_{LN}= 0.93. Moreover, the solution experiences a transition from a solitary wave to a snoidal wave.

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