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Collective scattering from turbulent plasmas, and particle motion

D. M. Grésillon [1], N. Lemoine [2]

[1]*LPTP Ecole Polytechnique – UMR 7648 du C.N.R.S. Palaiseau, France*

[2]*LPMIA Université Henri Poincaré – UMR 7040 du C.N.R.S. Nancy, France*

Collective scattering is a convenient means for quantitative measurements of plasma turbulence in space, tokamaks and the laboratory [1]. Experiments show large levels of fluctuations with a broad frequency spectrum, when the scattering wave vector **k** is mostly perpendicular to a confining magnetic field. When fluctuations are mostly convective, the random velocities shape the spectrum as the frequency Doppler shift, $\omega=\mathbf{k}\bullet\mathbf{v}$, of the velocity probability distribution function.

However it is not only the macroscopic plasma motion that can affect the scattered spectrum. A full account of the collective scattered signal must start from the elementary building blocks, which are the dressed particles and their structure factor at the scattering device wave vector [2]. The ion structure factor is the space Fourier transform of the electron cloud density around the test ion [3],

$$\delta n_i(\bar{k},\bar{v}) = \left.\frac{\chi_{e//}(\omega,\bar{k})}{\varepsilon_{//}(\omega,\bar{k})}\right|_{\omega=k_{//}v_{//}} J_0\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)$$

where v is the particle velocity ($v_{//}$, the component along B), $\chi_{e//}$ and $\varepsilon_{//}$ the electron susceptibility and longitudinal dielectric component parallel to the magnetic field [4]. An example of the variation of δn with $v_{//}=\omega/k_{//}c_S$, is shown in Fig. 1, when $k_{//}\,\lambda_D<< 1$, $k_{perp}\,\rho_s<1$ and Te/Ti=5. The upper (lower) graph is the real (imaginary) part.

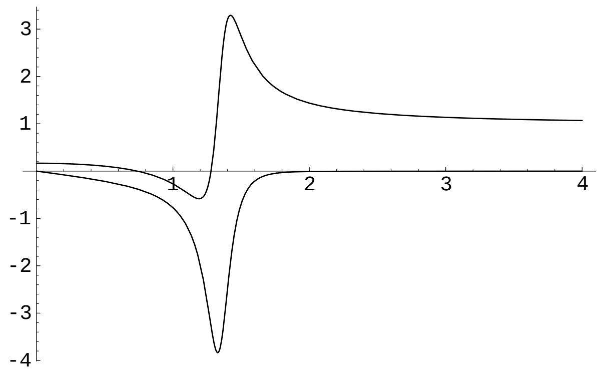

Figure 1 The ion structure factor δn as a function of the velocity $v_{//}/c_s$, for small $k_{//}\,\lambda_D$.

A strong variation occurs near the acoustic wave velocity c_s. For supersonic velocities, the structure factor is one, i.e. the total electron charge in the ion cloud is unity.

The elementary scattered field from particle j is

$$s_j(t) = \bar{E}_1(\bar{r}_1, t)\delta n_j(\bar{k}, \bar{v}_j)e^{-i\bar{k}\cdot\left[\bar{r}_{0j}+\bar{\Delta}(\bar{r}_{0j}|t)+\bar{v}_{//j}t\right]}$$

E_1 is the E field scattered by a fixed electron at the origin, r_{oj} the particle position at time 0, $v_{//j}$ its velocity parallel to B, and $\Delta(r_{oj}, t)$ is the macroscopic local plasma displacement from position r_{oj} during time t. In most experiments, the scattering electromagnetic wave system is made of finite extent, e.g. Gaussian beams, propagating across the B-field along the y axis.

$$exp[-2\frac{(x^2+z^2)}{w^2}]$$

where w is the beam "waist". The geometry is shown in Figure 2

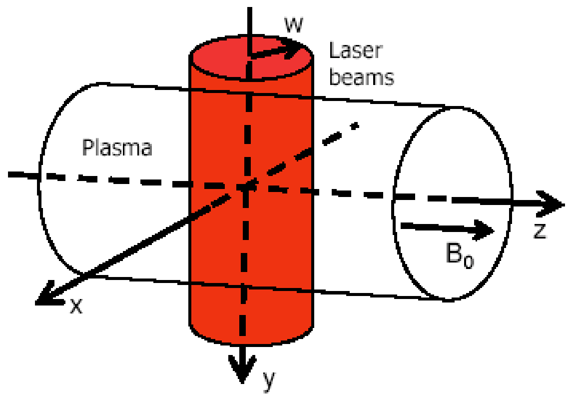

Figure 2. Plasma and scattering electromagnetic waves geometry (example for small angle forward scattering).

The total scattered signal is the sum of individual scattered signals, including the beam profile effect,

$$s(t) = \sum_{s,j} e^{-ik_\perp(x_{0j}+\Delta_x(\bar{r}_{0j}|t)} e^{-\frac{2}{w^2}[(x_{oj}+\Delta_x(\bar{r}_{0j}|t)]^2} \times$$
$$\times \delta n_{s,j}(\bar{k}, \bar{v}_j) e^{-ik_{//}(z_{oj}+v_{//j}t)} e^{-\frac{2}{w^2}(z_{oj}+v_{//j}t)^2}$$

Macroscopic density and motion are on the first line of this equation, while the second line only deals with microscopic structure factor and particle trajectory along the B axis. The time correlation is the averaged product $<s(t)s^*(t+\tau)>$. For "coherent scattering", the correlation retains only the product of same particles (j-j) [4]. Collective scattering deals with product of different particles (j≠l).

When macroscopic and microscopic motions are statistically independent, the signal correlation can be factorised into a product of a macroscopic (turbulent) and a microscopic (kinetic) part. The total microscopic correlation is calculated as an integral over the initial position z_j and as an average over the velocity distribution $f(v_j)$. A further time Fourier transform provides the microscopic spectrum as

$$S_\mu(\omega, \bar{k}) = \pi^2 w^2 exp[-\frac{k_{//}^2 w^2}{4}] \int d\bar{v}_j exp\{-\frac{\omega^2 w^2}{8 v_{j//}^2}\} f_{sj}(\bar{v}_j) \delta n_{sj}(\bar{k}, \bar{v}_j) \int d\bar{v}_l exp\{-\frac{\omega^2 w^2}{8 v_{l//}^2}\} f_{sl}(\bar{v}_l) \delta n_{sl}^*(\bar{k}, \bar{v}_l)$$

where j and l relate to two different particles.

Different components play a role in this equation. The structure factor $\delta n(k, v)$ is weighted by the velocity distribution function $f(v)$. When $Te/Ti=5$, the result, as a function of v, is shown in Fig.3. The main part of the velocity distribution enhances the structure factor at small velocities, while the resonance feature near $v/c_s=1$ is damped because of the few ions around this velocity.

The third important factor is the time-of-flight term, $\exp[-\omega^2 w^2/8v^2_{//}]$. The product of this term with the weighted structure factor is shown in Fig.4, when $\omega w/c_s = 0.3$, i.e. the transit time w/c_s is smaller than the period $2\pi/\omega$. Particles with small velocities, such that their transit time is large, do not contribute to the spectrum at frequency ω. The higher the frequency, the fewer the number of ions contributing to the frequency spectrum.

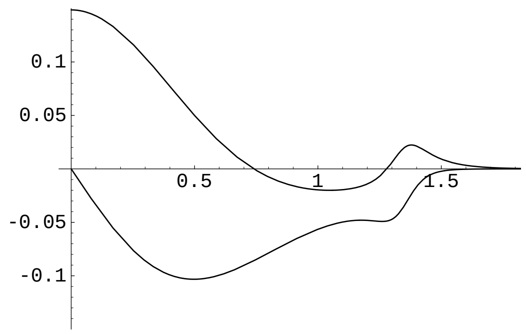

Figure 3 : The ion structure factor, weighted by the ion distribution function, as function of $v_{//}$

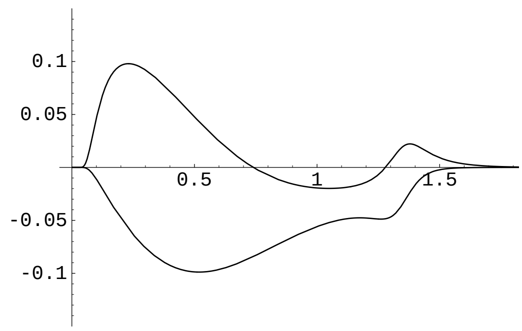

Figure 4. The structure factor as further weighed by the time-of-fligth term, as a function of $v_{//}$

The response at the given frequency ω is the integral over $v_{//}$ of the time-of-flight and velocity distribution weighted structure factor. This integral retains only the real part since it is an even function, while the imaginary part is an odd function of $v_{//}$. The microscopic spectrum is obtained as the square of this integral of the real part. Its variation is plotted in Figure 5 as a function of the frequency ω normalised to the transit frequency c_s/w (when $k_{//}\lambda_D=10^{-4}$ and $w/\lambda_D=300$). The spectrum is maximum at zero frequency and decays with a characteristic width $\Delta\omega$ on the order of 0.2 times the transit frequency of an ion at the acoustic velocity, c_s/w.

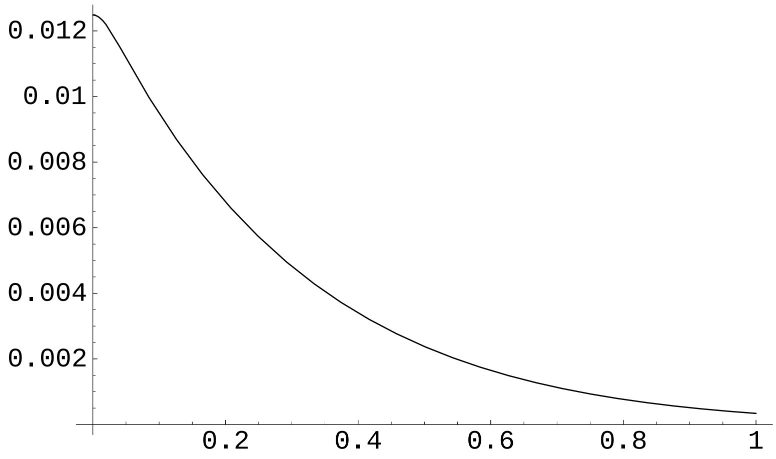

Figure 5. The ion microscopic spectrum due to transit time, as a function of ($\omega\, w / c_s$).

For warmer ions (as in Tokamaks), the ion structure factor at small velocities is larger, and so is the microscopic spectrum intensity and width.

When T_e>>T_i instead, the ion structure factor (at ion thermal velocities) decays as T_i/T_e and the ion scattering microscopic spectrum goes to vanishing intensity. In those cases, it is the electron scattering weighed structure factor that becomes dominant, with the wider characteristic frequency corresponding to a larger inverse electron transit time.

This investigation is relevant for the interpretation of collective scattering experiments. The observed spectrum results both from the macroscopic motion (Doppler profile) and from the microscopic spectrum as calculated above. The total spectrum is a convolution of the macroscopic spectrum with the microscopic spectrum. Since the microscopic spectrum frequency width is on the order of the inverse transit time of an ion particle sweeping across the scattering electromagnetic wave beam, the macroscopic spectrum is widened by this frequency. It should also be noted that this microscopic spectrum has no long tails because of its exponential, quickly vanishing component $\exp[-\omega^2 w^2/8v^2_{//}]$.

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