34th EPS Conference on Plasma Phys. Warsaw, 2 - 6 July 2007 ECA Vol.**31F**, P-5.094 (2007)

Bayesian diagnostic design of a multichannel interferometer

H. Dreier[1], A. Dinklage[1], R. Fischer[2], M. Hirsch[1], P. Kornejew[1]

Max–Planck–Institut für Plasmaphysik, EURATOM Association,

[1]*Teilinstitut Greifswald, D–17491 Greifswald /* [2]*D–85748 Garching*

Introduction

Fusion experiments are designed with respect to specific physical questions, e.g. the analysis of high confinement scenarios. The same physical questions can be used as optimisation criteria to design the diagnostics applied at these experiments. A possible approach is given by *Bayesian experimental design* (BED), which bases on the maximisation of an information measure.

In this paper, the method will be introduced and applied to the optimisation of a multichannel interferometer at the Wendelstein 7-X stellarator, currently under construction in Greifswald, Germany. Designs for different physical problems will be calculated and compared with respect to the physical questions to be addressed by the diagnostic.

Bayesian experimental design

Basic idea of the BED approach is to optimise a diagnostic by maximising the expected information gain about the parameters of interest from the future data. The information gain of a measurement can be described by an information measure, e.g. the Kullback–Leibler distance:

$$U_{KL}(D,\eta) = \int \mathrm{d}\theta\, p(\theta|D,\eta,I) \log \frac{p(\theta|D,\eta,I)}{p(\theta|I)} \tag{1}$$

Here, θ are the parameters of interest to be estimated by the future experiment, D are the (future) data and η the design parameters, e.g. the coordinates of a probing beam. The probability density function $p(\theta|D,\eta,I)$ encodes the knowledge (or uncertainty) about θ given D, η and our physical background knowledge expressed by I. The PDF $p(\theta|I)$ is our knowledge about θ without data.

In the design process, no measurements are available, so D is unknown. This can be overcome by integrating equation (1) over the expected data space (see [1] for detailed calculation). By applying Bayes' theorem, one finally obtains

$$EU(\eta) = \int \mathrm{d}D \int \mathrm{d}\theta\, p(D|\theta,\eta,I) \cdot p(\theta|I) \log \frac{p(D|\theta,\eta,I)}{p(D|I)}. \tag{2}$$

This *Expected Utility* function (EU) is an absolute measure for the expected information gain about θ of a future measurement. If the base-2 logarithm is used, it is given in *bit*. For BED, the EU is maximised with respect to the design parameter η.

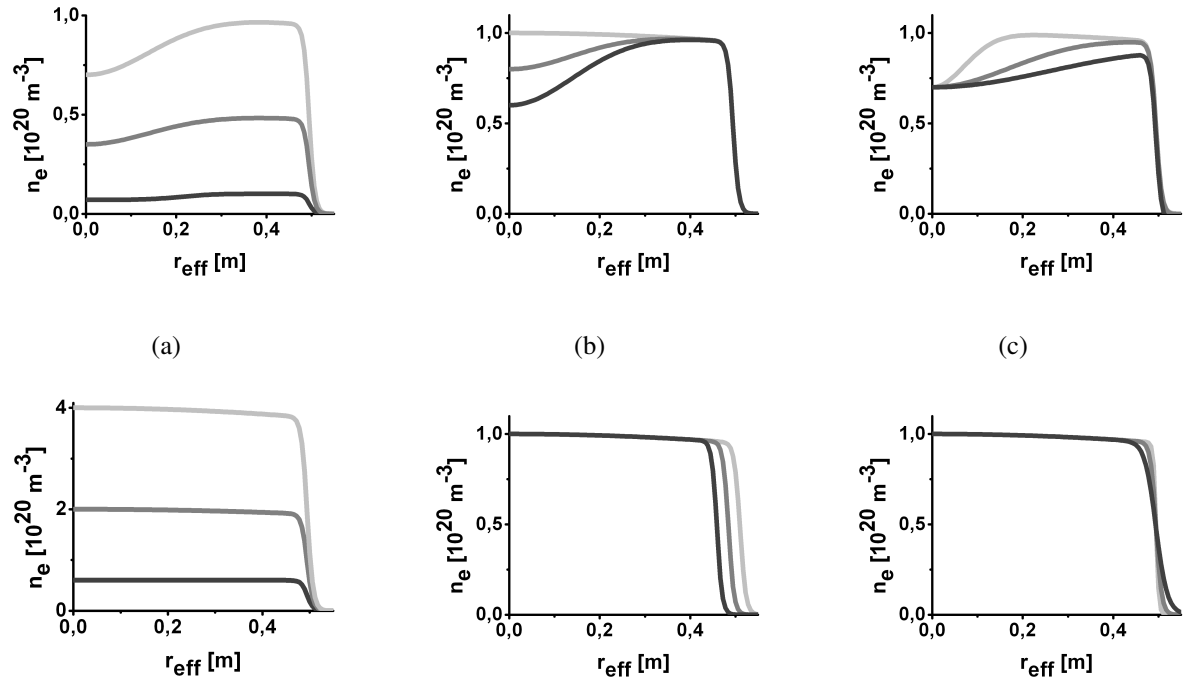

Figure 1: Assumed variations for the parameters of interest θ. Upper row: For the estimation of hollow profiles, the maximum density (a), the deepness (b) and the width of the hollow part are varied. Lower row: In case of high confinement regimes the parameters of interest are the maximum density (d),the position of the steepest gradient (e) and the steepness of the density decay (f).

To compute (2), two PDFs have to be provided by the physicist: First, the likelihood function $p(D|\theta,\eta,I)$, which includes a forward function of the experiment (”virtual diagnostic”) and the error statistics. Second, $p(\theta|I)$ describes the range of our physical interest: Upper and lower limits as well as a weighting of θ are included here. The expression $p(D|I)$ is calculated by marginalisation:

$$p(D|I) = \int \mathrm{d}\theta p(\theta|I) \cdot p(D|\theta,\eta,I). \tag{3}$$

Until this point, the approach is not restricted to a special kind of diagnostic or experiment. It can be applied for the optimisation of diagnostics as well as for the planning of experiments and experimental campaigns. The numerical values of (2) can be compared directly for different design configurations. And the physical question to be solved by the measurement is directly implemented in the process as a design criterion via $p(\theta|I)$.

The computational effort may become high if (2) cannot be solved directly, which is usually the case. Here, numerical integration algorithms have to be applied (e.g. Monte-Carlo methods), which may be time consuming and require appropriate hardware, which can be seen as a disadvantage of this method.

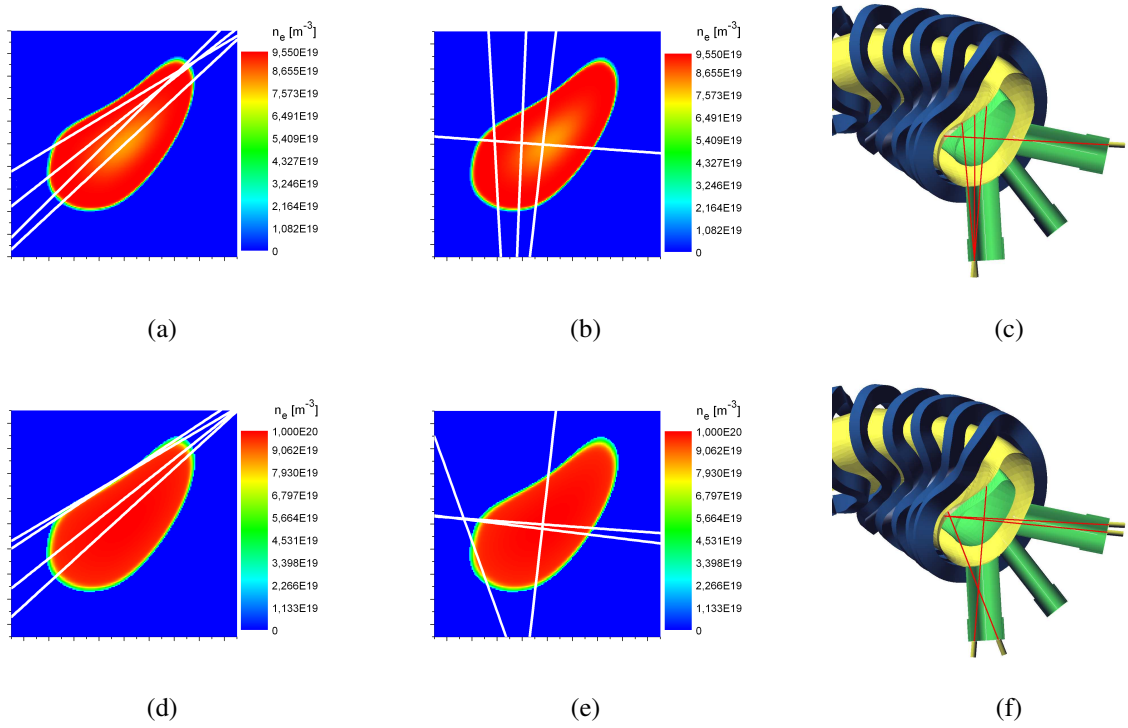

Figure 2: Design results for a four beam interferometer: Optimal configuration for the estimation of hollow profiles (upper row) and high confinement regimes (lower row). The design was calculated without (left) and including technical restrictions (middle) due to the port system (right).

Design of a multichannel interferometer

For W7-X, an infrared multichannel interferometer is planned [2]. The access to the plasma vessel is provided by a system of ports, which restricts the possible directions of the lines of sight. As no opposed ports are available, the beams are reflected by retro-reflectors, which have to fit to the structure of in-vessel components.

As an example, a four channel interferometer was optimised with respect to two different physical questions of interest: hollow density profiles, which may occur in case of Core Electron Root Confinement (CERC) scenarios, and the identification of high confinement regimes like optimal confinement (OC), H mode and HDH mode. The assumed variations of the parameters of interest are displayed in figure 1.

The optimal design was calculated for both physical questions, with and without respect to

physical question	unconstrained design	constrained design	difference
hollow profiles	$EU = 8.71 \pm 0.01$ *bit*	$EU = 6.43 \pm 0.01$ *bit*	2.28 *bit*
high confinement	$EU = 28.3 \pm 0.2$ *bit*	$EU = 8.53 \pm 0.01$ *bit*	19.8 *bit*

Table 1: Expected information gain for the unconstrained and constrained beamline design.

	hollow profiles	high confinement regimes	sum EU
hollow profiles design	$EU = 6.43 \pm 0.01$ *bit*	$EU = 7.40 \pm 0.01$ *bit*	13.83 *bit*
high confinement design	$EU = 5.95 \pm 0.01$ *bit*	$EU = 8.53 \pm 0.01$ *bit*	14.48 *bit*

Table 2: Comparison of both designs with respect to both physical questions.

the technical boundary conditions (port system, retro-reflectors). The resulting beamline configurations are shown in figure 2.

By comparing the expected information gain for the design with and without boundary conditions, the loss of information due to theses restrictions can be verified (see table 1). In case of the estimation of hollow profiles, the EU is reduced by approximately 26%. For the density changes as expected for high confinement scenarios, about 70% of the expected information gain is lost. This can be explained by the fact that the port system does not allow beamlines at the plasma edge, which cross the plasma on a long path (see fig. 2 (d) and (e)).

To identify the optimal design with respect to both problems, the expected information gain is calculated for every beamline configurations and both physical questions (table 2). It turns out that the design for the estimation of high confinement regimes is the more advantageous configuration for both problems: It is optimal in case of the confinement regimes, and its EU is only 0.5 *bit* smaller than for the optimal design for the estimation of hollow profiles. So, for the two problems analysed here, it would be the preferred design.

Conclusion

Bayesian experimental design was applied for the optimisation of a four-channel interferometer at W7-X. Two physical problems were used as design criteria, however, other questions of interest are possible, too. Different designs have been compared and the loss of information due to technical boundary conditions was quantified. The algorithm presented here is not related to a specific diagnostic, but can be seen as a general recipe for the design of diagnostics.

References

[1] H. Dreier, A. Dinklage, R. Fischer, M. Hirsch, P. Kornejew, and E. Pasch, Fusion Sci. Technol. **50**, 262 (2006).

[2] P. Kornejew, M. Hirsch, T. Bindemann, A. Dinklage, H. Dreier, and H.-J. Hartfuß, Rev. Sci. Instrum. **77**, 10F128 (2006).