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The Collective Energy Loss of The Relativistic Electron Beam Propagating Through Background Plasma

Oleg Polomarov[1], Igor Kaganovich[2] and, Gennady Shvets[1]

[1]*Department of Physics and Institute for Fusion Studies, The University of Texas at Austin, Austin, TX 78712*

[2]*Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543*

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We have developed a reduced computational model[1] describing beam-plasma interaction between kinetic beam particles and ambient plasma fluid. Using conservation of the canonical momentum for beam and plasma electrons in long beam pulses, we have numerically studied the dynamics of collisionless Weibel (filamentation) instability (WI) in the regime of very underdense electron beam. A physical model describing the structure and coalescence energetics of the beam/return current filaments produced as a result of the WI was developed. We emphasize the strongly nonlinear stage of the instability, during which the beam density of filaments is compressed to the background plasma density, and the ambient plasma is fully evacuated. Our analytic and numerical results using LSP code demonstrate that the beam filaments can carry super-Alfvenic currents with hollow current density profiles similar to the Hammer-Rostoker equilibrium. This has profound implications for the long-term evolution of the magnetic field and beam current and explains the long-standing puzzle: why magnetic field energy initially increases, but eventually decreases during the collisionless Weibel instability.

We developed an analytical model describing the structure and coalescence energetics of the beam/return current filaments produced as a result of the Weibel instability (WI) of an electron beam in the collisionless background plasma[1,2] . We identify three qualitatively different stages of the WI, as illustrated by Fig. 1: (i) the linear stage resulting in filaments formation and exponential growth of the magnetic field that saturates due to magnetic trapping of the beam particles; (ii) the non-linear stage corresponding to the merger of sub-Alfvenic ($I < I_A = \gamma\beta mc^3/e$) beam filaments and *increase* of the magnetic field energy that saturates when the filaments' beam current reaches approximately

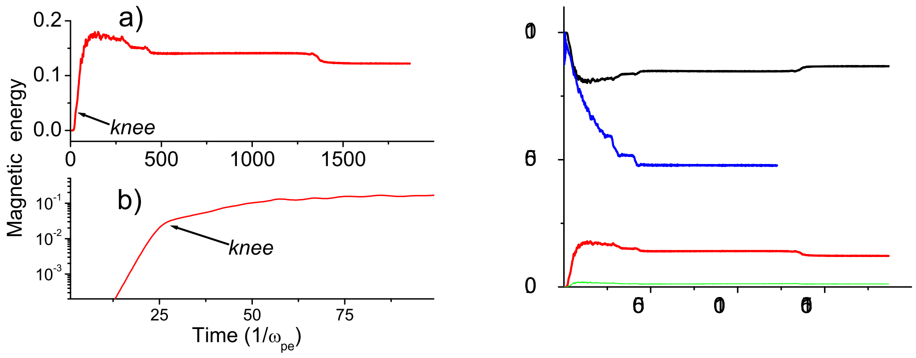

FIG. 1: (Colour on-line.) Left: PIC simulations[4] of the dynamics of the magnetic field energy: (a)long-term evolution, (b) short-time zoom-in for the WI of a relativistic beam in plasma. Beam parameters: $n_{b0}/n_0 = 0.2$ and $\beta_{b0} = 0.95$. Beam initially fills the simulated $40c/\omega_{pe} \times 40c/\omega_{pe}$ computational domain. The magnetic field energy is normalized to the initial beam energy. Right: PIC-simulated normalized magnetic (red), particle (black), electric (green) energies and normalized (to its initial value) current.

I_A, and (iii) the highly nonlinear stage, corresponding to the merger of super-Alfvenic ($I > I_A$) filaments and *decreasing* magnetic field energy. The filaments with hollow beam current structure similar to the classic Hammer-Rostoker (HR) equilibrium[3] with the outer return current develop and coalesce during stage (iii). We analytically predict that the magnetic energy encreases/decreases when sub/super-Alfvenic filaments coalesce, thereby explaining the recently observed magnetic field decay during the late stages of WI.

We start by estimating the magnetic energy of a single evacuated cylindrical beam filament with the beam density $n_b = n_0$ for $r < R$ and $n_b = 0$ for $r > R$. The beam filament is surrounded by the return current of the ambient plasma with density $n_e = n_0$ for $r > R$ and $n_e = 0$ for $r < R$. The dominant electromagnetic field is the in-plane magnetic field $\vec{B}_\perp = -\vec{e}_z \times \vec{\nabla}\psi$, where $\vec{A}_z \equiv \vec{e}_z\psi$ is the vector potential [1]. We assume that the initial electron beam with the forward momentum $\vec{P} = \vec{e}_z p_{b0}$ is under-dense with respect to the quiescent ambient plasma: $n_{b0} < n_0$. The following normalized quantities are used below: $\tilde{x} = \omega_{pe}x/c$, $\tilde{t} = \omega_{pe}t$, $\tilde{n} = n/n_0$, $\beta_{e,bz} = v_{e,bz}/c$, $\tilde{p}_{e,bz} = p_{e,bz}/mc$, $\tilde{\psi} = e\psi/mc^2$, $\tilde{B} = eB/mc\omega_{pe}$, and $\tilde{E} = eE/mc\omega_{pe}$. Tildes are dropped in what follows. Using conservation of the longitudinal canonical momentum, fluid momenta of the beam and the plasma in the filament are given by $p_{bz}(r) = p_{b0} + \psi(r)$ and $p_{ez} = \psi(r)^1$. The

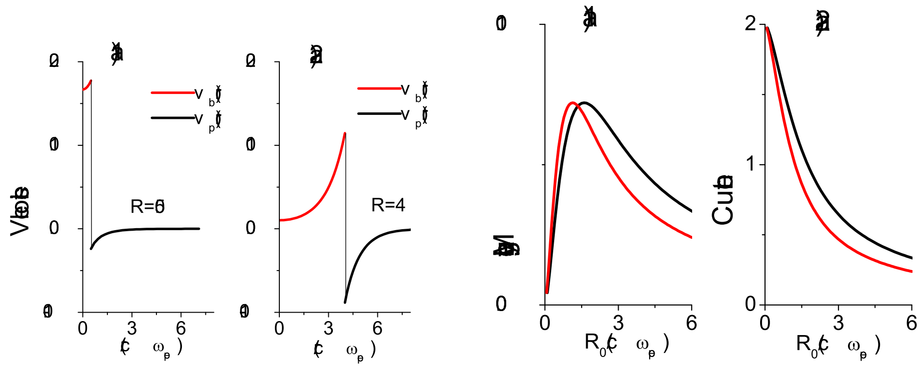

FIG. 2: (Colour on-line.) Left:Normalized velocities β_{bz} and β_{ez} of the beam (red) and plasma (black) for $R = 0.5$ and $R = 4$ with $\beta_{bz0} = 0.2$. Right: Normalized magnetic energy $2\bar{U}_B(R_0)$ and normalized current $2\bar{I}_b(R_0)$ for *two* filaments (black) with radii R_0, and $\bar{U}_B(R_1)$ and $\bar{I}_b(R_1)$ for *one* filament (red) with $R_1 = \sqrt{2}R_0$.

Ampere's law for the magnetic field flux ψ inside and around the evacuated beam filament can be expressed as $\nabla^2\psi = n_e\beta_{ez} + n_b\beta_{bz}$.

In the simplest case of non-relativistic beam and plasma, Ampere's law becomes $\nabla^2\psi - \psi = \theta(r)\beta_{b0}$, where $\theta(r < R) = 1$ and $\theta(r > R) = 0$. Requiring the continuity of ψ and $B_\theta = \partial_r\psi$ at $r = R$ and solving for θ results in the analytic expressions for the beam's and plasma's axial velocities: $\beta_{bz}(r) = \beta_{b0}I_0(r)K_1(R)R$ and $\beta_{ez}(r) = -\beta_{b0}K_0(r)I_1(R)R$. These are plotted in Fig. 2 for a thin ($R = 0.5$) and thick ($R = 4$) filaments. It is apparent that the beam velocity is strongly peaked at the filament's periphery whenever $R \gg 1$. The resulting hollow current distribution for the beam part of the filament corresponds to the Hammer-Rostoker[3] equilibrium of a charge-neutralized beam with a flat-top density distribution. Note that an electron beam filament with $R \gg 1$ carries a super-Alfvenic current.

Note that the beam and plasma velocities as well as the filaments' beam current (equal by construction to the plasma return current) and the magnetic energy are parameterized by only two parameters: the initial beam velocity β_{b0} and the filament's radius R. These quantities are calculated for *two* non-interacting filaments of radius R_0 before merger and *one* resulting (coalesced) filament with radius $R_1 = \sqrt{2}R_0$. Normalized beam current and filament magnetic field energy (with magnetic energy of the return current) are expressed as $\bar{I}_b(R) = 2\int_0^R\, drr\beta_{bz}/\beta_{b0}R^2$ and $\bar{U}_B(R) = \int_0^\infty\, drr|\partial_r\psi|^2/R^2$. The initial (pre-merger) current and energy are given, respectively, by $2\bar{I}_b(R_0)$ and $2\bar{U}_B(R_0)$. The same post-merger quantities are given by $\bar{I}_b(R_1)$ and $\bar{U}_B(R_1)$. Both pre-merger (black curves) and post-merger (red curves) quantities are plotted in Fig. 2(right). Independently of the filaments' radius R_0, the total current of the merged filament is decreased during the merger, see Fig. 1 and Fig. 2.

This happens because the beam loses longitudinal momentum due to inductive electric and changing magnetic fields in going from one equilibrium to another. Remarkably, the total magnetic energy can either increase (for $R < R_{crit} \approx 1.5$) or decrease (for $R > R_{crit}$) as the result of the merger, as shown in Fig. 2 and Fig 1. Qualitatively, the merger of two small sub-Alfvenic beam filaments results in the addition of the single filament currents: $\bar{I}_b(R_1) \approx 2\bar{I}_b(R_0)$. This is because for $R < 1$, the filament's current is uniformly distributed across the filament and is therefore approximately proportional to its area.

It is then straightforward to demonstrate that the total magnetic field energy approximately doubles as the result of the merger. The situation drastically changes for $R > 1$ wide super-Alfvenic filaments: the beam current is concentrated within the skin depth c/ω_{pe} of its periphery. Therefore, the total current of the merged filaments decreases by a factor $\sim \sqrt{2}$ and consequently the total magnetic field energy decreases approximately by the same factor during the merger. The transition from magnetic energy growth to decay is clearly seen in Fig. 1.

In conclusion, we have developed a robust and transparent analytical model describing the energetics of merging between two hollow super-Alfvenic current filaments in the ambient plasma. Our model predicts, and PIC simulations confirm, that such mergers are responsible for the magnetic energy decay during the late stages of the Weibel instability.
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[4] Simulations were performed with LSP-PIC code which is a product of ATK Mission Research, Albuquerque, NM 87110.