

Eulerian and Lagrangian statistical analysis of SOL turbulent transport

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Among the open issues to be addressed by ITER is the control of plasma wall interaction and the divertor operation at very high power. In this strongly non-linear process, the turbulent transport is observed to be a key element with intermittent-like behaviour and, as such, requires a full ab-initio investigation. In particular, Scrape-Off Layer turbulent transport exhibits long range radial transport that significantly depart from the diffusive like transport assumption. In that respect it is important to analyse the statistical properties of SOL turbulence, both for a proper understanding of the ongoing turbulence, but also to provide elements for a predictive description of transport.

In the present paper, a statistical numerical analysis of the eulerian and lagrangian velocity fluctuations of SOL turbulence is achieved. The present statistical results are computed with data from TOKAM-2D simulations of SOL turbulence [1].

This model is based on the interchange instability in the flute and isothermal approximation. The equations used in this analysis are very similar to the one describing the Rayleigh-Bénard experiment. It is a convenient description of SOL turbulence since it is a very efficient simulations tool that reproduces the "blobby" transport and show an extremely intermittent behavior as reported experimentally. From the simulations, a temporal series of the electric potential and of density maps are obtained (figure 1) from which Eulerian and Lagrangian velocities are respectively computed.

The Eulerian velocity is derived at each position

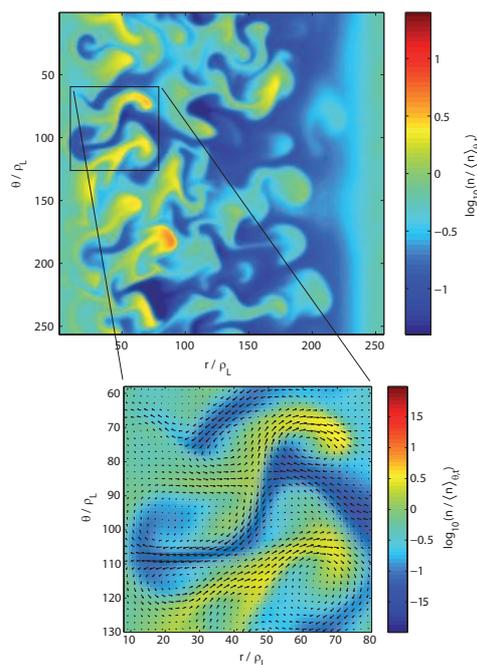


Figure 1: 2D density map derived from Tokam-2D. The radial r and poloidal θ directions are normalized by the Larmor radius ρ_L . The simulation domain is $256 \rho_L \times 256 \rho_L$. Note the logscale of the normalized density. Blow-up : velocity vectors (black arrows) are superimposed to the density field.

from the electric potential maps as being the electric drift velocity, $\vec{v}_E = \vec{\nabla}\phi \wedge \vec{B}/B^2$. The measurements volume is restricted to $r = 15\rho_L$ to $r = 160\rho_L$ in which turbulence is fully developed. The resulting velocity field is a 145×255 vectors array respectively in the radial and poloidal directions. From the temporal fluctuations of the electric drift velocity, the temporal Eulerian velocity increments are computed as $\delta_t u = u(r, \theta, t + \delta t) - u(r, \theta, t)$, where u is restrict to the radial velocity component.

The plasma Lagrangian velocity, on the other hand, is obtained from the density maps by tracking “structures” of density. Fronts and density structures can be clearly identified using a simple thresholding method on the density value. 50 % of the absolute density maximum value was chosen as the threshold and both sub-dense and over-dense structures are tracked. Typically, around ten structures are identified on a given map. In order to take into account the structure deformation, we compute their “center of mass”, $\sum_{r,\theta} \vec{r} n / \sum_{r,\theta} n$. The velocity components are computed as the displacement of the structure’s center during the time delay between two maps $\tau = 50/\Omega_i$, where Ω_i is the ion cyclotron frequency. The average displacement of a structure during τ is of the order of 1 Larmor radius ρ_L . Since the extension of density structures is about ~ 30 and $\sim 10 \rho_L$ respectively in the radial and poloidal directions, an overlap technique is employed to recognize an identical structure from one map to another. It consists to verify that the center of mass of the “ $\tau + 1$ ” structure is always included into the “ τ ” structure. If a stretched structure dislocates itself into smaller structures, the latter are considered as being new structures. A temporal sequence is associated with each identified structure. Statistics of the lagrangian velocity increments are computed for each structure n as $\delta_t^{(n)} u = u^{(n)}(t + \delta t) - u^{(n)}(t)$.

Four probability distribution functions (pdf) of the lagrangian velocity increments for different time delays are plotted in figure 2. The shape of the pdf is found to significantly vary as a function of the time scale δt . At large time scale, the pdfs are found to be close to gaussianity. Conversely, at small separation time scale, the pdfs depart from a Gaussian distribution of same variance and exhibit

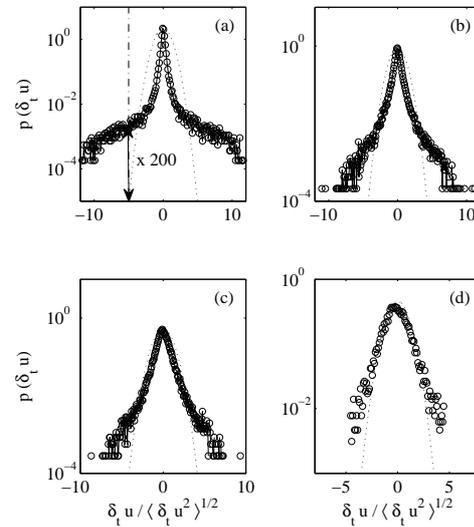


Figure 2: *LAGRANGIAN. Probability density functions of the radial Lagrangian velocity increments for different time delays (a) $\delta t = 1 \tau$, (b) $\delta t = 4 \tau$, (c) $\delta t = 10 \tau$ and (d) $\delta t = 30 \tau$. Black dotted lines correspond to a Gaussian distribution with same variance.*

heavy tails with respect to their central peak. A velocity acceleration $\partial u/\partial t$ equal to 5 times the root-mean-square (rms) value is 200 times more probable than for a Gaussian distribution. This non-gaussian behavior at small scale is an essential feature of the dynamic of the turbulence and is a signature of intermittency.

The pdfs of the temporal eulerian velocity increments for different time delays δt are plotted in figure 3. As for the lagrangian velocity, the pdfs are Gaussian at large time delay and depart significantly to Gaussian distribution at small δt . As an illustration, a velocity acceleration of 5 times the rms value is 80 times more probable than a Gaussian.

To further quantify the variable flattening shape of distribution functions, the flatness factor (or kurtosis) is defined as the normalized fourth-order structure function $\langle [\delta_t u]^4 \rangle$,

$$F(\delta t) = \frac{\langle [\delta_t u]^4 \rangle}{\langle [\delta_t u]^2 \rangle^2}, \quad (1)$$

where the structure function of order q is defined as $\langle [\delta_t u]^q \rangle = \int (\delta_t u)^q p(\delta_t u) d(\delta_t u)$. $F = 3$ corresponds to a Gaussian distribution. Large values of F correspond to highly intermittent signals in which $\delta_t u$ is nearly zero much of the time and periodically burst into life.

In fluid turbulence, values of the flatness factor of the velocity increment are found to be in the range 4-40 depending on the turbulence intensity [2]. Furthermore the structure functions are assumed to follow a scaling law in the inertial range $\langle [\delta u]^q \rangle \sim r^{\zeta_q}$, where r is the scale (spatial and temporal scales can be linked through the Taylor hypothesis $r \sim U_{rms} \delta t$) and ζ_q is the structure function exponent. It is now clearly established that ζ_q present a non linear variation as a function of the order q [2]. The departure of ζ_q to linearity is the signature of intermittency. Kolmogorov [3] proposed a non linear variation of ζ_q as $\zeta_q = c_1 q - c_2 q^2/2$, where c_2 is the intermittency coefficient and is universal in 3D turbulence. The flatness exponent in the inertial range, $F \sim r^{\zeta_4 - 2\zeta_2}$ according equation (1), is connected to the c_2 coefficient and is given by $\zeta_4 - 2\zeta_2 = -4 c_2$. The flatness exponent is found to be $4 c_2 = 0.1 \pm 0.012$ for eulerian measurements whereas in lagrangian turbulence $4 c_2$ is equal to 0.36 ± 0.08 . Lagrangian increments are thus found

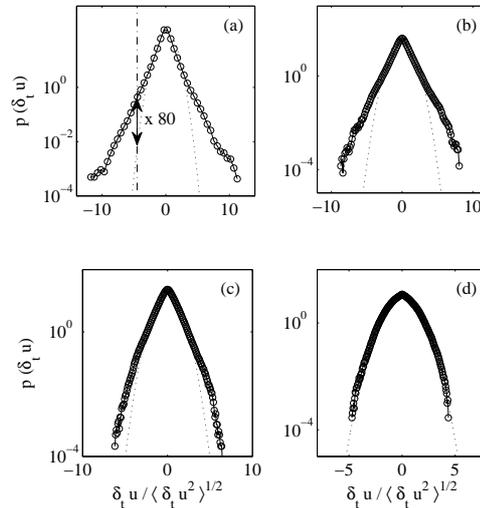


Figure 3: *EULERIAN. Probability density functions of the radial Eulerian velocity increments for different time delays (a) $\delta t = 1 \tau$, (b) $\delta t = 4 \tau$, (c) $\delta t = 10 \tau$ and (d) $\delta t = 30 \tau$. Black dotted lines correspond to Gaussian distribution with same variance.*

to be ~ 3.5 times more intermittent than Eulerian ones in fluid turbulence.

The Lagrangian and the Eulerian flatness, normalized by 3, are plotted in figure 4, as a function of the time delay δt normalized by, respectively, the life time of Lagrangian density structures and the correlation time of the Eulerian velocity. At large time delay δt , the flatness factor is close to 3 as expected for Gaussian distributions. Conversely, at small δt , F increases up to values larger than 20, which is characteristic of strongly intermittent signal. The growth of the Eulerian flatness (data \circ), with decreasing δt , is found to be sharper than the expected slope (i.e. -0.1) in fluid turbulence whereas the increase of the Lagrangian flatness (data \times) seems to be weaker than the expected slope, i.e. -0.36. We do not claim that the flatness

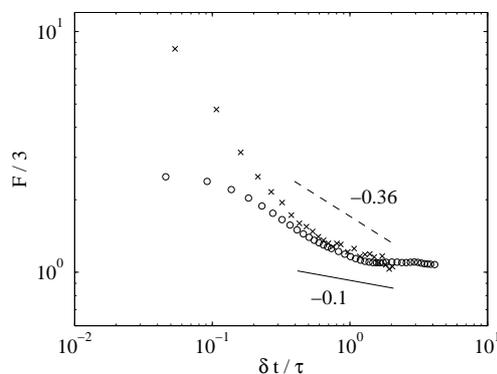


Figure 4: Eulerian (data \circ) and Lagrangian (data \times) flatness of the velocity increments. The full lines correspond to the expected power law of the Eulerian flatness in fluid turbulence whereas the dashed lines correspond to the expected increases of the Lagrangian flatness.

grows as a power law of the time delay. This non self-similar growth is consistent with the fact that a clear initial range is not recovered in SOL turbulence which is may due to the presence of several injection scales. However the growth of the Lagrangian flatness at small time delay seems to be slightly sharper than the one of the Eulerian flatness and this effect is compatible with a more intermittent Lagrangian velocities as previously observed in fluid turbulence.

A comparison with experimental data from Doppler backscattering [4] will be achieved in a near future.

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