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Interplay between magnetic shear and flow shear

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Introduction Turbulence regulation by $\mathbf{E}\times\mathbf{B}$ shear flow is considered to be indispensable for eliminating anomalous transport and thus for the confinement of fusion plasmas. For low-beta plasma, the profile of coherent magnetic fields – the so-called magnetic shear (q profile) – is also a crucial element that controls anomalous transport [1]. On one hand, magnetic shear has a direct impact on transport by modifying the onset of linear instabilities of various modes on microscales – the stabilization of curvature driven instability by negative magnetic shear is one example. On the other hand, it can have an indirect impact through modifying the structure of nonlinearly saturated state such as the localization of modes and inducing toroidal mode coupling for large magnetic shear; the localization of modes can enhance the overall transport by inhibiting the generation of zonal flows, which is a main player in turbulence regulation. A similar effect of magnetic shear can result from toroidal mode coupling for high magnetic shear. The importance of $\mathbf{E}\times\mathbf{B}$ flow shear and magnetic shear in the formation of transport barrier has actively been studied recently (e.g., see [1] and references therein). In particular, [1] has derived an empirical criterion for the formation of internal transport barrier in ion temperature gradient (ITG) turbulence as $\Omega/\gamma_{ITG} > 0.68s - 0.095$, where Ω, s and γ_{ITG} are the $\mathbf{E}\times\mathbf{B}$ flow shearing rate, magnetic shear, and ITG linear growth rate, respectively. It is thus crucial to understand how these main players in transport – flow and magnetic shear – interact each other and influence the overall transport.

The purpose of this paper is to investigate the interplay between flow shear and magnetic shear. Specifically, by consistently incorporating the effects of flow shear and magnetic shear, we analytically show that magnetic shear can interfere with turbulence/transport suppression by flow shear, thereby weakening its effect on turbulence regulation. This is because the shearing process by shear flows becomes less effective due to magnetic perturbation induced by magnetic shear. This result – less efficient turbulence regulation by shear flows (e.g., zonal flows) for higher magnetic shear – thus suggests that particle/heat transport and the generation (or damping) of shear would be more efficient in high magnetic shear regime for a fixed flow shear.

Formulation of the problem To elucidate the essential physics in the simplest way, we consider the three dimensional (3D) reduced magnetohydrodynamic (RMHD) turbulence in the local cartesian coordinates near resonant surface ($\mathbf{B}\cdot\mathbf{k} = 0$), where x, y, and z represent the local radial, poloidal, and toroidal directions, respectively. We assume that a large-scale sheared magnetic field $\mathbf{B} = B_0\hat{y} = xB_0'\hat{y}$ is aligned with shear flow $\mathbf{U} = U_0\hat{y} = -x\Omega\hat{y}$ in the poloidal (y) direction around the resonant surface at $x = 0$. Here, Ω and B_0' are flow shear and magnetic shear, respectively; Ω is assumed to be positive constant without loss of generality. The linear equations for fluctuating magnetic vector potential a, electric potential ϕ, and test particle (or passive scalar field) density n are then given by

$$[\partial_t - x\Omega\partial_y]\,\omega = -xB_0{}'\partial_y\nabla_\perp^2 a + \nu\nabla_\perp^2\omega + F_\omega\,, \tag{1}$$

$$[\partial_t - x\Omega\partial_y]\,a = xB_0'\partial_y\phi + \eta\nabla_\perp^2 a + F_a\,, \tag{2}$$

$$[\partial_t - x\Omega\partial_y]\,n = -u_x\partial_x n_0 + D\nabla_\perp^2 n\,. \tag{3}$$

Here, the fluctuating magnetic vector potential a is related to magnetic fluctuations $\mathbf{b}$ as $\mathbf{b} = \nabla\times a\hat{z} = (\partial_y a, -\partial_x a, 0)$; ω is the fluctuating vorticity related to electric potential ϕ as $\omega = -\nabla_\perp^2\phi$ and velocity $\mathbf{u} = \nabla\times\phi\hat{z}$ as $\omega\hat{z} = \nabla_\perp\times\mathbf{u} = (\partial_x u_y - \partial_y u_x)\hat{z}$; ν, η, and D are viscosity and Ohmic and particle diffusivities, respectively; $\nabla_\perp^2 = \partial_{xx} + \partial_{yy}$ is the two-dimensional Laplacian; F_ω and F_a are small–scale random forcings acting on the fluid and magnetic field, respectively. Note that physically, F_ω and F_a can be due to instabilities which drive strong electrostatic and magnetic perturbations, respectively. Turbulence driven by the forcings F_ω and F_a in Eqs. (1)–(2) leads to anomalous (turbulent) transport of momentum and particles. In the absence of flow shear Ω and magnetic shear B_0', this turbulent transport is fast (much faster than neo-classical prediction). Our previous works have shown how this fast transport is quenched by strong flow shear in a few different turbulence models [2] and by flow shear and uniform magnetic fields in MHD [3]. The key question here is what would happen in the presence of flow shear and magnetic shear. In order to answer this question, we need to incorporate both effects non-perturbatively in our analysis. To this end, we solve Eqs. (1)-(3), by using the time dependent Fourier transformation for fluctuation φ [2, 3]:

$$\varphi(\mathbf{x},t) = \tilde{\varphi}(\mathbf{k},t)\exp\{i(k_x(t)x + k_y y)\}\,, \tag{4}$$

with k_x satisfying an eikonal equation $\partial_t k_x(t) = k_y\Omega$. It can easily be shown that under the transformation (4), the term of the form $(\partial_t - x\Omega\partial_y)\varphi$ on the left hand side in Eqs. (1)–(3) becomes $\partial_t\tilde{\varphi}$ while the term of the form $x\varphi$ on the right hand side involving magnetic shear becomes $i\partial_{k_x}\tilde{\varphi}$. Therefore, in terms of the new variables $\tau = k_x(t)/k_y$, $S = B_0'/\Omega$, and

$$\tilde{\psi}_1 = \tilde{\omega} + Sk_y^2(1+\tau^2)\tilde{a}\,, \tilde{\psi}_2 = S\tilde{\omega}/(1+\tau^2) + k_y^2\tilde{a}\,, \tag{5}$$

Eqs. (1) and (2) can be cast into the following form:

$$\partial_\tau \tilde{\psi}_1 = \alpha_\nu[S(1+\tau^2)^2\tilde{\psi}_2 - (1+\tau^2)\tilde{\psi}_1] + \tilde{F}_\omega/\Omega \quad (6)$$

$$\partial_\tau \tilde{\psi}_2 = \alpha_\eta[S\tilde{\psi}_1 - (1+\tau^2)\tilde{\psi}_2] + k_y^2\tilde{F}_a/\Omega. \quad (7)$$

Here, $\alpha_\nu = \xi_\nu/(1-S^2)$, $\alpha_\eta = \xi_\eta/(1-S^2)$, $\xi_\nu = \nu k_y^2/\Omega$, and $\xi_\eta = \eta k_y^2/\Omega$.

Eqs. (5)–(7) [or, Eqs. (1)–(2)] immediately show that the evolution of vorticity and magnetic fluctuations decouples in the absence of magnetic shear. Furthermore, one can easily show that homogeneous part of coupled equations (6) and (7) exhibits an instability for $(1-S^2) < 0$ in the presence of dissipation. That is, for a given strength of flow shear, the system becomes unstable in the presence of finite dissipation (i.e., $\xi_\eta \neq 0$, or $\xi_\nu \neq 0$) as magnetic shear becomes stronger than flow shear such that $|S| > 1$, reminiscent of resistive tearing instability. Since our purpose here is to examine how the interaction between flow shear and magnetic shear affects turbulent transport, in the following, we focus on the stable situation with $|S| < 1$ and investigate how the reduction in momentum and particle transport by flow shear is influenced by magnetic shear. Of our particular interest is the strong shear limit, that is, only when the effect of the flow shear dominates that of dissipations, i.e., $\xi_\eta = \eta k_y^2/\Omega \ll 1$ and $\xi_\nu = \nu k_y^2/\Omega \ll 1$. Here, k_y is the characteristic wavenumber of the driving forcing. Thus, ξ_η is treated as a small parameter, characterizing a strong shear limit while ξ_ν is assumed to vanish, for simplicity.

In the case of of fluid forcing with $F_a = 0$, the solutions to Eqs. (6) and (7) are

$$\tilde{\psi}_1(\tau) = \frac{1}{\Omega}\int^\tau d\tau_1 \tilde{F}_\omega(\tau_1), \quad (8)$$

$$\tilde{\psi}_2(\tau) \sim 0, \quad (9)$$

in the strong shear limit ($\xi_\eta \ll 1$) to leading order in $O(\xi_\eta)$. Eqs. (3), (5), (8), and (9) then give us the eddy viscosity ν_T defined by $\langle u_x u_y - b_x b_y\rangle = \nu_T \Omega$ and turbulent particle diffusivity defined by $\langle n u_x\rangle = -D_T \partial_x n_0$ as:

$$\nu_T = -\frac{\tau_f}{2\Omega^2}\frac{1}{1-S^2}\int \frac{d^2k}{(2\pi)^2}\frac{\phi_\omega(\mathbf{k})}{k_y^2}, \quad (10)$$

$$D_T \simeq \frac{\tau_f}{\Omega^2}\frac{1}{(1-S^2)^2}\frac{\pi^2}{8}\int \frac{d^2k}{(2\pi)^2}\frac{\phi_\omega(\mathbf{k})}{k_y^2}. \quad (11)$$

Here, $D \ll \eta$ was assumed to obtain Eq. (11); ϕ_ω is the power spectrum of the fluid forcing F_ω, which is assumed to be homogeneous and stationary with a short correlation time τ_f (e.g., see [3]). Eqs. (10)–(11) clearly show that magnetic shear plays the role of the coupling of electric to magnetic fluctuations and the weakening of the effect of flow shear. First, recall that for $F_a = 0$, fluctuating vorticity is generated by fluid forcing, while weak magnetic fluctuation is induced due to (weak) magnetic shear. Thus, in the absence of magnetic shear, there are no magnetic fluctuations, thereby recovering the 2D hydrodynamic results. For instance, for $S = 0$, the conservation of enstropy in the 2D hydrodynamic turbulence leads to the amplification of shear flow, with a negative viscosity $\propto -1/\Omega^2$ (see, e.g. [3]). This amplification process is slowed down $\propto \Omega^{-2}$ as Ω increases, manifesting a self-regulation. In this case, turbulent particle transport in (11) is quenched as $\propto \Omega^{-2}$ due to flow shear. As the magnetic shear increases ($S \neq 0$), the magnitude of both eddy viscosity and particle diffusivity in Eqs. (10)–(11), however, increases as the quenching effect of flow shear is somewhat compensated by magnetic shear. This can physically be understood since the shearing and distortion of turbulent eddies by flow shear becomes inefficient due to magnetic perturbation induced by magnetic shear. In particular, perturbations on different magnetic field lines propagate at different speeds, undergoing phase mixing. As a result, both the particle mixing and the generation of shear flow are less reduced, compared to the case without magnetic shear. However, for large S, the increase in D_T in Eq. (10) is faster than that in ν_T in Eq. (11). These results thus imply faster particle mixing in the regime with higher magnetic shear. That is, for a given shear flow (as in the case of a shear flow driven by pressure gradient, or externally), particle transport is expected to faster in higher magnetic shear regime. A similar result is also found in the case of magnetic forcing $F_a \neq 0$ and $F_\omega = 0$ [4].

Conclusion we have shown that magnetic shear can inhibit transport quenching by flow shear, leading to more efficient transport, compared to the case without magnetic shear. Specifically, transport of particles is shown to be more efficient in the regime of higher magnetic shear for a fixed flow shear. These results thus suggest a low magnetic shear regime as a favorable location for the formation of transport barrier. This could be relevant to understanding experimental observations of the negative effect of high (positive) magnetic shear on the formation of ITB in plasma core or to reversed field pinch plasmas where the effect of magnetic fluctuations is not negligible.

References

[1] T.J.J. Tala, et al, Plasma Phys. Control. Fusion **43**, 507 (2001).

[2] E. Kim, Phys. Plasmas **12**, 090902 (2005); E. Kim, Astron. & Astrophys. **441**, 763 (2005); A. Newton & E. Kim, Phys. Plasmas, **14**, 122306 (2007).

[3] E. Kim, Phys. Rev. Lett, **96**, 084504 (2006); E. Kim and B. Dubrulle, Phys. Plasmas, **8**, 813 (2001); J. Douglas, E. Kim and A. Thyagaraja, Phys. Plasmas, **15**, 052301 (2008).

[4] E. Kim, Phys. Plasmas, **14**, 084504 (2007).