

Generation of electromagnetic structures by drift wave interactions

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Introduction

Existence of coherent meso-scale structures at the edge of magnetic fusion devices had long been known and reported in experimental observations. Plasma polarization (caused by magnetic field curvature) has been suggested as an underlying mechanism for the formation and convection of blobs (see review paper [1] and references therein). It has recently been proposed also that nonlinear plasma polarization due to Reynolds stresses associated with small scale turbulence can be an important factor in the formation of blobs [2]. In particular, it was shown that interplay of the nonlinear stresses due to drift wave turbulence and interchange drive may lead to the formation of electromagnetic meso-scale perturbations with transverse dimensions larger than the characteristic length scale of underlying plasma turbulence, yet smaller than the characteristic scales of the equilibrium plasma parameters. Such structures have a finite poloidal mode number m and finite length along the equilibrium magnetic field. Respectively, one can define the poloidal and parallel wave vectors, $q_y \simeq m/r_s$ and $q_z \simeq m/qR$, where m is a poloidal mode number, r_s is a minor radius, q is the safety factor, and R is a major radius. The purpose of this paper is to consider the process of turbulent blob generation by using the approach of four-wave modulational interaction. Another goal of this work is to take into account the finite ion temperature effects both in the wave dispersion and the interchange drive.

Nonlinear equations

In the low frequency, $\omega < \omega_c$, and long wavelength, $k_{\perp}^2 \rho_i^2 < 1$, approximations, basic nonlinear equations consist of the electron continuity equation, quasineutrality equation and the parallel component of Ohm's law. The first two equations are standard, while in the ion continuity equation we have to take into account the inertial polarization drift as

well as the ion drift due to gyroviscosity. Both of these effects are of the same order in a finite T_i plasma and both contribute to the wave dispersion and to Reynolds stress, which is responsible for generation of large scale structures. The continuity equation has the form

$$-\frac{1}{e}\nabla_{\parallel}J - \frac{1}{e}\nabla \cdot J_p + n_0\rho_i^2\nabla_{\perp} \cdot \frac{d_0}{dt}\nabla_{\perp} \left(\frac{e\phi}{T_i} + \frac{p}{p_i} \right) = 0 \quad (1)$$

Here, the divergence of the diamagnetic current is calculated as

$$\nabla \cdot J_p = -\frac{2}{B}\hat{\mathbf{z}} \times \nabla (p_e + p_i) \cdot \nabla \ln B. \quad (2)$$

Separation of scales

We assume scale separation between the large scale modes and small scale fluctuations (presumably existing as a turbulent bath). All perturbed quantities can be represented as a sum of large and small scale components, $X = X_k + X_q$, $q < k$. An analogous separation of scale is assumed in the time domain, $\Omega < \omega_k$. Here, ω_k is the eigen-frequency of small fluctuations, $X_k(t) \sim \exp(-i\omega_k t)$ and Ω is the frequency of the large scale modes, $X_q \sim \exp(-i\Omega t)$.

We consider a simplest drift-wave type fluctuations assuming that primary modes are electrostatic and that the condition $\omega < k_{\parallel}v_{Te}$ is satisfied. We assume that ion temperature gradient is not too large so that the ITG instability does not occur, perturbations of ion temperature remain "slaved" to the potential fluctuations and do not affect directly the mode stability nor the dispersion of primary modes. The temperature fluctuations however are important as they contribute to the Reynolds stress drive. They also modify the dispersion properties of large scale perturbations.

Nonlinear interaction of primary fluctuations with large scale components leads to the excitation of the side-bands of the perturbed quantities. Side-bands of density and pressure are driven by the nonlinear convection

$$-i\omega^{\pm}n_{\pm k+q} - \frac{icn_0'}{B_0}(\pm k_y + q_y)\phi_{\pm k+q} \mp \frac{c}{B_0}\hat{\mathbf{z}} \cdot \mathbf{k} \times \mathbf{q}(\phi_{\pm k}n_q - \phi_q n_{\pm k}) = 0, \quad (3)$$

$$-i\omega^{\pm}p_{\pm k} - \frac{icp_{0i}'}{B_0}(\pm k_y + q_y)\phi_{\pm k+q} \mp \frac{c}{B_0}\hat{\mathbf{z}} \cdot \mathbf{k} \times \mathbf{q}(\phi_{\pm k}p_q - \phi_q p_{\pm k}) = 0, \quad (4)$$

where $\omega^{\pm} = \Omega \pm \omega_{*k}$. Respectively, one has similar equations for the temperature side-bands.

We assume that the small scale fluctuations are electrostatic: n_k , ϕ_k and $A_k = 0$ and the large scale electromagnetic so that the large scale perturbations of the electrostatic potential ϕ_q , magnetic potential A_q and density n_q are present.

Dynamics of large scale perturbations and turbulent generation of electromagnetic blobs

In neglect of dispersive corrections, which leads to $A_{\pm k+q} = 0$, the large scale component of the parallel momentum balance remains linear

$$\frac{1}{c} \frac{\partial A_q}{\partial z} + \frac{\partial}{\partial z} \left(\phi_q - \frac{T_e \tilde{n}_q}{e n_0} \right) + \frac{T_e}{e n_0 B_0} \hat{\mathbf{z}} \cdot \nabla A_q \times \nabla n_0 = 0. \quad (5)$$

It is assumed here that $\omega < q_z v_{Te}$ so the of the electron inertia are not important and electron temperature fluctuations are determined by $\nabla_{\parallel} T_e = 0$.

Equation for large scale component of the electron density is

$$\left(\frac{\partial n_q}{\partial t} + \mathbf{V}_{E_q} \cdot \nabla n_0 \right) + \left(\tilde{\mathbf{V}}_E \cdot \nabla \tilde{n} \right)_q - 2 \left[(n \mathbf{V}_E)_q + (n \mathbf{V}_{pe})_q \right] \cdot \nabla \ln B - \frac{1}{e} \frac{\partial}{\partial z} J_q = 0. \quad (6)$$

There is no contribution from the Maxwell stress because we assume that primary fluctuations are electrostatic. The total Reynolds stress is represented as $R_1 = R_1^a + R_1^b$,

$$R_1^a = 2i \left(\frac{c}{B_0} \right)^2 |\hat{\mathbf{z}} \cdot \mathbf{k} \times \mathbf{q}|^2 |\phi_k|^2 \frac{q^2}{\Omega - q_y V_*} \times (\tau^{-1} + 1 + \eta_i) \left[\frac{e \phi_q}{T_{0e}} - \frac{n_q}{n_0} \right], \quad (7)$$

$$R_1^b = 2i \left(\frac{c}{B_0} \right)^2 |\hat{\mathbf{z}} \cdot \mathbf{k} \times \mathbf{q}|^2 |\phi_k|^2 \times \left(\frac{\mathbf{k} \cdot \mathbf{q} \omega_* - q^2 \Omega}{\Omega^2 - \omega_{*k}^2} \right) \left[\frac{T_q}{T_{0i}} - \eta_i \frac{n_q}{n_0} \right]. \quad (8)$$

These expressions are valid both for zonal flow and streamer perturbations provided the background turbulence is purely electrostatic and waves are dispersionless.

To complete the dispersion relation for large scale modes we need to determine the perturbations of ion and electron temperatures and the relation between the perturbations of vector and electrostatic potentials. Note, that in our model, large scale perturbations of density and potential are finite.

Expression for the Reynolds stress in the main order becomes

$$R_1^a = 2i \left(\frac{c}{B_0} \right)^2 |\widehat{\mathbf{z}} \cdot \mathbf{k} \times \mathbf{q}|^2 |\phi_k|^2 \frac{q^2}{\Omega} (\tau^{-1} + 1 + \eta_i) \frac{e\phi_q}{T_{0e}}, \quad (9)$$

and $R_1^b = 0$. These are the lowest order expressions neglecting the dispersion in the large scale components.

The final dispersion relation for the meso-scale perturbations.

$$\Omega^2 + \Omega \Omega_* \tau (1 + \eta_i) - \frac{\Omega_{Di} \Omega_*}{q_{\perp}^2 \rho_s^2} [1 + \eta_i + \tau^{-1} (1 + \eta_e)] - q_z^2 v_A^2 = \quad (10)$$

$$-2 \left(\frac{c}{B_0} \right)^2 (1 + \tau + \tau \eta_i) |\widehat{\mathbf{z}} \cdot \mathbf{k} \times \mathbf{q}|^2 |\phi_k|^2 \quad (11)$$

The second term on the left hand side describes the drift stabilization due to finite ion temperature, the third term is the interchange drive, and the term on the right-hand side is the Reynolds stress drive that takes into account the diamagnetic contributions due to finite ion temperature. This equation is a principal result of this paper. Equation (10) is fully consistent with results of the previous analysis by using the wave kinetic equation approach[2]. Note that the growth rate of this electromagnetic instability is a factor $q\rho_s$ larger than for electrostatic modes with a finite q_y .

Summary

We have considered the instability of large scale electromagnetic structures (such as blobs) driven by modulational interactions of primary electrostatic fluctuations. This model, based on four-wave interaction approach, fully recovers the results based on the wave kinetic equation[2]. As it was described in Ref. [2], synergy of the interchange drive and nonlinear effects associated with drift wave turbulence may lead to generation of filamentary electromagnetic structures in edge plasmas of tokamaks. Such structures are born out at the intermediate (meso-) scales suggested by the combination of the interchange drive, stabilization due to finite ion temperature and the inverse cascade process.

References

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