Study of the spectral characteristics and the nonlinear evolution of ELMs on JET using a wavelet analysis

F.M. Poli[1], S.E. Sharapov[2] and JET-EFDA Contributors*

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

[1]Physics Department, University of Warwick, Coventry CV4 7AL, UK
[2]Euratom/UKAEA Fusion Assoc., Culham Science Centre, Abingdon OX14 3DB, UK

See Appendix of M. Watkins et al., Fusion Energy (Proc. 21st Int. Conf. Chengdu, 2006) IAEA

INTRODUCTION: The H-mode in tokamaks is characterized by the occurrence of large bursts of MHD activity at the plasma edge known as Edge Localized Modes (ELMs) causing particle and energy losses, which may result in a reduction of the energy confinement time as large as 20% [1]. On the JET tokamak, type-I and type-III ELMs are seen in the edge Mirnov pickup coils as MHD events lasting from 200 μs to 1 ms. Such a short characteristic time scales involved in the growth and collapse of ELMs make the analysis of the temporal evolution of the amplitude and mode numbers of ELMs difficult to perform with the standard Fourier analysis. A spectral analysis designed for transients is required for such short-lived perturbations. We show in the present paper that the Morlet wavelet transform is one of the best options for investigating spectral properties of ELMs, including their toroidal mode numbers n, as time resolution comparable to the acquisition time can be achieved. We also compare the mode numbers n involved in ELMs with those of ELM pre-cursors and post-cursors also often observed on JET, which have a lifetime up to hundredths of ms [2, 3], so that conventional Fourier analysis and the Morlet transform can be applied to them.

THE METHOD: The computation of the Windowed Fourier Transform (WFT) of a time series requires N points, where N is chosen in order to optimize the time-frequency resolution. The resulting power spectrum is an average of the spectral components over a time window of length $T=N\delta t$, where δt is the acquisition time (1 μs on JET) [4] and T~4 ms for typical spectrograms on JET. The time averaging that is implicit in the WFT makes therefore it difficult to detect variations in spectral quantities, such as the amplitude, the frequency and the mode number, that occur over time scales shorter than T, as in the case of ELMs. It was previously shown [5] that significant advantages in the study of the spectral properties of ELMs and of ELM precursors are introduced by the use of wavelet functions. Not only precursors with lifetime shorter than 1 ms are easily detected in the wavelet spectrum, but the time evolution of their amplitude, frequency and toroidal mode number can be followed with a time resolution comparable to the wave period. In addition, due to the lower noise level typical of the wavelet transform, the determination of *n*'s is much less affected by random phase oscillations. We report herein results of the analysis of spectral features of ELMs, ELM precursors and post-cursors by means of the Morlet wavelet, a sinusoidal function modulated by a Gaussian envelope (see [5] and references therein):

$$\psi(t) = \pi^{-1/4} e^{-t^2/\sigma^2} e^{i\omega_0 t}. \tag{1}$$

The continuous wavelet transform (CWT) of a discrete time series x_n, sampled at the rate δt^{-1}, is defined as the convolution product of x_n with $\psi_{s\tau}(t)$:

$$W_m(s) = \frac{1}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \psi^* \left(\frac{[n-m]\delta t}{s} \right), \tag{2}$$

where $\psi_{s\tau}(t)$ is constructed from (1) by shifting ($t \rightarrow t-\tau$) and dilating ($t \rightarrow t/s$). Apart from the normalization factors, the only difference between (2) and the WFT is that the windowing is intrinsic in the wavelet transform and it depends on scale *s*. An example of the magnetic spectrogram computed from (2) is shown in Fig.1(b) for JET pulse #58982, an ELMy H-mode discharge. In the time window of interest a series of ELMs occurs, with the first ELM having amplitude larger than the following ones and power spectrum extending from low frequencies (~ 1-5 kHz) up to the Nyquist frequency (500 kHz).

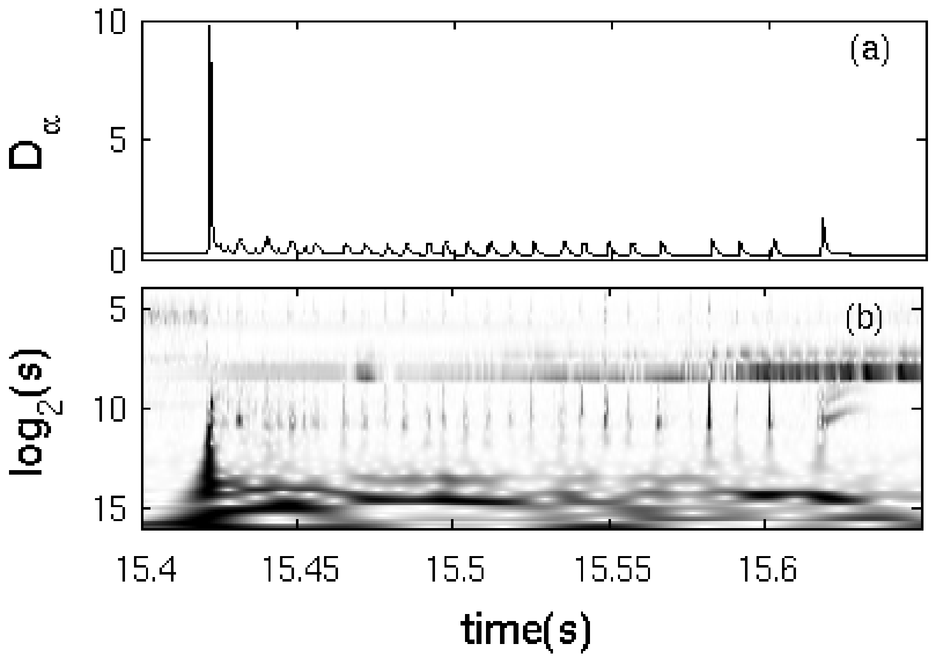

***Figure 1**. JET pulse #58982:(a) D_α emission signal; (b) Normalised wavelet coefficients, computed from the Mirnov coil data. Frequency is related to the scale* s *via* $\omega = 2\pi/(s\cdot\delta t)$.

The last ELM in the series triggers a post-cursor, dubbed Palm Tree Mode because of its shaping in frequency [3]. The frequency of the post-cursor, initially very low, increases in time, while its amplitude decreases, indicating strong damping of the mode [3]. The toroidal mode number n is extracted from the phase shift between magnetic perturbations measured by two edge Mirnov pickup coils, located at R = 3.884 m, at z =1.013 m above the equatorial plane and toroidally separated by 10.17 degrees [4]. Figure 2 shows the spectrum of n's associated with the spectral components with largest amplitude. In the case of the first ELM, the toroidal mode number evolves in time from low values of n at low frequencies to higher values of n associated with higher frequency components. We have chosen a constant level of amplitude in the normalized wavelet coefficients (Fig.1b) and extracted the time evolution of the toroidal mode number for that level of amplitude. The results are shown in Fig.3. The value of n gradually increases from 1 to 12 in a time window between 5 and 7 ms before the ELM crash then it decreases over longer time scales.

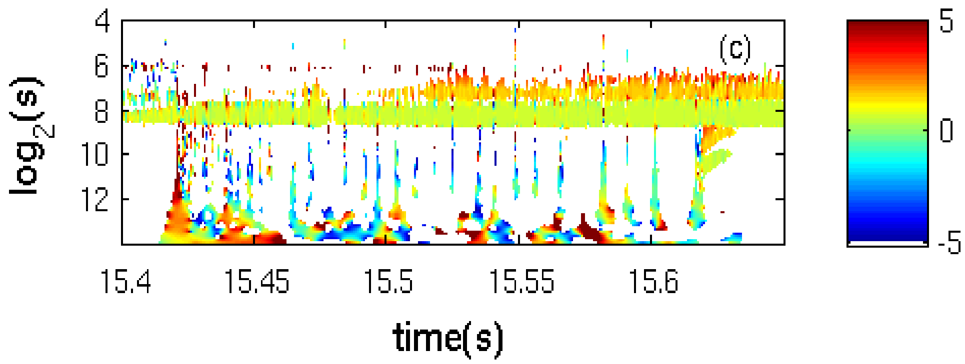

Figure 2. *Toroidal mode number n, extracted from the phase shift between magnetic perturbation measured with two edge Mirnov coils with toroidal separation of 10.17 degrees. The color scale is saturated between [-5,5], modes with n>5 are shown as deep red.*

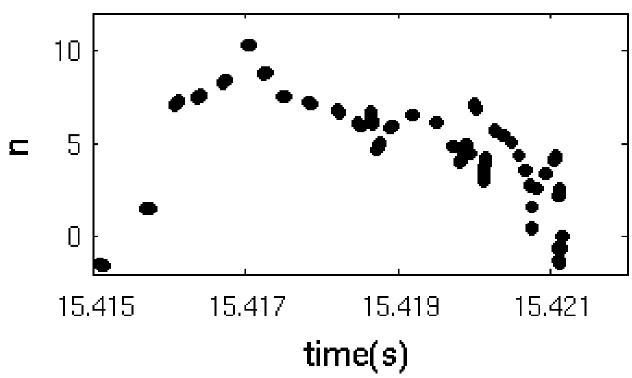

Figure 3*. Time evolution of the toroidal mode number for a constant level of power spectral amplitude extracted from of two Mirnov coils toroidally separated by 10.17 degrees.*

The bispectrum of magnetic perturbations (not shown) confirms that during the ELM evolution the total level of nonlinear three-wave interactions reaches a maximum in time and that wave coupling starts from the lowest frequency components and gradually extends to spectral components with higher frequencies.

ELMs VERSUS ELM PRE-CURSORS AND ELM POST-CURSORS: We can now compare the ELM characteristics with those of ELM pre-cursors and ELM post-cursors. The Palm Tree Mode post-cursors detected at ~15.62 sec in the power spectrum shown in Fig.2, are found to have toroidal mode numbers, n = 1,2,3, i.e. they are close to the ELM mode number. However, the pre-cursor shown in the example in Fig. 4 has toroidal mode number n= 8, distinctly different than the ELM itself with n~ 1-2.

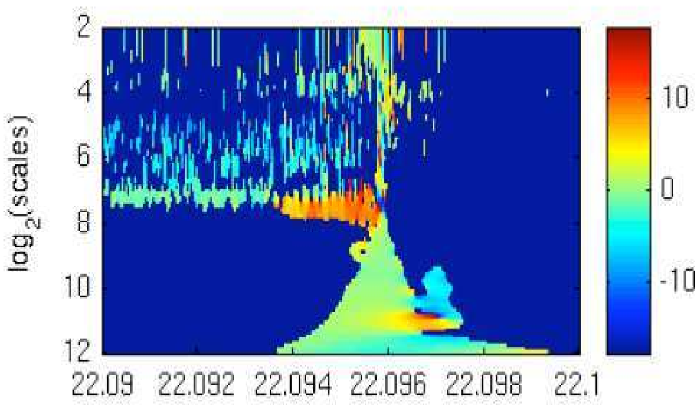

Figure 4. *Morlet spectrogram of magnetic perturbations showing the toroidal mode numbers of an ELM precursor and and ELM in JET pulse #53062 [5].*

CONCLUSIONS: The spectra of magnetic perturbations computed with the Morlet wavelet reveal that type I ELMs have a complex structure in toroidal mode numbers, with the value of n increasing from 1 up to 12 between 7 ms and 5 ms before the crash. From the wavelet spectrum of frequencies and mode number of ELMs it is possible to get information about their dynamics on time scales not accessible to standard Fourier techniques.

ACKNOWLEDGMENTS: *This work has been conducted under the European Fusion Development Agreement and funded by Euratom and the UK EPSRC. The views and opinions expressed herein do not necessarily reflect those of the European Commission.*

REFERENCES:

[1] M. Zohm, *Plasma Phys. Control. Fusion*, **38**, 105 (1996);

[2] C.P. Perez *et al*, *Nucl. Fusion*, **44**, 609 (2004);

[3] HR Koslowski *et al*, *Nucl. Fusion*, **45**, 201 (2005);

[4] R.F. Heeter, A.F. Fasoli, *et al, Rev. Sci. Instrum*, **71**, 4092 (2000);

[5] F.M. Poli, S.E. Sharapov *et al*, submitted to *Plasma Phys. Control. Fusion* (2008).