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# Tokamak plasma response to asymmetric magnetic perturbations

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## 1. Introduction

Analysis of the resistive wall mode (RWM) feedback stabilization in tokamaks requires knowledge of the plasma reaction to slowly varying external helical perturbations. Recently an accurate solution became necessary for coupling the MHD solver to a 3D eddy current solver into the CarMa code [1–3]. Similar problem has been treated in [4]. In both cases the key element was the separation of the plasma-produced magnetic field from the total field found from the equilibrium calculations. This was differently done in [1–3] and [4] through quite complicated procedures, especially in [4]. Here the easiest and direct way of such separation is shown with a general solution for the codes such as IPEC or CarMa which allows extension of the approach to other codes or analytical applications.

## 2. Formulation of the problem

The plasma equation of motion

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j}\times\mathbf{B} + \mathbf{f} , \qquad (1)$$

where $\rho$, $\mathbf{v}$, $p$, $\mathbf{j}$ and $\mathbf{f}$ are the plasma mass density, velocity, pressure, current density and the viscose force, contains the total magnetic field $\mathbf{B} = \mathbf{B}^{pl} + \mathbf{B}^{out}$. Here $\mathbf{B}^{pl}$ is the contribution from the plasma and $\mathbf{B}^{out}$ due to all outer sources. In the ideal MHD, $\mathbf{f} = 0$ which was analysed in [4]. In the CarMa code [1, 2] the nonvanishing $\mathbf{f}$ includes various kinetic effects.

Assume, as in [1–4], that equation (1) is solved with some boundary conditions and restrictions, and $\mathbf{B}$ is known in the plasma and in some narrow vacuum region outside. Then to find the external field necessary to maintain the configuration one has to separate the contributions in $\mathbf{B}$, which is a vital part in the CarMa and IPEC codes [1–4].

The separation could be done by calculating directly the magnetic field produced by the currents in the plasma:

$$\mathbf{B}^{pl} = \frac{\mu_0}{4\pi}\nabla\times \int\limits_{plasma} \frac{\mathbf{j}(\mathbf{r}_{pl})}{|\mathbf{r}-\mathbf{r}_{pl}|} dV_{pl} = \frac{\mu_0}{4\pi} \int\limits_{plasma} \mathbf{j}(\mathbf{r}_{pl})\times \frac{\mathbf{r}-\mathbf{r}_{pl}}{|\mathbf{r}-\mathbf{r}_{pl}|^3} dV_{pl} . \qquad (2)$$

The integration here is performed over the plasma with the current density to be found from $\mu_0\mathbf{j} = \nabla\times\mathbf{B}$ with $\mathbf{B}$ a solution of (1) at the presence of a given stationary perturbation.

On the other hand, the magnetic perturbation $\mathbf{b} = \nabla\varphi$ in the plasma-wall vacuum gap can be found by solving the Laplace equation $\nabla^2\varphi = 0$ with proper boundary conditions depending on the solution inside the plasma. The latter strategy is accepted in both the IPEC and CarMa codes. The algorithms there include solving of the equation $\nabla^2\varphi = 0$ inside and outside some "control" surface $S$ around the plasma with given $\mathbf{n}\cdot\nabla\varphi = \mathbf{n}\cdot\mathbf{b}$ at $S$, introduction of a surface current at $S$ and calculation of the magnetic field produced by this current.

The latter scheme looks straightforward, but its practical realization [1–4] turns into large additional block of numerical calculations. Instead, the magnetic fields from the inner and outer sources can be directly and explicitly separated by means of (2) and proper transformation of the volume integral to the surface one, which is shown below.

**3. Solution of the problem**

Using the method described in [5], but without the constraint $\mathbf{n}\cdot\mathbf{B} = 0$ we obtain the equality

$$\mu_0\int_V (\mathbf{j}\times\nabla f)dV = \int_S \{(\mathbf{n}\times\mathbf{B})\times\nabla f + \nabla f(\mathbf{n}\cdot\mathbf{B})\}dS - \int_V \mathbf{B}\nabla^2 f dV\,, \tag{3}$$

where $\mathbf{n}$ is the unit outward normal to $S$. With

$$f = \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{4}$$

which satisfies $\nabla^2 f(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$, and (2) this gives, when integrated over the plasma,

$$\mathbf{B}^{pl}(\mathbf{r}) = \frac{1}{4\pi}\int_S \left\{(\mathbf{n}\times\mathbf{B})\times\frac{\mathbf{r}-\mathbf{r}_s}{|\mathbf{r}-\mathbf{r}_s|^3} + (\mathbf{n}\cdot\mathbf{B})\frac{\mathbf{r}-\mathbf{r}_s}{|\mathbf{r}-\mathbf{r}_s|^3}\right\}dS + \nu\mathbf{B}(\mathbf{r})\,, \tag{5}$$

with $\nu = 1$ inside, 0 outside and 0.5 at the boundary $S$. The integration here can be extended to a larger volume (say, bounded by some axisymmetric surface in the vacuum) because a vacuum region with $\mathbf{j} = 0$ does not contribute to $\mathbf{B}^{pl}$.

Equation (5) gives, in an explicit form, a necessary relation between the plasma-produced magnetic field $\mathbf{B}^{pl}$ and the total field $\mathbf{B}$, the latter at the surface $S$ which may be a plasma boundary or an arbitrary toroidal surface enclosing the plasma. Here the field $\mathbf{B}$ at $S$ must be considered as known: the procedures described in [1–4] include the solvers providing $\mathbf{B}$ for given $\mathbf{n}\cdot\mathbf{B}$ at the surface $S$ and fixed plasma parameters.

In a general case, $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, where $\mathbf{B}_0$ is the unperturbed (or equilibrium) magnetic field and $\mathbf{b}$ the perturbation, and similarly $\mathbf{B}^{pl} = \mathbf{B}_0^{pl} + \mathbf{b}^{pl}$. Since (5) is a linear form with respect to both $\mathbf{B}$ and $\mathbf{B}^{pl}$, it can be split on the 'equilibrium' part and that with perturbations. In other words, in (5) we can replace $\mathbf{B}$ and $\mathbf{B}^{pl}$ by $\mathbf{b}$ and $\mathbf{b}^{pl}$, respectively, and obtain $\mathbf{b}^{pl}$ as a function of $\mathbf{b}$ given at the surface $S$. This dependence is exactly what is needed in the CarMa and IPEC codes.

Equation (5) obtained by integrating $\mu_0 \mathbf{j} = \nabla \times \mathbf{B}$ with account of $\nabla \cdot \mathbf{B} = 0$ is a purely electromagnetic relation that can be used with any plasma model. It does not even require the plasma to be in equilibrium. Note that in [1–4] the plasma was considered as a system with linear response to external perturbations. Equation (5) is not restricted by this assumption and can be used, as well, with nonlinear plasma models.

**4. Applications**

We need an expression for the plasma magnetic response to the externally applied field. The latter (in terms of perturbations) is, obviously, $\mathbf{b}^{out} = \mathbf{b} - \mathbf{b}^{pl}$. Assume that the plasma response is linear, which is a natural first-step approximation used in numerical and analytical RWM and related studies. Then, calculating the perturbed equilibria for fixed plasma parameters (and, therefore, fixed $\mathbf{B}_0$) and a number of perturbations $\mathbf{b}^{out}$ (the typical “modes” with given amplitudes) one can use the obtained ratios (or matrices) $\mathbf{n} \cdot \mathbf{b} / \mathbf{n} \cdot \mathbf{b}^{out}$ as proportionality coefficients for predicting the response in a general case. This is the approach accepted in [1–4].

Briefly, a dependence of $\mathbf{n} \cdot \mathbf{b}^{out}$ on $\mathbf{n} \cdot \mathbf{b}$ at $S$ is needed, which is obtained numerically by some procedure in [1–4]. Here (5) gives us an explicit solution for $\mathbf{b}^{out}$ through $\mathbf{b}$:

$$\mathbf{b}^{out} = \mathbf{b} - \mathbf{b}^{pl} = \mathbf{b}(1-\nu) - \frac{1}{4\pi}\int_S \left\{ (\mathbf{n} \times \mathbf{b}) \times \frac{\mathbf{r} - \mathbf{r}_s}{|\mathbf{r} - \mathbf{r}_s|^3} + (\mathbf{n} \cdot \mathbf{b}) \frac{\mathbf{r} - \mathbf{r}_s}{|\mathbf{r} - \mathbf{r}_s|^3} \right\} dS \,. \tag{6}$$

This completely solves the separation problem addressed in [1–4]: with known $\mathbf{b}$ at $S$ we get $\mathbf{b}^{out}$ by the direct integration in (6).

Equation (5) also allows to eliminate an excessive step in calculations [2] and [3]. Assume that there is no currents inside $S$, while $\mathbf{n} \cdot \mathbf{b}$ at $S$ is the same as that with a plasma. In other words, consider a vacuum field $\mathbf{b}_v$ (with sources somewhere behind $S$) with $\mathbf{n} \cdot \mathbf{b}_v = \mathbf{n} \cdot \mathbf{b}$. Then equation (5) yields

$$0 = \frac{1}{4\pi}\int_S \left\{ (\mathbf{n} \times \mathbf{b}_v) \times \frac{\mathbf{r} - \mathbf{r}_s}{|\mathbf{r} - \mathbf{r}_s|^3} + (\mathbf{n} \cdot \mathbf{b}_v) \frac{\mathbf{r} - \mathbf{r}_s}{|\mathbf{r} - \mathbf{r}_s|^3} \right\} dS + \nu \mathbf{b}_v \,, \tag{7}$$

and after subtracting this from the original equation (5), with $\mathbf{B}$ replaced by $\mathbf{b}$, we obtain

$$\mathbf{b}^{pl}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_S \mathbf{i} \times \frac{\mathbf{r} - \mathbf{r}_s}{|\mathbf{r} - \mathbf{r}_s|^3} dS + \nu(\mathbf{b} - \mathbf{b}_v) \,, \tag{8}$$

where

$$\mu_0 \mathbf{i} = \mathbf{n} \times (\mathbf{b} - \mathbf{b}_v) . \tag{9}$$

This shows, in particular, that the surface current $\mathbf{i}$ on $S$ would provide the same magnetic perturbation outside $S$ ($\nu = 0$ there) as the plasma does, for given $\mathbf{n} \cdot \mathbf{b}$ at $S$. This is exactly the key result of [2] expressed there by Eq. (10) with the related comments.

To use (8), one has to know $\mathbf{b}$ and $\mathbf{b}_v$ at $S$. The field $\mathbf{b}$ has to be found with plasma, and $\mathbf{b}_v$ without plasma. In both cases $\mathbf{n} \cdot \mathbf{b} = \mathbf{n} \cdot \mathbf{b}_v$ at $S$ is given. Calculation of $\mathbf{b}_v$ is an excessive step since equation (5) gives $\mathbf{b}^{pl}$ when only $\mathbf{b}$ is known. Note that equation (8) can be also considered as the necessary proof of the CarMa coupling strategy described in [1–3].

## 5. Conclusions

When $\mathbf{b}$ is known at the plasma boundary or some surface $S$ outside the plasma and we have to find $\mathbf{b}^{out}$ or $\mathbf{b}^{pl} = \mathbf{b} - \mathbf{b}^{out}$, there is no need in calculation of the artificial currents, such as described in [1–4, 6], because equation (6) gives us the relation between $\mathbf{b}^{out}$ and $\mathbf{b}$. Being based on the first principles, the equations here are universal and applicable to any plasma model and toroidal geometry. These equations give, in particular, a complete solution of the 'interface' problem addressed in [4]. The regular method described above is much more simple and clear than the multi-step procedure with a long chain of derivations in [4]. It is also more general since neither axial symmetry, nor special geometry of $S$ is assumed here. We operate with real physical quantities, no fictitious sources and artificial jumps at $S$, contrary to [4].

Equations (8) and (9) confirm that the representation of the plasma by superficial current density on $S$ in the CarMa procedure [1–3] (see Eq. (10) in [2] and Eqs. (6) and (7) in [3]) is correct. However, the result can be obtained in a more simple way which is shown here by the explicit solutions (5) and (6) for $\mathbf{b}^{pl}$ and $\mathbf{b}^{out}$ as functions of known $\mathbf{b}$ on $S$.

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