Cherenkov emission of electron cyclotron waves
by a magnetized satellite orbiting the ionosphere

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Introduction

The space can be a good laboratory for boundary-free plasma physics experiments. The big variety of probes sent in the last 50 years into the space has given us a good knowledge of the earth ionosphere. Nevertheless, this knowledge is far to be complete.

In this work, we study the Cherenkov emission of plasma waves by a magnetized satellite orbiting the ionosphere. The idea is that of an *active* experiment, conceptually different from most of the space plasma physics experiments performed so far. In a traditional ionospheric measurement experiment, a satellite acts as a probe in an environment which has to be studied being perturbed as little as possible. In an active experiment, on the other hand, the effort is that of making the satellite to interact as much as possible with the environment, to get informations about the satellite-ionosphere system, and indirectly about the ionosphere itself. With this aim, the Active Magnetic Experiment (AcME) project was born [1], which consists in a orbiting satellite, with a current loop inside, generating a dipole magnetic field. This magnetic bubble can interact with the ionosphere much more efficiently than an un-magnetized one, and provide novel phenomena to be studied, especially Hall current dominated effects and EMHD phenomena, that cannot be realized in a laboratory.

The clearest sign of interaction between a magnetized satellite and the ionosphere is the generation of waves. Like a boat crossing the sea, a magnetized body traveling through the magnetized ionospheric plasma generates waves via Cherenkov effect, with a Mach cone whose width depends on the ratio between the waves phase velocity and the body speed. Due to the dimensions and the keplerian speed of a typical satellite, the frequency range is located between the ion and electron cyclotron frequency. Such an infra-thermal range will be shown to give rise to the excitations of wave of the whistlers type, close to the electron-cyclotron resonance.

The scheme of the paper is the following: firstly, we recall the study of the orbit heights range [1] in order to maximize the collective response of the ionospheric plasma to the passage of the satellite and we study the dispersion relation of a plasma with the characteristics of the chosen heights range, imposing the Cherenkov condition and consequently providing the Mach cone structure; secondly, we study the differential power emitted in waves by the satellite, as a

function of the emission direction, and integrating we find the total radiated power.

Orbits characteristics, dispersion relation and Cherenkov emission

In order to have collective phenomena, the orbit range has been chosen to be $500\,km < h < 2000\,km$, where h is the height above the earth surface [1]. This is because at those heights, the Debye length is smaller than the satellite size (typically 1 m), the neutral atmospheric concentration is not affecting relevantly the plasma phenomena and both ions and electrons are magnetized (they perform tens of gyro-orbits before colliding). The reference plasma parameters at those heights are the following: the ion Larmor radius is $\rho_i \simeq 6\,m$, the electron Larmor radius is $\rho_e \simeq 10\,cm$, the ion cyclotron frequency is $\Omega_i \simeq 2.1 \cdot 10^2\,rad/s$, the electron cyclotron frequency is $\Omega_e \simeq 6.1 \cdot 10^6\,rad/s$ and the plasma frequency is $\omega_p \simeq 4 \cdot 10^7\,rad/s$. These are the ranges for standard EMHD treatment. The dimensionless plasma parameter is $g = (n\lambda_D^3)^{-1} \sim 10^{-8}$, where n is the plasma density, of the order of $n \sim 10^5\,cm^{-3}$ at the chosen heights. Such a small g parameter means that the plasma phenomena are collective phenomena, rather than particle phenomena.

The transit time τ of a satellite with a size of 1 m, orbiting the ionosphere at the chosen range of heights is calculated considering that the keplerian velocity is $v = 7.8\,km/s$. We can focus on the range of frequency around $\omega_\tau = 2\pi/\tau = 2\pi v/L \simeq 5 \cdot 10^4\,rad/s$, that lies between Ω_i and Ω_e. Frequen-

Figure 1: Mach cone for $\omega/\Omega_i = 200$.

cies higher than the ion-cyclotron one allow us to assume a model with static ions. the physics of this system therefore can be described considering free electrons moving in a uniformly charged background. the convenient dispersion relation is the Appleton-Hartree one [2]:

$$n^2 = 1 - \frac{2\alpha\omega^2(1-\alpha)}{2\omega^2(1-\alpha) - \Omega_e^2 \sin^2\theta \pm \Omega_e\Delta} \tag{1}$$

where

$$\Delta = \left(\Omega_e^2 \sin^4\theta + 4\omega^2(1-\alpha)^2 \cos^2\theta \right)^{1/2} \tag{2}$$

and $n = kc/\omega$ is the refraction index, $\alpha = \omega_p^2/\omega^2$ and θ is the angle between $\boldsymbol{k}$ and $\boldsymbol{B_0}$, the background magnetic field (with intensity $B_0 = 0.2G$ at those heights). The approxi-
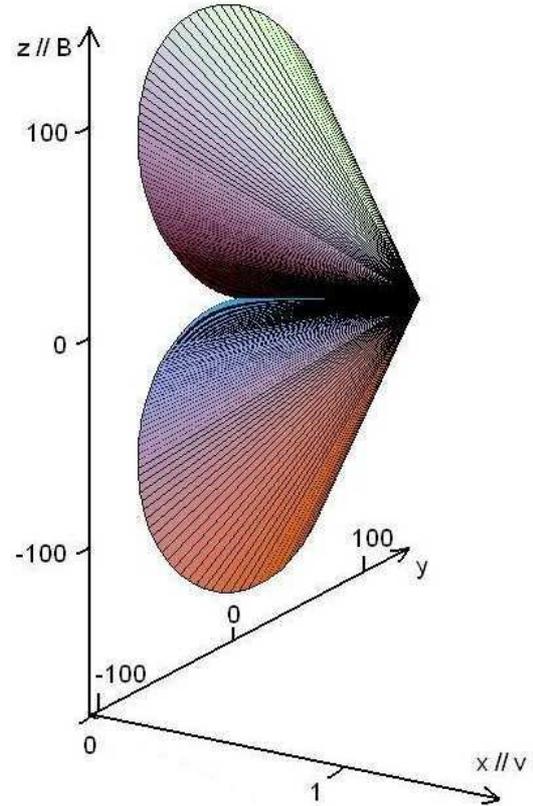
mation $\Omega_e^2 \sin^4 \theta \ll 4\omega^2 (1-\alpha)^2 \cos^2 \theta$ can be made if we neglect the small angle range $|\theta - 90| < 0.006$. Assuming also $\Omega_e^2 \sin^2 \theta \ll 2\omega^2 |(1-\alpha)|$, valid in the same range, we get two solutions. One is evanescent at the considered frequencies, the other one is the whistler branch, which takes the form

$$n^2(\omega, \theta) \simeq 1 - \frac{\alpha\omega}{\omega + \Omega_e |\cos \theta|} \tag{3}$$

The Cherenkov condition, describing the waves emission from a body moving with a speed v in a medium, reads $v_{ph} = \omega/k = v \sin \gamma$, where γ is the angle between $\boldsymbol{k}$ and $\boldsymbol{v}$. If we choose for simplicity $\boldsymbol{v} \perp \boldsymbol{B_0}$ and use spherical angle coordinates (θ, ϕ), where ϕ is the angle with $\boldsymbol{v}$, measured in the plane perpendicular to $\boldsymbol{B_0}$, we can write the Cherenkov condition in the form:

$$n^2(\theta, \phi) = \frac{c^2}{v^2 \sin^2 \theta \cos^2 \phi} \tag{4}$$

Suing the dispersion relation and the Cherenkov condition, we can calculate the Mach cone section equation in an orthogonal coordinate system (x, y, z) with $x \parallel \boldsymbol{v}$ and $z \parallel \boldsymbol{B_0}$:

$$y(\phi) = \frac{1}{2} \frac{\sin^2 \theta(\phi) \, \sin 2\phi}{1 - \sin^2 \theta(\phi) \, \cos^2 \phi} \tag{5}$$

$$z(\phi) = \frac{1}{2} \frac{\sin 2\theta(\phi) \, \cos \phi}{1 - \sin^2 \theta(\phi) \, \cos^2 \phi} \tag{6}$$

Power emitted in plasma waves

The power radiated by the satellite can be calculated in the reference frame of the environment plasma, with the satellite current acting as external current $\boldsymbol{J}^{ext}$ moving with velocity $\boldsymbol{v}$. We mainly follow the conceptual scheme of reference [3], which faces the problem of a long tether orbiting the ionosphere. Considering a plasma with dielectric tensor

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \tag{7}$$

the electric field $\boldsymbol{E}$ generated by the satellite current J^{ext} and by the currents induced in the plasma, is given by

$$\Lambda_{tj}(\mathbf{k}, \omega) E_j(\mathbf{k}, \omega) = -\frac{4\pi i}{\omega} J_t^{ext}(\mathbf{k}, \omega) \tag{8}$$

where

$$\Lambda_{tj}(\mathbf{k}, \omega) = n^2 \left(\frac{k_t k_j}{k^2} - \delta_{tj} \right) + \varepsilon_{tj}(\mathbf{k}, \omega) \tag{9}$$

δ_{tj} being the Kronecker delta (with value 1 if $t = j$ and 0 in the other cases). The general solution of Eq. (8) is:

$$E_t(\mathbf{k}, \omega) = -\frac{4\pi i}{\omega} \left\{ \mathscr{P}[\frac{1}{\Lambda(\mathbf{k}, \omega)}] - \pi i \delta[\Lambda(\mathbf{k}, \omega)] \right\} \lambda_{tj}(\mathbf{k}, \omega) J_j^{ext}(\mathbf{k}, \omega) \tag{10}$$

where $\mathscr{P}$ is the Cauchy principal value, Λ is the determinant of Λ_{tj} and λ_{tj} is the *cofactor* matrix, defined by $\Lambda_{tj}\lambda_{jl} = \Lambda\delta_{tl}$.

Known $\boldsymbol{E}$ and J^{ext}, we can calculate the radiated power with the Ohm's law:

$$P = -\lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\Re[J_j^{ext}(\mathbf{k},\omega)E_j^*(\mathbf{k},\omega)] \tag{11}$$

We use for simplicity a current distributed in a square circuit in the plane perpendicular to $\boldsymbol{B_0}$ and calculate the normalized differential radiation resistance $\hat{r}$ defined by

$$P = I_0^2 r_0 \int_0^{2\pi} d\theta \int_{-\pi/2}^{\pi/2} d\phi \ \hat{r}(\theta,\phi) \tag{12}$$

The solution is

$$\hat{r}(\theta,\phi) = \frac{G_\sigma H_{L,2}}{\left| \left(\frac{v}{c}\right)\frac{\partial n_\sigma}{\partial\xi}\Big|_{\xi=\xi_{\sigma,c}} \xi_{\sigma,c} \right| F} \tag{13}$$

where

$$G_\sigma(n_\sigma^2(\omega,\theta);\omega,\theta) = |\varepsilon_1\varepsilon_3 - n_\sigma^2\varepsilon_1 sen^2\theta - n_\sigma^2\varepsilon_3 cos^2\theta|$$

$$F(\omega,\theta) = \left|\frac{\partial\Lambda}{\partial n^2}\right| = \sqrt{B^2 - 4AC}$$

$$H_{L,2}(n_\sigma^2(\omega,\theta);\omega,\theta,\phi) = \frac{2\sin^2\left(\frac{n_\sigma\omega}{c}\frac{L}{2}\sin\theta\sin\phi\right)\left(1-\cos(\frac{n_\sigma\omega}{c}L\sin\theta\cos\phi)\right)}{n_\sigma|\sin^3\theta||\cos^3\phi|\sin^2\phi}$$

The main contribution to the total radiation power is given by the angle range close to the direction of $\boldsymbol{v}$ and wide about 2. The peak frequency corresponds to $\omega \simeq 200\Omega_i$, like guessed by the transit time calculation. The total radiation resistance is $r \simeq 0.6 \cdot 10^{-7} Ohm$.

In order to calculate the total power we estimate the current in the satellite assuming a solenoid with $N = 10^4$ loops and $I_0 = 10A$. We get

$$P_{rad} = r I_{tot}^2 = r n^2 I_0^2 \simeq 600\,W \tag{14}$$

which is up to two order of magnitude greater than the power emitted by typical telecommunications satellites. The radiation emitted by the magnetized satellite could be revealed by a diagnostic system composed by several sub-satellites or by ground-based detectors.

References

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