Non-fluid Micro-Reconnecting Modes and Experimental Observations*

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Introduction and Motivation

In this paper we study the properties of the microscopic electromagnetic instability driven by the electron temperature gradient in the presence of magnetic shear which was previously derived [1] and hereafter referred to as Micro-Reconnecting Mode (MRM). The mode has a natural transverse (to the magnetic field) scale distance of the order of $d_e = c/\omega_{pe}$ and is not radially localized by finite electron gyroradius effects. The mode produces adjacent strings [1, 2] of magnetic islands, which can significantly alter the effective thermal diffusion coefficients. A proper treatment of the mode requires a phase space (drift-kinetic) description for the electron population dynamics. The mode is charaterized by $\omega \sim k_\parallel \mathrm{v}_{te}$, ω being the mode complex frequency that is of the order of $k_\perp c T_e/(eBr_{Te})$, and $1/r_{Te} \equiv -(\mathrm{d}\log T_e/\mathrm{d}r)$. The implied ordering, $\beta_e \sim 2r_{Te}^2/L_s^2$ where β_e, the ratio of electron thermal energy density to the magnetic energy density, is regarded as relevant to current experiments such as those carried out by the NSTX device where modes with transverse scale distances of the order of d_e have been identified [4].

The MRM does not produce an appreciable particle transport while the relevant effective perpendicular thermal diffusion coefficient $D_{e\perp}^{th}$ is estimated to be of the order of $(d_e/r_{Te})cT_e/(eB)$. A significant density gradient can depress the MRM considerably, while the trapped electron mode does not require a temperature gradient [3]. Ultimately, both modes should be envisioned. As explained in Reference [1] the MRM can also significantly reduce the effective $D_{e\parallel}^{th}$ and allow the possibility to excite mesoscopic drift-tearing modes [1, 2], which depends critically on the ratio $D_{e\perp}^{th}/D_{e\parallel}^{th}$. In fact, if this ratio were determined by the well known collisional theory, drift-tearing modes would not be observed in existing high temperature experiments.

Theoretical Formulation

In this section we give a brief outline of the derivation of the MRM dispersion equation. We consider a sheared slab equilibrium, $\vec{B} = B_0\,[\vec{e}_z + x/L_s\vec{e}_y]$, where $L_s > 0$ quantifies the magnetic shear. For perpendicular scale-lengths greater than the electron gyroradius ($k_\perp \rho_e < 1$) the electrons are well described by the drift-kinetic equation with the guiding center drift velocity taken to be the $\vec{E}\times\vec{B}$ velocity. The perturbed electromagnetic fields are represented

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with potentials using the standard electromagnetic approximation, *i.e.* $\vec{B} \simeq \vec{\nabla}\hat{A}_{\parallel} \times \vec{B}/B$ and $\vec{E} \simeq -\vec{\nabla}\hat{\Phi} - (1/c)(\partial\hat{A}_{\parallel}/\partial t)\vec{B}/B$. Upon linearization, neglecting derivatives of the equilibrium magnetic field compared to derivatives of perturbed quantities, and representing perturbed quantities using Fourier-Laplace transforms as $\hat{\Phi} = \tilde{\Phi}(x)\exp(ik_y y + ik_z z - i\omega t)$ the drift-kinetic equation can be written as

$$\left[\omega - k_{\parallel}v_{\parallel}\right]\tilde{f}_e \simeq k_y\left[v_{\parallel}\tilde{A}_{\parallel} - c\tilde{\Phi}\right]\frac{1}{B}\frac{\partial f_e}{\partial x} - \frac{e}{mc}\left[ck_{\parallel}\tilde{\Phi} - \omega\tilde{A}_{\parallel}\right]\frac{\partial f_e}{\partial v_{\parallel}} \tag{1}$$

For the ions we consider an "adiabatic" response, valid in the limit $k_{\perp}\rho_i > 1$, such that $\tilde{f}_i \simeq -(e\tilde{\Phi}/T_i)f_i$. Considering the electrons and ions to have Maxwellian equilibrium distribution functions with densities and temperatures depending on x, and integrating over the perpendicular velocity coordinates, the quasi-neutrality condition becomes

$$\left[1 + \frac{T_e}{T_i}\right]\tilde{\Phi} \simeq \left[\frac{m_e}{2\pi T_e}\right]^{1/2}\int_{-\infty}^{\infty}\mathrm{d}v_{\parallel}\, e^{-\frac{m_e v_{\parallel}^2}{2T_e}}\left[\frac{\omega_{*e} + \omega_{*Te}(\frac{m_e v_{\parallel}^2}{2T_e} - \frac{1}{2}) - \omega}{\omega - k_{\parallel}v_{\parallel}}\right]\left[\frac{v_{\parallel}}{c}\tilde{A}_{\parallel} - \tilde{\Phi}\right] \tag{2}$$

and similarly the parallel component of Ampére's law

$$\left[d_e^2\frac{\mathrm{d}^2}{\mathrm{d}x^2} - k_y^2 d_e^2\right]\tilde{A}_{\parallel} \simeq$$
$$-2\left[\frac{m_e}{2T_e}\right]^{3/2}\frac{c}{\sqrt{\pi}}\int_{-\infty}^{\infty}\mathrm{d}v_{\parallel}\, v_{\parallel}e^{-\frac{m_e v_{\parallel}^2}{2T_e}}\left[\frac{\omega_{*e} + \omega_{*Te}(\frac{m_e v_{\parallel}^2}{2T_e} - \frac{1}{2}) - \omega}{\omega - k_{\parallel}v_{\parallel}}\right]\left[\frac{v_{\parallel}}{c}\tilde{A}_{\parallel} - \tilde{\Phi}\right] \tag{3}$$

where $\omega_{*e} \equiv (ck_yT_e)/(eB)\mathrm{d}\log(n)/\mathrm{d}x$ and $\omega_{*Te} \equiv (ck_yT_e)(eB)\mathrm{d}\log(T_e)/\mathrm{d}x$ are the diamagnetic drift frequencies. From these equations the following dispersion equation may be derived:

$$\left[\omega^3\left[1 + \frac{T_i}{T_e}\frac{\omega_{*e}}{\omega}\right] + \frac{1}{2}k_{\parallel}^2 c_{se}^2\Omega_*\right]\left[d_e^2\frac{\mathrm{d}^2}{\mathrm{d}x^2} - k_y^2 d_e^2\right]\tilde{A}_{\parallel} \simeq -\omega^2\Omega_*\left[1 + \frac{T_i}{T_e}\frac{\omega_{*e}}{\omega}\right]\tilde{A}_{\parallel} \tag{4}$$

where $c_{se} = (2T_i/m_e)^{1/2}$ is the electron sound velocity, and we have introduced the following definition,

$$\Omega_* \equiv \frac{\omega^2}{\sqrt{\pi}}\int_{-\infty}^{\infty}\mathrm{d}u\, 2u^2 e^{-u^2}\left[\frac{\omega_{*e} + \omega_{*Te}(u^2 - \frac{1}{2}) - \omega}{\omega^2 - k_{\parallel}^2 v_{te}^2 u^2}\right] \tag{5}$$

where $v_{te} = (2T_e/m_e)^{1/2}$ is the electron thermal velocity. In the "fluid" limit explored previously [2], which involves $|\omega| > |k_{\parallel}v_{te}|$, $(k_{\parallel}^2 v_{te}^2/\omega^2)(\omega_{*Te}/\omega) \sim (T_e/T_i) \sim (\omega_{*e}/\omega)$ and $k_{\perp}^2 d_e^2 \sim (\omega_{*Te}/\omega)$ the integral operator reduces to $\Omega_* \simeq \omega_{*Te}$ in Eq. (4), and a quadratic form may be derived which yields a cubic dispersion equation from which numerical and analytical computations confirm the results of the quadratic form. In the present kinetic case, no simple and useful quadratic form may be derived and a more detailed analysis is required.

Analysis of Micro-Reconnecting Modes

Here we consider the dominant spatial dependence to come from $k_\parallel \simeq k_y x/L_s$ and introduce the following dimensionless variables and parameters,

$$D_*^2 \equiv \frac{k_y v_{te}}{\omega_{*Te}}\frac{d_e}{L_s} \qquad \bar{x}^2 \equiv \frac{x^2}{d_e^2}\frac{1}{D_*^2} \qquad \bar{\omega} \equiv \frac{\omega}{\omega_{*Te}}\frac{1}{D_*^2} \qquad k_0^2 \equiv k_y^2 d_e^2 D_* \qquad \alpha \equiv \frac{\omega_{*e}}{\omega_{*Te}}\frac{1}{D_*^2} \qquad \tau \equiv \frac{T_i}{T_e}$$

Such that the dispersion equation becomes

$$\left\{\bar{\omega}^3 + \tau\alpha\bar{\omega}^2 + \frac{\tau}{2}\bar{x}^2\left[(\alpha-\bar{\omega})D_*\mathscr{F}_0^0 + \mathscr{G}_0^0\right]\right\}\left\{\frac{\mathrm{d}^2}{\mathrm{d}\bar{x}^2} - k_0^2\right\}\tilde{A}_\parallel \simeq \\ -\left[(\alpha-\bar{\omega})D_*\mathscr{F}_0^0 + \mathscr{G}_0^0\right]\bar{\omega}^2\left[1 + \frac{\tau\alpha}{\bar{\omega}}\right] \quad (6)$$

where $\mathscr{F}_0^0$ and $\mathscr{G}_0^0$ are functions of $\lambda \equiv \bar{\omega}/(D_*\bar{x})$ and may be numerically evaluated using the plasma dispersion functions through the relations $\mathscr{F}_0^0 = -2\lambda^2[1+\lambda Z(\lambda)]$ and $\mathscr{G}_0^0 = -\lambda^2[1+2\lambda^2+2\lambda^3 Z(\lambda)] - 1/2\mathscr{F}_0^0$. We solve Eq. (6) by the shooting method. We integrate numerically from $\bar{x}=0$ using either the odd ($\tilde{A}_\parallel = 0$, $\mathrm{d}\tilde{A}_\parallel/\mathrm{d}\bar{x} = 1$) or even ($\tilde{A}_\parallel = 1$, $\mathrm{d}\tilde{A}_\parallel/\mathrm{d}\bar{x} = 0$) boundary conditions to large values of $\bar{x}$ until the numerical solution has reached the asymptotic solution $\tilde{A}_\parallel \sim C_1\exp(-\bar{x}) + C_2\exp(\bar{x})$ where the constant C_2 is then determined. By using a complex Newton's method, we determine the value of $\bar{\omega}$ which minimizes C_2.

In Figure 1 the numerically determined unstable complex eigenmode frequencies are plotted in the complex plane as a function of the parameter k_0 for $\alpha = 0$ (*i.e.* negligible density gradient), $\tau = 1$ and different values of D_*. For small D_* the results agree well with the previously reported solutions in the "fluid" limit. In figure 2 an example unstable eigenfunction is plotted as a function of the radial coordinate $\bar{x}$ where it is evident that the mode is primarily electromagnetic for $v_{te}/c < 1$ and that the scale distance in the "radial" direction is of the order of the electron collisionless skin depth (d_e).

References

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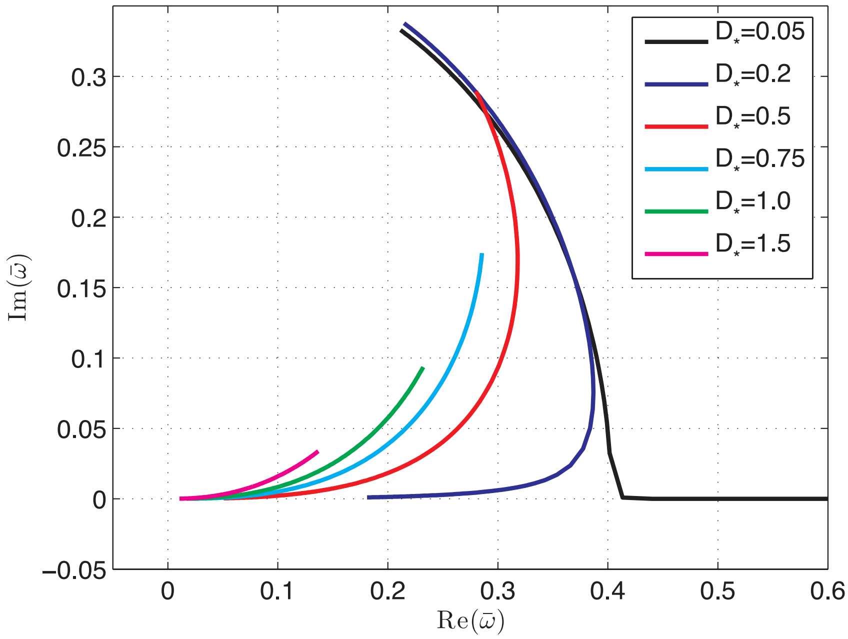

Figure 1: Numerically determined unstable complex eigenmode frequencies as a function of k_0 for $\alpha = 0$, $\tau = 1$ and different values of D_* and even parity *i.e.* $\mathrm{d}\tilde{A}_{\|}(\bar{x} = 0)/\mathrm{d}\bar{x} = 0$.

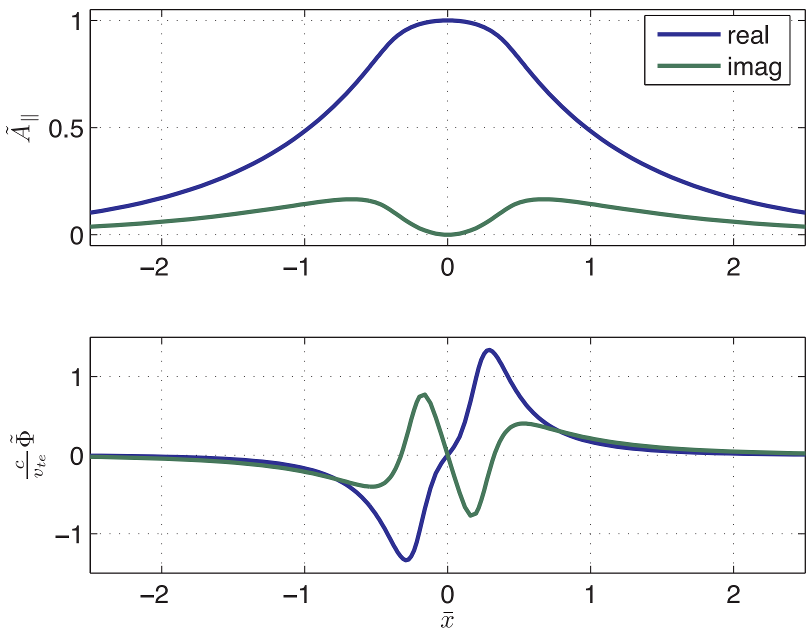

Figure 2: Numerically determined unstable eigenfunction (top) and corresponding electrostatic potential $(c/v_{te})\tilde{\Phi}$ (bottom) for $D_* = 0.5$, $\alpha = 0$, $k_0 \simeq 1$, $\tau = 1$, and $\bar{\omega} \simeq 0.31 + 0.23i$.