Statistical properties of edge plasma turbulence in the Mega-Amp Spherical Tokamak and the Large Helical Device

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Introduction. The extent to which universality is displayed by edge plasma turbulence in toroidal magnetically confined plasmas is an important but unresolved issue [1, 2]. The identification of universal features requires the comparison of turbulence properties for a range of operational regimes in a range of confinement systems. Here, we focus on the statistical properties of measurements of ion saturation current I_{sat} made by Langmuir probes in the edge region of the Mega-Amp Spherical Tokamak (MAST) and the Large Helical Device (LHD) [3, 4, 5]. We utilise modern techniques for the statistical analysis of nonlinear time series [6].

Data sets. Typical MAST and LHD I_{sat} time series are shown in figure 1. Some LHD time series contain a few low frequency coherent modes which are filtered out [7]. Table 1 gives a summary of the properties of MAST and LHD and the data sets studied. All time series studied are bursty and intermittent and contain structures of many temporal scales. MAST time series are dominated by large positive intermittent bursts. For LHD, tip 16 is dominated by positive intermittent bursts, tip 17 is dominated by negative intermittent bursts and the time series for tip 18 is almost symmetric.
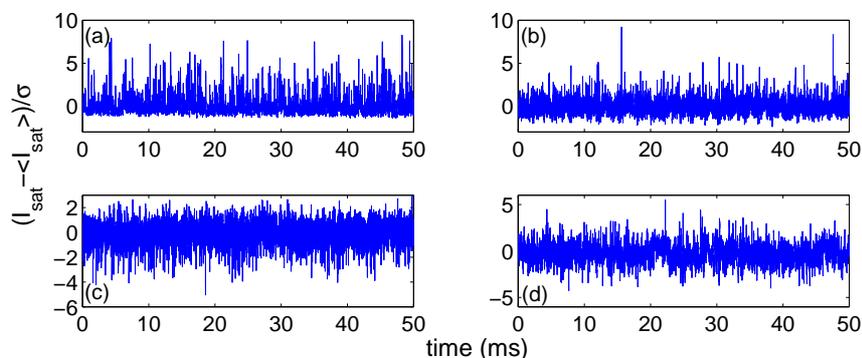

Figure 1: Typical time series: (a) MAST (b) LHD tip 16 (c) LHD tip 17 (d) LHD tip 18.

Scaling properties. We treat I_{sat} fluctuations as stochastic increments on a temporal scale τ_{min}, the time between consecutive measurements. Fluctuations on longer time scales are ob-

|	MAST	LHD
Device type	Spherical tokamak	Heliotron-type stellarator
Major and radius, R / a	0.85m / 0.65m	3.9m / 0.65m
Typical B field strength	0.5T	2.5T
Probe type	Reciprocating probe, outboard midplane	3 pins, 6mm apart in divertor plate
Sampling frequency	500kHz	250kHz
Typical time series length	50ms / 25,000 samples	1s / 250,000 samples
Discharge numbers	14218, 14219, 14220, 14222, 14260, 14264	44190, 44191

Table 1: Summary of MAST and LHD properties

tained by summing over a window of length τ [8], $\delta I_{sat}(t,\tau) = \sum_{t'=t}^{t+\tau-\tau_{min}}(I_{sat}(t') - <I_{sat}>_t)/\sigma$, where $<I_{sat}>_t$ and σ are the mean and standard deviation of the I_{sat} signal calculated over all times. We examine the scaling properties of the absolute moments of these fluctuations, $S_m(\tau) \equiv \langle |\delta I_{sat}(t,\tau)|^m \rangle \propto \tau^{\zeta(m)}$ and obtain scaling exponents $\zeta(m)$. For all the data sets studied, we find two regions of scaling with the break between the scaling regimes at $30-50\mu s$. For example, figure 2 shows absolute moments of order 1 to 4 for MAST plasma 14218 and LHD plasma 44190 tip 16. In MAST, the scaling break time scale is comparable to the life time of filaments [9]. For each scaling region, we apply a linear fit $\zeta(m) = \alpha m$ with $\zeta(0) = 0$ to extract a single scaling exponent α. Figure 3 shows scaling exponents $\zeta(m)$ and the best linear fit for MAST and LHD plasma 44190 tip 16. In region I, we find the mean value of the scaling exponent is $\alpha_I = 0.94 \pm 0.07$ for MAST and $\alpha_I = 0.85 \pm 0.05$ for LHD. In region II, mean values are $\alpha_{II} = 0.56 \pm 0.08$ for MAST and $\alpha_{II} = 0.58 \pm 0.01$ for LHD. This suggests coherent behaviour in the short time scale region.

We now examine the origin of the observed scaling behaviour by letting $y_{ij} = \delta I_{sat}(t,\tau)$ and $x_j = (I_{sat}(t') - <I_{sat}>_t)/\sigma$. Then, $y_{ij} = \sum_{k=j}^{i+j-1} x_k$ and the second order moment can be written as,

$$S_2 = \langle \left(\sum_{k=j}^{i+j-1} x_k \right)^2 \rangle_{N-i+1} = \langle \sum_{k=j}^{i+j-1} x_k^2 \rangle_{N-i+1} + 2\langle \sum_{k>l}^{i+j-1} x_k x_l \rangle_{N-i+1}. \qquad (1)$$

The second term on the right hand side is related to correlations. We denote the two terms as $\sigma^2(\tau)$ and $\gamma(\tau)$ respectively, $S_2(\tau) = \sigma^2(\tau) + \gamma(\tau)$. In figure 4 we plot $S_2(\tau)$ and its components
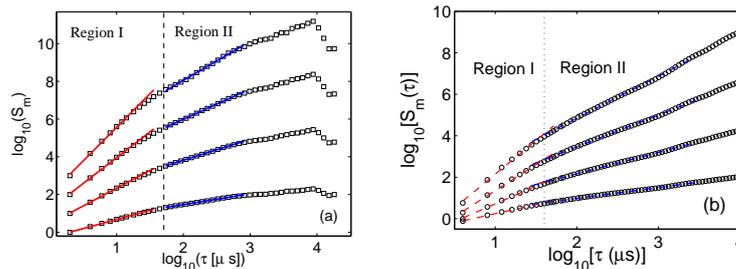
Figure 2: Absolute moments $1 \leq m \leq 4$ for (a) MAST plasma 14218 (b) LHD plasma 44190 tip 16.
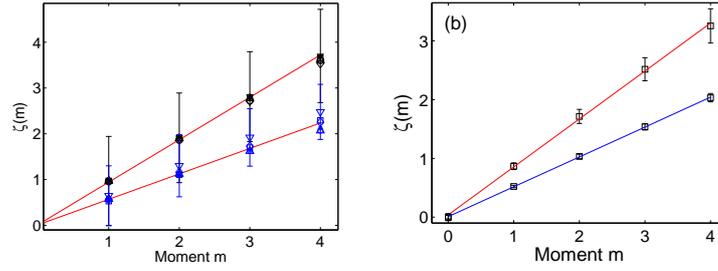
Figure 3: Scaling exponents $\zeta(m)$ for (a) MAST plasmas: 14218, 14219 and 14220 (b) LHD plasma 44190 tip 16.

$\sigma^2(\tau)$ and $\gamma(\tau)$ for LHD plasma 44190 tip 16 as an example. We see that short time scale correlations modify the scaling in region I while the underlying scaling is recovered in region II. This suggests that the observed dual scaling is due to short time scale correlations of coherent density blobs / filaments in the time series.
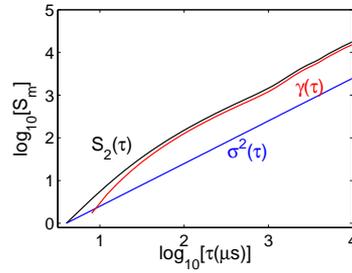

Figure 4: $S_2(\tau)$, $\sigma^2(\tau)$ and $\gamma(\tau)$ for LHD plasma 44190 tip 16.

Probability density function. We now consider continuous models for the $\delta I_{\mathrm{sat}}(t,\tau)$ distributions on different temporal scales. We consider the Fréchet $P_F(x,k) = C_F \exp(-x^{-k})/(x^{1+k})$ and Gumbel $P_G(x,a) = C_G \exp[-a(x+e^{-x})]$ distributions which are derived from extreme value statistics [10]; k and a are fitting parameters. Figures 5 and 6 show the PDFs $P(\delta I_{\mathrm{sat}},\tau)$, normalised to the standard deviation, for $\tau = \tau_{\min}$ and $\tau = 64\mu$s for MAST and LHD respectively. The PDF for $\tau = \tau_{\min}$ is in scaling region I; $\tau = 64\mu$s is in scaling region II. For all the MAST plasmas considered, the δI_{sat} distribution on timescale $\tau = \tau_{\min}$ is well fitted by a Fréchet with index $k \approx 1.25$. For $\tau = 64\mu$s the Gumbel distribution with $a = 1.4$ gives a satisfactory description of the entire PDF. For LHD, this fitting works well only for tip 16 in region I; we find the best fits to be $k = 1.025$ and $a = 1.3$. However, neither distribution captures the full discrete
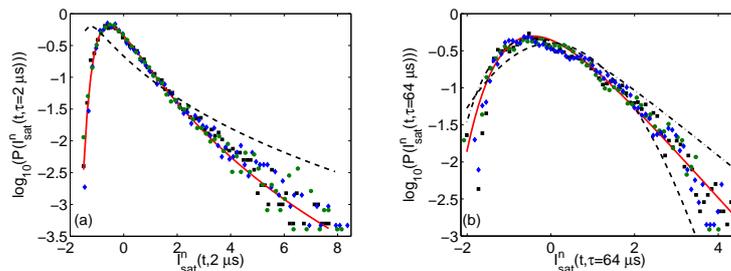

Figure 5: PDFs of $\delta I_{\mathrm{sat}}(t,\tau)$ for MAST plasmas; 14218, 14219 and 14220 on temporal scale (a) $\tau = \tau_{min}$ (b) $\tau = 64\mu$s. Solid lines represent (a) Fréchet fit (b) Gumbel fit. For comparison, dashed lines show: (a) log-normal distribution (b) normal distribution.

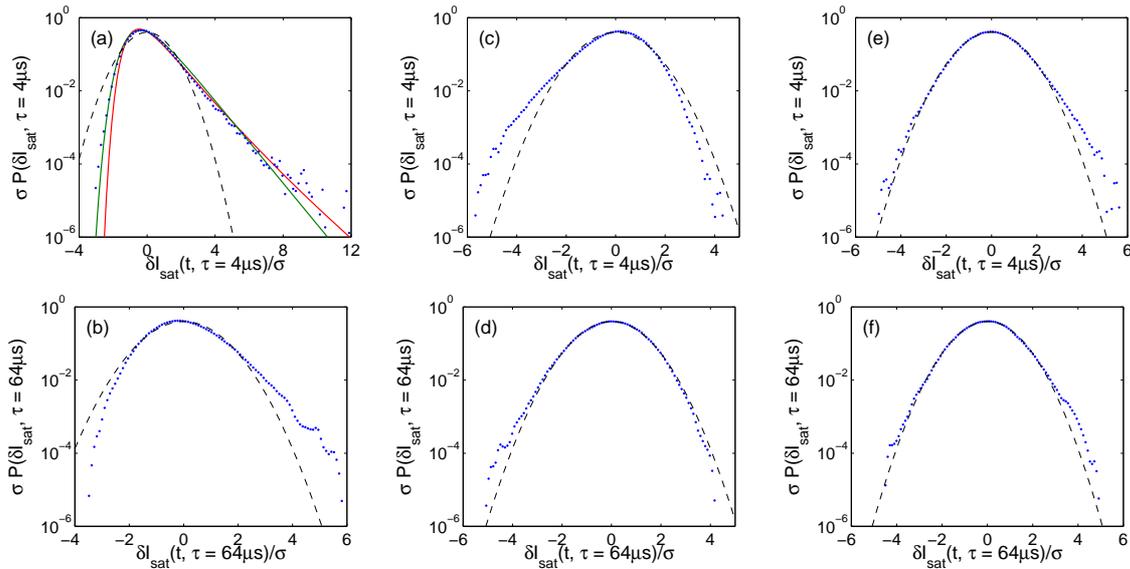Figure 6: PDFs of $\delta I_{sat}(t,\tau)$ for LHD plasma 44190 on temporal scale $\tau = \tau_{min}$ (top row) and $\tau = 64\mu s$ (bottom row): (a) & (b) tip 16, (c) & (d) tip 17, (e) & (f) tip 18. In (a), solid lines represent Fréchet (red) and Gumbel (green) fits. For comparison, dashed lines show normal distributions.

PDF: the positive tail is better represented by Fréchet while negative values are closer to the Gumbel distribution.

Conclusions We have compared statistical properties of edge plasma turbulence as measured by Langmuir probes in MAST and LHD. Analysis of absolute moments reveals two region of scaling separated at $30 - 50\mu s$. We suggest that this transition is due to the dominance of coherent blob structures in the short time scale region. On time scales greater than $50\mu s$, similar values of scaling exponents α_{II} suggest universality of the edge plasma turbulence.

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