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Full-wave simulations of lower hybrid wave propagation in toroidal plasma with nonthermal electron distributions*

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The computational challenge in simulating heating and current drive in the lower hybrid frequency range is formidable, because the perpendicular wavelength is very much shorter than the plasma size ($k_\perp R \sim 10^3$ in current devices, approaching 10^4 in ITER). Furthermore, when driven current and plasma heating are significant, wave-induced electron velocity space diffusion considerably alters the shape of $f_e(\psi,\mu,\varepsilon)$ from a Maxwellian. Results obtained previously in the geometric optics limit with the GENRAY code [1], combined with a self-consistent solution for f from the CQL3D Fokker Planck code [2] have provided considerable insight. However, in order to assess the importance of coherent effects such as diffraction, caustics, focii, etc, a more general description is required [3].

The full-wave code, TORICLH, solves the linearized Maxwell-Vlasov equations to compute the vector wave field $\mathbf{E} = \mathbf{E}(\mathbf{r}_\perp)\exp i(n\phi - \omega t)$ in an axisymmetric ($\partial/\partial\phi = 0$, $\mathbf{r}_\perp \cdot \hat{\phi} = 0$) toroidal plasma. Specifically, TORICLH uses a Fourier basis set in poloidal angle (size M) and a finite element representation in ψ (size N_ψ). The wave equation is recast as a block tri-diagonal matrix system [4] with block size $(4M)^2$ [5]. Development effort is underway to enable computation of fields and distribution functions, self-consisently evolved under the action of collisions and quasilinear diffusion (with bounce-averaged diffusion tensor $\langle \mathbf{D}_{\rm ql}(\psi,\mu,\varepsilon)\rangle$ [6, 7]), as has been achieved for full-wave simulations of IC heating and current drive [8]. Since $\partial f/\partial\phi = 0$, the solution for each toroidal component of the wave field proceeds independently, given the antenna spectral strength, and the results are summed to construct $\langle \mathbf{D}_{\rm ql}\rangle$. The iteration loop is thus: $f_e \rightarrow$ TORICLH $\rightarrow \langle \mathbf{D}_{\rm ql}\rangle \rightarrow$ CQL3D $\rightarrow f_e$. The original formulation of the required linearized hot-plasma dielectric response was for Maxwellian distribution functions, with ψ dependent density and temperature. Previous extensions allowed specification of non-Maxwellian, distribution functions of general functional form [9].

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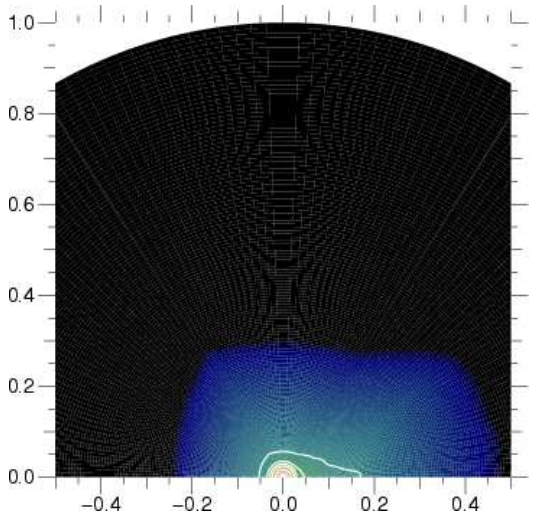
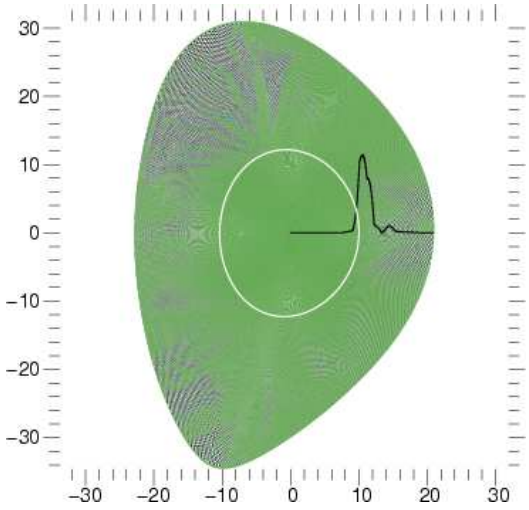


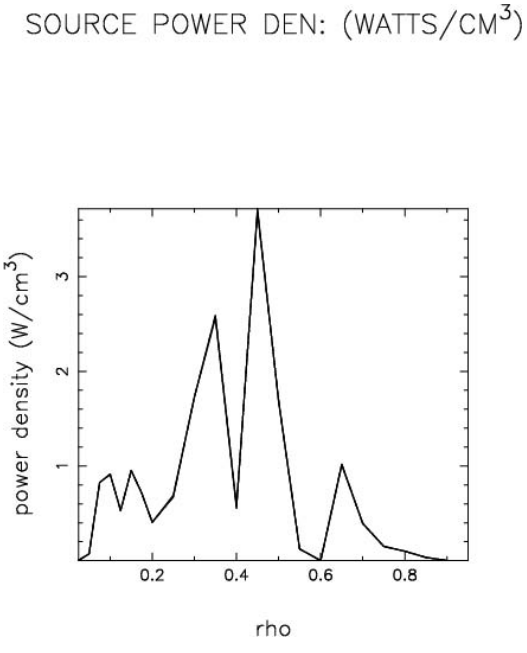
(1.a) $f_e(\mathbf{u})|_{r=10.\,\mathrm{cm}}$



(1.b) $E_{\|}(R,Z)$

The first step in this iteration loop has been completed by solving for the wave fields using numerical $f_{ql}(u,\theta,\psi)$ computed by CQL3D, given as a function of momentum per mass u, angle θ between $\mathbf{u}$ and $\mathbf{B}$, at the point of minimum $|\mathbf{B}|$ on each flux surface ψ. This is mapped onto $f(v_{\|},v_{\perp},\vartheta,\psi)$, a function of velocity parallel and perpendicular to $\mathbf{B}$, poloidal angle ϑ, and ψ, as is required by TORICLH. The code was tested by importing f_e from a CQL3D/GENRAY simulation of CMOD shot 1060728 at 1100 ms. The resolution needed to obtain convergence for the fields is $N_\vartheta = 1024$, $(M = 511)$, $N_\psi = 480$. The numerical equilbrium geometry was constructed using data from magnetic diagnostic measurements. Central density and temperature are $7\cdot 10^{13}\,\mathrm{cm}^{-1}$ and $2.34\,\mathrm{keV}$, and the toroidal field $B|_{r=0} = 5.4\,\mathrm{T}$.

The launched $n_{\|} = 2.03$. Figure 1.a is a colorcell/contour plot of $f_e(\mathbf{u},\psi_e)$, evaluated where the minor radius $r(\psi_e) = 10\,\mathrm{cm}$ at the outside midplane. Each of the 5 level curves represent a successive 10-fold decrease in f. The horizontal (vertical) axis is $u_{\|}$ ($u_{\perp}$), with energy $E_{u=1} = 2.5\,\mathrm{MeV}$. A colorcell plot of $E_{\|}(R,Z)$ is shown in Fig. (1.b) along with the outboard radial dependence of the electron Landau damping (principal absorption mechanism) shown in black. The white contour shows the flux surface for which f_e is plotted. The power deposition profile illustrates the



(2.) Ray tracing power deposition

exclusion of wave energy from regions much below $r = 10\,\mathrm{cm}$, in contrast to the ray results, Figure 2, in which power penetrates and is absorbed up to much smaller radii. The reason for this difference is under investigation [10].

The solution of the full-wave equations for lower hybrid wave propagation in laboratory-, and ultimately reactor-, sized toroidal plasma is computationally demanding. Parallelization [11] has shown excellent scaling with processor count on the midsized MIT/PSFC LOKI cluster (256 cores). However, given the block-structure, the operation count for the direct solution scales with poloidal resolution as M^3, making increased algorithmic efficiency extremely desirable in order to achieve ITER-scale simulations. The fact that the equilibrium poloidal variation is smooth (dominantly low poloidal modes with appreciable amplitude within a spectral width $\Delta M << M$) suggests that the effectively banded structure can be used as the basis for an iterative solver, for which the operation count per iteration scales as $\Delta M M^2$. Initial investigations of such an iterative solver have shown promise: The block structure

$$-B_k u_{k+1} + A_k u_k - C_k u_{k-1} = r_k\,,$$

where $\psi = \psi_k$, and A_k, B_k, C_k are size $4M \times 4M$ is solved by Gaussian elimination with the substitution

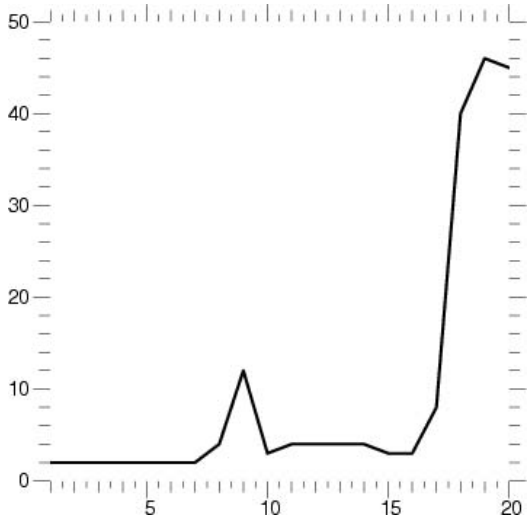
$$u_k = G_k u_{k+1} - y_k\,.$$

This requires the solution of

$$\begin{aligned} D_k G_k &= B_k\,, \\ D_k y_k &= r_k - C_k y_{k-1}\,, \end{aligned}$$

with $D_k = A_k - C_k G_{k-1}$. The banded structure is exploited by decomposing $D_k = D_{k,b} + \delta D_k$ with $D_{k,b}(i,j) = D_k(i,j)$ if $|i-j| < \Delta M_k$, and zero otherwise. The bandwidth is computed locally, $\Delta M_k = \mathrm{const}\, S_k^{1/2}$, with

$$S_k = \frac{\sum_{i,j}(i-j)^2 |D_k(i,j)|}{\sum_{i,j}|D_k(i,j)|}\,.$$



(3.) RMS bandwidth vs radial index

As an example, $S_k^{1/2}$ vs k is shown for a case where $4M = 124$ in Figure 3. The iterative solution for G_k proceeds as

$$\begin{aligned} D_{k,b} G_k^{(1)} &= B_k\,, \\ D_{k,b} G_k^{(n)} &= B_k - \delta D_k G_k^{(n-1)}\,, \end{aligned}$$

and correspondingly for y_k. Convergence is rapid. After $n = 3$ iterations, the relative rms difference between the iterative and direct solutions is $5 \cdot 10^{-4}$. Application of similar techniques throughout the computation is being

pursued in order to achieve significant overall speedup.

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[5] Nominally the block size would be $(6M)^2$, since each of the three components of **E** and their first ψ derivatives are solved for. For current drive applications, with frequencies $\omega > \omega_{\rm lh}$, the lower hybrid frequency, the ion plasma mode does not propagate, allowing for the solution of E_ψ and $\partial E_\psi/\partial\psi$ in terms of the other components.

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